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Load frequency control in multi area electric power system using genetic scaled fuzzy logic

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In multi area electric power systems, if a large load is suddenly connected (or disconnected) to the system, or if a generating unit is suddenly disconnected by the protection equipment, there will be a long-term distortion in the power balance between that delivered by the turbines and that consumed by the loads. This imbalance is initially covered from the kinetic energy of rotating rotors of turbines, generators and motors and, as a result, the frequency in the system will change. Therefore The Load Frequency Control (LFC) problem is one of the most important subjects in the electric power system operation and control. In practical systems, the conventional PI type controllers are applied for Load Frequency Control. In order to overcome the drawbacks of the conventional PI controllers, numerous techniques have been proposed in literatures. In this paper, a new Fuzzy type controller is considered for Load Frequency Control problem. In this new Fuzzy technique, the upper and lower bounds of the Fuzzy membership functions are obtained using genetic algorithms optimization method and so this Fuzzy method is called "scaled-Fuzzy". A multi area electric power system with a wide range of parametric uncertainties is given, to illustrate proposed method. To show effectiveness of the proposed method, a classical PI type controller optimized by genetic algorithms (GA) was designed in order to make comparison with the proposed scaled Fuzzy method. The simulation results visibly show the validity of scaled Fuzzy method, in comparison with the traditional PI type method.

Key words: Multi area electric power system, Load Frequency Control, scaled fuzzy logic, genetic algorithms.

INTRODUCTION

For large scale electric power systems with interconnected areas, Load Frequency Control (LFC) is important to keep the system frequency and the interarea tie power as near to the scheduled values as possible. The input mechanical power to the generators is used to control the frequency of output electrical power and to maintain the power exchange between the areas as scheduled. A well designed and operated electric power system must cope with changes in the load and with system disturbances, and it should provide acceptable high level of quality while power maintaining

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both voltage and frequency within tolerable limits. Many control strategies for Load Frequency Control in electric power systems have been proposed by researchers over the past decades.

This extensive research is due to fact that, Load Frequency Control constitutes an important function of electric power system operation where the main objective is to regulate the output power of each generator at prescribed levels while keeping the frequency fluctuations within pre-specifies limits. Robust adaptive control schemes have been developed by Lim et al. (1996), Wang et al. (1998) and Stankovic et al. (1998) to deal with changes in system parametrics under Load Frequency Control strategies. A different algorithm has been presented by Taher et al. (2008) to improve the performance of multi area electric power systems. 378 Int. J. Phys. Sci.



(2003) and Liu et al. (2003). This paper deals with a design method for Load Frequency Control in a multi area electric power system, using a new scaled Fuzzy type controller whose membership functions boundaries are tuned by genetic algorithms optimization method. In order to show effectiveness of the new scaled Fuzzy Load Frequency Control, this method is compared with the conventional PI type controller for Load Frequency Control. Simulation results show that the proposed method guarantees robust performance under a wide range of operating conditions and system uncertainties.

Plant model

A four-area electric power system is considered as a test system and shown in Figure 1. The block diagram for each area of interconnected areas is shown in Figure 2 (Wood et al., 2003). The parameters in Figure 2 are defined as follows:

 Δ : Deviation from nominal value M_i=2H: Constant of inertia of ith area D_i: Damping constant of ith area R_i: Gain of speed droop feedback loop of ith area T_{ti}: Turbine Time constant of ith area T_{Gi}: Governor Time constant of ith area G_i: Controller of ith area ΔP_{Di} : Load change of ith area u_i: Reference load of ith area $B_i=(1/R_i)+D_i$: Frequency bias factor of ith area ΔP_{tie} ij: Inter area tie power interchange from ith area to ith area. Where: i=1. 2. 3. 4 j=1, 2, 3, 4 and i≠i The inter-area tie power interchange is as (1) (Wood et al., 2003).

$$\Delta \mathsf{P}_{\mathsf{tie}} \mathsf{ij} = (\Delta \omega_{\mathsf{i}} - \Delta \omega_{\mathsf{j}}) \times (\mathsf{T}_{\mathsf{ij}}/\mathsf{S}) \tag{1}$$

Where:

 $T_{ii}=377 \times (1/X_{tie}ij)$ (for a 60 Hz system)

X_{tie}ij: impedance of transmission line between i and j areas.

The ΔP_{tie} ij block diagram is shown as Figure 3. Figure 2 shows the block diagram of ith area and Figure 3 shows the method of interconnection between ith and jth areas.



Figure 1. Four-area electric power system with interconnections.



Figure 2. Block diagram for one area of system (ith area).

Viewing a multi area electric power system under Load Frequency Control as a decentralized control design for a multi-input multi-output system, it has been shown by Yamashita et al. (1991) that, a group of local controllers with tuning parameters can guarantee the overall system stability and performance.

The reported results demonstrate clearly the importance of robustness and stability issue in Load Frequency Control design. In addition, several practical and theoretical issues have been addressed by Xiaofeng et al. (2004), Doolla et al. (2006), Grigor'ev et al. (2005) and Gvozdev et al. (2005) which include recent technology, utilized by vertically integrated utilities, augmentation of filtered area control error with Load Frequency Control schemes and hybrid Load Frequency Control, that encompasses an independent system operator and bilateral Load Frequency Control. The applications of artificial neural network, genetic algorithms and optimal control to Load Frequency Control have been reported by Hematti et al. (2008), Rerkpreedapong et al.

The state space model of four-area interconnected power system is as (2) (Wood et al., 2003).

 $\begin{cases} \bullet \\ X = AX + BU \\ Y = CX \end{cases}$ (2)

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Figure 3. Block diagram of inter area tie power ($\Delta P_{tie}ij$).



Figure 4. Scaled Fuzzy controller.

Where:

The matrixes A and B in (2) and the typical values of system parameters for the nominal operating condition are given in appendix.

DESIGN METHODOLOGY

As mentioned before, in this paper a new scaled Fuzzy controller is considered for Load Frequency Control problem. Fuzzy method has three major headings as membership functions, rule bases and defuzzification. In classical Fuzzy methods, the boundaries of the membership functions are adjusted based on expert person's experiences that may be with trial and error and does not guarantee performance of the system. To solve this problem, in this paper the boundaries of the membership functions are tuned by an optimal search for achieving the best boundaries. Therefore, the boundaries of input and output membership functions are considered as uncertain and then the optimal boundaries are obtained by genetic algorithms (Cordon et al., 2001). Here the proposed Fuzzy controller block

diagram is given in Figure 4. In fact, it is a nonlinear PI-type Fuzzy logic controller with two inputs and one output. The inputs are filtered by washout block to eliminate the DC components.

Also, there are three parameters denoted by K_{in1} , K_{in2} and K_{out} which are defined over an uncertain range and then obtained by genetic algorithms optimization method. Therefore, the boundaries of 380 Int. J. Phys. Sci.

inputs and output signals are tuned on an optimal value. Though the Fuzzy controller accepts these inputs, it has to convert them into fuzzified inputs before the rules can be evaluated. To accomplish this, one of the most important and critical blocks in the whole Fuzzy controllers should be built and it is the 'Knowledge Base'. It consists of two more blocks namely the 'Data Base' and

Table 1. The linguistic variables for inputs and output.

Big Positive (BP)	Medium Positive (MP)	Small Positive (SP)
Big Negative (BN)	Medium Negative (MN)	Small Negative (SN)
Zero (ZE)		



Figure 5. Membership function of inputs and output.

Δω	BN	MN	SN	7F	SP	MP	BP
d(Δω)/dt			0.1		0.		
BN	BN	BN	BN	BN	MN	SN	ZE
MN	BN	MN	MN	MN	SN	ZE	SP
SN	BN	MN	SN	SN	ZE	SP	SP
ZE	MN	MN	SN	ZE	SP	MP	MP
SP	SN	SN	ZE	SP	SP	MP	BP
MP	SN	ZE	SP	MP	MP	MP	BP
BP	ZE	SP	MP	BP	BP	BP	BP

the 'Rule Base' (Rajase et al., 2007).

Data base consists of the membership function for input and output variables described by linguistic variables shown in Table 1 (Rajase et al., 2007). The "triangular membership functions" are used as membership functions for the input and output variables. The Figure 5 illustrates this in detail, indicating the range of the variable. This range

Data base

is defined as default and then tuned via cascade K parameters (K_{in1}, K_{in2} and K_{out}) and adjusted on the optimal value. **Rule base**

The other half of the knowledge base is the Rule Base, which consists of all the rules formulated by the experts. The Fuzzy rules which are used in this scheme are listed in Table 2.

Methodologies adopted in fuzzy inference engine

Many methodologies have been mentioned in evaluating the various expressions like Fuzzy union (OR operation) and Fuzzy intersection (AND operation) with varying degree of complexity. Here in Fuzzy Boroujeni et al. 381

Table 3. Obtained values K_{in1}, K_{in2} and K_{out} for Fuzzy controllers.

	K _{in1}	K _{in2}	Kout
First area Fuzzy parameters	1.4490	0.7482	1.3677
Second area Fuzzy parameters	1.0136	0.5715	0.8600
Third area Fuzzy parameters	0.5251	1.2349	1.1181
Fourth area Fuzzy parameters	1.1886	1.1297	1.1172

scheme, the most widely used methods for evaluating such expressions are used. The function used for evaluating OR is "MAX", which is the maximum of the two operands and similarly the AND is evaluated using "MIN" function which is defined as the minimum of the two operands. It should be noted that in the present research paper, the equal importance is assigned to all the rules (Rajase et al., 2007).

Defuzzification method

The defuzzification method followed in this study is the "Center of Area Method" or "Gravity method". This method is discussed in (Rajase et al., 2007). As mentioned before, in this paper the boundaries of the membership functions are adjusted by genetic algorithms. Subsequently, a brief introduction about genetic algorithms is presented.

Genetic algorithms

Genetic algorithms are global search techniques, based on the operations observed in natural selection and genetics (Randy and Sue, 2004). They operate on a population of current approximations (the individuals) initially drawn at random, from which improvement is sought. Individuals are encoded as strings (chromosomes) constructed over some particular alphabet, e.g., the binary alphabet {0.1}, so that chromosomes values are uniquely mapped onto the decision variable domain. Once the decision variable domain representation of the current population is calculated, individual performance is assumed according to the objective function which characterizes the problem to be solved. It is also possible to use the variable parameters directly to represent the chromosomes in the GA solution. At the reproduction stage, a fitness value is derived from the raw individual performance measure given by the objective function and used to bias the selection process. Highly fit individuals will have increasing opportunities to pass on genetically important material to successive generations. In this way, the genetic algorithms search from many points in the search space at once and yet continually narrow the focus of the search to the areas of the observed best performance. The selected individuals are then modified through the application of genetic operators. In order to obtain the next generation genetic operators manipulate the characters (genes) that constitute the chromosomes directly, following the assumption that certain genes code, on average, for fitter individuals than other genes. Genetic operators can be divided into three main categories (Randy and Sue, 2004):

1. Reproduction: Selects the fittest individuals in the current population to be used in generating the next population;

2. Cross-over: Causes pairs, or larger groups of individuals to exchange genetic information with one another;

3. Mutation: Causes individual genetic representations to be changed according to some probabilistic rule.

FUZZY CONTROLLER TUNING USING GENETIC ALGORITHMS

Here, the membership functions of the proposed scaled Fuzzy controller are tuned by K parameters (K_{in1} , K_{in2} and K_{out}). These K parameters are obtained based on genetic algorithms optimization method. Next, the system controllers are shown in Figure 2 as G_i. Here, these controllers are substituted by scaled Fuzzy controllers shown in Figure 4 and the optimum values of K_{in1} , K_{in2} and K_{out} in scaled Fuzzy controllers are accurately computed using genetic algorithms. In genetic algorithms optimization method, the first step is to define a performance index for optimal search. In this study, the performance index is considered as (3). In fact, the performance index is the Integral of the 'Time multiplied Absolute value of the Error (*ITAE*)'.

$$ITAE = \int_{0}^{t} t \left| \Delta \omega_{1} \right| dt + \int_{0}^{t} t \left| \Delta \omega_{2} \right| dt + \int_{0}^{t} t \left| \Delta \omega_{3} \right| dt + \int_{0}^{t} t \left| \Delta \omega_{4} \right| dt$$
(3)

The parameter "t" in ITAE is the simulation time. A 100 s time period is considered for simulation. It is clear to understand that the controller with lower ITAE is better than the other controllers. To compute the optimum parameter values, a 10% step change in ΔP_{D1} is assumed and the performance index is minimized using genetic algorithms. The following genetic algorithm parameters have been used in the present research:

- 1. Number of chromosomes: 12; Population size: 48
- 2. Crossover rate: 0.5; Mutation rate: 0.08

The optimum values of the parameters K_{in1} , K_{in2} and K_{out} are obtained using genetic algorithms and summarized in Table 3. The boundaries of k parameters for optimal search are as follows:

$$0.1 \le K_{in1} \le 5$$
 $0.1 \le K_{in2} \le 5$ $0.1 \le K_{out} \le 10$

In the controller design for multi-area electric power systems, some areas have more importance than the others for tie-power and also frequency control; but in this paper the importance of areas is considered as equal.

RESULTS AND DISCUSSION

Here, the proposed scaled Fuzzy controller is applied to the system for Load Frequency Control. In order to make comparison and show the effectiveness of the proposed method, a classical PI type controller optimized by genetic algorithms was designed for Load Frequency Control. The structure of PI type controller is shown in Figure 6. The optimum value of the parameters K_P and K_I for PI controllers optimized using genetic algorithms have

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Figure 6. The structure of PI type controller.

Table 4. Optimum values of KP and KI for PI controllers.

	Κ _P	Kı
First area controller (G1)	1.0624	4.6021
Second area controller (G ₂)	3.1791	4.2829
Third area controller (G ₃)	2.4916	2.6287
Fourth area controller (G ₄)	1.8912	5.8094

been obtained and summarized in Table 4 (Randy and Sue, 2004). In order to study and make analysis of system performance under system uncertainties (controller robustness), three operating conditions are considered as follows:

i. Nominal operating condition;

ii. Heavy operating condition (20% changing parameters from their typical values);

iii. Very heavy operating condition (50% changing parameters from their typical values).

In order to demonstrate the robustness performance of the proposed method, the *ITAE* is calculated following step change in the different demands (ΔP_D) at all operating conditions (nominal, heavy and very heavy) and results are shown in Tables 5 and 6. Following step change, the optimal scaled Fuzzy controller has better performance than the optimized PI controller at all operating conditions. Fuzzy logic method has a nonlinear characteristic and therefore, with changing system parameters and system operating conditions, the fuzzy rule bases and the controller change with system conditions. In fuzzy method, instead of a fix performance, an intelligent controller with dynamic performance is applied to control system and therefore, the system with fuzzy controller has a softer and better performance than the other method. Although the performance index results are enough to compare the methods, it can be more useful to show responses in figures. Figure 7 shows $\Delta \omega_1$ at nominal, heavy and very heavy operating conditions, following 10% step change in the demand of first area (ΔP_{D1}). It is clear to see that, the scaled Fuzzy has better performance than the other method at all operating conditions.



Figure 7. Dynamic response $\Delta \omega 1$ following step change in the demand of first area ($\Delta PD1$). (a) Nominal operating

Conclusions

In this paper, a new scaled Fuzzy approach for Load Frequency Control has been successfully proposed. The proposed method was applied to a typical four-area electric power system containing system parametric uncertainties and various load conditions. Simulation

results demonstrated that the designed controllers capable to guarantee the robust stability and robust performance, such as precise reference frequency tracking and disturbance attenuation, under a wide range of uncertainties and load conditions. Also, the simulation results showed that the scaled Fuzzy approach is robust to change the system parameters and it has better performance than the conventional PI type controller at all operating conditions.

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APPENDIX

The typical values of system parameters for the nominal operating condition are as follows:

1st area parameters

T _{T1} =0.03	T _{G1} =0.08	M ₁ =0.1667	R ₁ =2.4
D ₁ =0.0083	B ₁ =0.401	T ₁₂ =0.425	T ₁₃ =0.500
$T_{14} = 0.400$	T ₂₃ = 0.455	T ₂₄ = 0.523	T ₃₄ =0.600

2nd area parameters

$T_{T2}=0.025$	T _{G2} =0.091	M ₂ =0.1552	R ₂ =2.1
D ₂ =0.009	B ₂ =0.300	T ₁₂ =0.425	T ₁₃ =0.500
T ₁₄ = 0.400	T ₂₃ = 0.455	T ₂₄ = 0.523	T ₃₄ =0.600

3rd area parameters

T _{T3} =0.044	T _{G3} =0.072	M ₃ =0.178	R ₃ =2.9
D ₃ =0.0074	B ₃ =0.480	T ₁₂ =0.425	T ₁₃ =0.500
T ₁₄ = 0.400	T ₂₃ = 0.455	T ₂₄ = 0.523	T ₃₄ =0.600

4th area parameters

$T_{T4}=0.033$	T _{G4} =0.085	M ₄ =0.1500	R ₄ =1.995
D ₄ =0.0094	B ₄ =0.3908	T ₁₂ =0.425	T ₁₃ =0.500
$T_{14} = 0.400$	T ₂₃ = 0.455	$T_{24} = 0.523$	$T_{34}=0.600$
. 14 0 00	.23 000	.24 0.010	. 34 0.000

Also the matrixes A and B in (2) are as follows:

0	0	$\frac{1}{M_1}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	$\frac{1}{M_2}$	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	$\frac{1}{M_3}$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	$\frac{1}{M_4}$	0	0	0	0	0	0
$\frac{1}{T_{G1}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	$\frac{1}{T_{G2}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	$\frac{1}{T_{G3}}$	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	$\frac{1}{T_{G4}}$	0	0	0	0	0	0	0	0
	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{T_{G1}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{T_{G1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \frac{1}{M_1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{T_{G1}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 &$	$\begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 &$	$ \begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0$	$ \begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{M_2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0$	$ \begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{M_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 &$	$ \left[\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0$	$ \begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0$	$ \begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 &$	$ \begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & $	$ \begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & $	$ \begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & $	$ \begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & $	$ \begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & $	$ \begin{bmatrix} 0 & 0 & \frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & $

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	$\left[\frac{-1}{T_{C1}}\right]$	0	$\frac{-1}{R_1T_{G1}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	$\frac{1}{T_{TT}}$	$\frac{-1}{T_{TT}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	$\frac{1}{M_1}$	$\frac{-D_1}{M_1}$	0	0	0	0	0	0	0	0	0	$\frac{-1}{M_1}$	$\frac{-1}{M_1}$	$\frac{-1}{M_1}$	0	0	0	
	0	0	0	$\frac{-1}{T_{G2}}$	0	$\frac{-1}{R_2T_{G2}}$	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	$\frac{1}{T_{T2}}$	$\frac{-1}{T_{T2}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	$\frac{1}{M_2}$	$\frac{-D_2}{M_2}$	0	0	0	0	0	0	$\frac{1}{M_2}$	0	0	$\frac{-1}{M_2}$	$\frac{-1}{M_2}$	0	
	0	0	0	0	0	0	$\frac{-1}{T_{G3}}$	0	$\frac{-1}{R_3T_{G3}}$	0	0	0	0	0	0	0	0	0	
A =	0	0	0	0	0	0	$\frac{1}{T_{T3}}$	$\frac{-1}{T_{T3}}$	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	$\frac{1}{M_3}$	$\frac{-D_3}{M_3}$	0	0	0	0	$\frac{1}{M_3}$	0	$\frac{1}{M_3}$	0	$\frac{-1}{M_3}$	
	0	0	0	0	0	0	0	0	0	$\frac{-1}{T_{G4}}$	0	$\frac{-1}{R_4T_{G4}}$	0	0	0	0	0	00	
	0	0	0	0	0	0	0	0	0	$\frac{1}{T_{T4}}$	$\frac{-1}{T_{T4}}$	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	$\frac{1}{M_4}$	$\frac{-D_4}{M_4}$	0	0	$\frac{1}{M_4}$	0	$\frac{1}{M_4}$	$\frac{1}{M_4}$	
	0	0	T ₁₂	0	0	$-T_{12}$	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	T ₁₃	0	0	0	0	0	- T ₁₃	0	0	0	0	0	0	0	0	0	
	0	0	T ₁₄	0	0	0	0	0	0	0	0	$-T_{14}$	0	0	0	0	0	0	
	0	0	0	0	0	T ₂₃	0	0	- T ₂₃	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	T ₂₄	0	0	0	0	0	- T ₂₄	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	T ₃₄	0	0	- T ₃₄	0	0	0	0	0	0	