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Comparison of experimental internal forces with full and partial interaction theories in steel-concrete-steel sandwich beams

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Steel-concrete-steel sandwich beam (double skin composite beam - DSC) experimental results including axial forces in steel plates and shear forces between the layers are compared with full and partial interaction theories. The flexibility of shear stud connectors on both tension and compression faces are taken into account in the partial interaction analysis including the influence of frictional forces between the concrete and external steel plates at the supports and load points. Quasi-static test results on DSC beams are compared with the theoretical solutions based on partial interaction theory assuming realistic material and shear connector properties. The comparison of results indicates that the proposed theoretical method shows good correlation with real behaviour and may be reliably used for the analysis of simply supported DSC beams.

Key words: Steel-concrete-steel sandwich beams, double skin composite construction, partial interaction, full interaction, shear connectors, frictional force, quasi-static loading.

INTRODUCTION

Double skin composite (DSC) construction consists of a layer of a plain concrete, sandwiched between two layers of relatively thin steel plate, connected to the concrete by welded stud shear connectors. This construction acts in a similar way to doubly reinforced concrete elements but the flexibility of connection between the steel plates and concrete gives rise to interface slip and additional overall element deflection. This results in a strong and efficient structure with certain potential advantages over conventional forms of construction.

Steel–concrete composite systems generally consist of steel plate, concrete and reinforcement. Shear connectors are usually utilized to develop the composite action between steel and concrete. In steel-concrete composite members, the natural bonding, friction, and mechanical interlocking actions of shear connectors have a significant influence on the degree of interaction

(Veljkovic, 1996; Oehlers et al., 2000).

It is known that the degree of interaction between steel and concrete influences the shear flow and strain distribution. Also, it has an impact on the structural performance such as strength, stiffness, and failure mode. The degree of interaction in steel–concrete composite systems can be evaluated as full-interaction, partial-interaction, and no-interaction (Veljkovic, 1996; Oehlers et al., 2000). The assumption of full-interaction may result in an overestimation of the structural performance while the assumption of no-interaction may cause an underestimation of the structural performance. Therefore, the partial-interaction assumption and analysis with a degree of interaction becomes more practical and seems to be essential for a precise prediction of behaviour. Actually, the steel–concrete composite members generally show partial-interaction due to the deformation of shear connectors and slip at the interface under the applied loads (Johnson, 1994; Dogan, 1997; Roberts and Dogan, 1998; Oehlers and Bradford, 1999; Jeong et al., 2005; Ranzi et al., 2006; Gara et al., 2006; Queiroza et al., 2007; Ranzi and Bradford, 2007; Jeong,

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2008; Ranzi, 2008; Girhammar et al., 2009; Sousa Jr. et al., 2010).

In many situations, slip and its influence on the structural behaviour of steel-concrete composite systems may be small enough to be neglected in the analysis (that is, full interaction). However, in some cases it may be feasible to use either fewer connectors than are required for complete shear connection or connectors which possess a relatively low stiffness. In such situations the influence of slip may not be negligible and result in reduced stiffness of the system (that is, partial interaction). In general, the stiffness of the connectors has a significant influence on both the slip and deformations of a composite beam. The stiffness of the shear connectors may be determined experimentally from so called push-shear tests.

An important aspect of the design of double skin composite (DSC) beams is the design of the so called shear connectors, which transfer both shear and normal forces between the concrete infill and external steel plates. The shear connection is sometimes defined as complete when the bending strength can not be increased by the provision of additional connectors. However, all connectors possess finite stiffness and therefore slip must occur between the concrete and steel plates if the shearing forces are to develop. Slip results in a strain discontinuity at the steel-concrete interface, with a corresponding reduction in flexural stiffness.

Analysis of the influence of slip in composite beams, assuming both linear and non-linear material and shear connector behaviour (Knowles, 1973; Yam, 1981; Newmark et al., 1951; Yam and Chapman, 1968; Yam and Chapman, 1972; Johnson, 1975; Johnson, 1981) has generally been based upon an approach attributed to Newmark et al. (1951). The equilibrium and compatibility equations for an element of the beam are reduced to a single second order differential equation in terms of either the resultant axial force in the concrete or the interface slip. Solutions for the axial force or interface slip are substituted back into the basic equilibrium and compatibility equations, which can then be solved to give displacements and strains throughout the beam.

Newmark et al. (1951) presented the results of tests and analysis for evaluating the load deflection behaviour of simply supported, partially interactive, composite concrete and steel T-beams. The theoretical analysis was based upon the assumption that a continuous imperfect connection existed between the two elements. A second order differential equation expressing the relationship between the longitudinal forces, transmitted through the shear connection from the concrete slab to the steel beam, and the applied bending moment, was derived and solved for the case of a beam loaded with a concentrated load.

Newmark et al.'s approach has been developed by Yam and Chapman (1968, 1971) and Yam (1981) to incorporate non-linear material and shear connector behaviour. The resulting non-linear differential equations were solved iteratively and the influence of slip on the ultimate flexural strength of composite beams was studied.

Johnson (1975, 1981) have presented modified versions of Newmark et al's theory in which the differential equations are formulated in terms of interface slip. The equations were used as the basis of an extensive theoretical study of the loss of interaction in short-span composite beams and slabs.

An alternative approach to the analysis of composite beams with partial interaction has been presented by Roberts (1985), in which the basic equilibrium and compatibility equations are formulated in terms of the displacements of the layers. The resulting differential equations are then solved simultaneously by expressing the displacement derivatives in finite difference form. The development of this approach to incorporate non-linear material and shear connector behaviour has been described by Al-Amery and Roberts (1990). The resulting non-linear differential equations are expressed in finite difference form and solved iteratively.

Experimental and theoretical studies of the behaviour of DSC beams with partial interaction have been reported by Oduyemi and Wright (1989), Wright et al. (1991a, 1991b), and Narayanan et al. (1982). Wright and Oduyemi (1991) presented closed form solutions for the partial interaction analysis of simply supported DSC beams. The analysis takes into account the flexibility of the connection on both the tension and compression faces, and incorporates the influence of concrete cracking and non-linear connector behaviour using a step-wise linearization technique. Two coupled differential equations for the axial forces in the tension and compression plates were formulated and closed form solutions found for various load combinations. The solutions were compared with the results of tests on several DSC beams and good agreement between the theory and experiment was found.

The objective of this research is to compare experimental results of double skin composite beams (DSC) reported by Dogan (1997) with full and partial interaction theories originally introduced by Wright and Oduyemi (1991) and later modified by Dogan (1997). The partial interaction analysis is extended to incorporate the influence of frictional forces between the concrete and external steel plates, at the supports and load points (Dogan, 1997; 2010). The theoretical solutions derived by Dogan (1997) are compared with the experimental results including steel plate axial forces and stud shear forces on DSC beams reported by Dogan (1997).

Governing differential equations

Full interaction

Full interaction analysis of DSC beams is based on the

following assumptions: both steel and concrete are linearly elastic materials, concrete subjected to tensile strain is cracked and ineffective in resisting load, the shear connection between the concrete and steel is sufficiently stiff to ensure that slip is negligible, and plane sections remain plane.

The assumed linear strain distribution over the depth of a DSC section subjected to bending is shown in Figure 1a, together with the associated resisting forces in the concrete and steel plates. Figure 1b shows the assumed positive conventions for displacements u and v in the x and y directions, moments M, shear forces V and curvature k.

Axial forces in steel plates for full interaction are given by

$$F_{sc} = \rho_1 M \tag{1}$$

$$F_{st} = \rho_2 M \tag{2}$$

where referring to Figure 1a, Fsc is compression force in steel compression plate, Fst is tension force in steel tension plate, ρ1 and ρ2 are parameters defined as follows:

$$\rho_1 = \frac{E_{sc} A_{sc}}{\sum EI (1 + \alpha)} \left(d_{cu} + \frac{t_{sc}}{2} \right) \tag{3}$$

$$\rho_2 = \frac{E_{st} A_{st}}{\sum EI (1 + \alpha)} \left(d_c - d_{cu} + \frac{t_{st}}{2} \right) \tag{4}$$

in which Esc is Young's modulus of steel compression plate, Asc is cross-section area of steel compression plate and equal to b*tsc, tsc is the thickness of steel plate in compression, dcu is uncracked depth of concrete section, Est is Young's modulus of steel tension plate, Ast is cross-section area of steel tension plate and equal to b*tst, tst is the thickness of steel plate in tension, dc is the depth of concrete section, EI is the rigidity of the beam and α is a parameter related with the cross section and material properties (Dogan, 1997, 2010).

The interface shear forces per unit length qsc and qst are equal to the rate of change of the axial forces in the steel plates (Figure 2a). Hence

$$q_{sc} = -\frac{dF_{sc}}{dx} \tag{5}$$

$$q_{st} = -\frac{dF_{st}}{dx} \tag{6}$$

Partial interaction

The partial interaction analysis introduced by Wright and

Oduyemi (1991) is extended to incorporate the influence of frictional forces between the concrete and external steel plates, at the supports and load points. Theories of partial interaction are based on the following simplifying assumptions: (a) both steel and concrete are linearly elastic materials, (b) deflections are small, (c) shear deformations within each material are negligible, (d) the shear connection between the concrete and steel plates is continuous along the beam that is,. the discrete stud connectors act as a continuous (smeared) connection, (e) the shear stiffness of the connection is linear, (f) the distribution of strain throughout the depth of each individual layer is linear, (g) at every section of the beam, each layer is bent to the same radius of curvature that is,. each layer deflects by the same amount and no buckling or separation of layers occurs, (h) the concrete subjected to tensile strain is cracked and ineffective in resisting load and (i) the depth of the neutral axis is constant and related to the beam geometry and material properties.

General solution of axial forces in steel plates for partial interaction are given by

$$F_{sc} = A_1 \cosh \sqrt{m_1}\, x + A_2 \sinh \sqrt{m_1}\, x + A_3 \cosh \sqrt{m_2}\, x + A_4 \sinh \sqrt{m_2}\, x + g_1 M + g_2 D^2 M \tag{7}$$

$$F_{st} = A_1 g_3 \cosh \sqrt{m_1}\, x + A_2 g_3 \sinh \sqrt{m_1}\, x + A_3 g_4 \cosh \sqrt{m_2}\, x + A_4 g_4 \sinh \sqrt{m_2}\, x + g_5 M + g_6 D^2 M \tag{8}$$

where A1 to A4 are constants determined by boundary conditions, m1, m2, and g1 to g6 are coefficients representing material and section properties of beam and stud connectors (Dogan, 1997; Roberts and Dogan, 2010).

The interface shear forces per unit length qsc and qst for partial interaction are the same as Equations 5 and 6.

Assumptions and material properties

The behaviour of DSC beams is extremely complex and therefore various assumptions are used in full and partial interaction analysis, to simplify the system as discussed before.

In this study, the distance between the symmetrical loads is reduced to zero, to obtain solutions for a simply supported beam with a point load at midspan, as shown in Figures 1 - 3. Various parameters are investigated, in particular the stiffness of the shear connection and frictional forces between the steel plates and concrete infill. For the beam shown in Figures 1 - 3 the applied bending moment diagram is symmetrical about midspan and therefore only half of the beam need be considered.

For all the beams a frictional coefficient g of about 0.25 was found to give close agreement between theoretical and experimental results. In the analysis, the influence of studs outside of the supports was represented by an axial tensile force in the tension steel plate, deduced from the experimental results at the appropriate applied load level.

The assumed beam geometry for the comparison of full and partial interaction theories are span L = 1400 mm, breadth b = 200 mm, concrete core depth dc = 150 mm, top and bottom steel plate
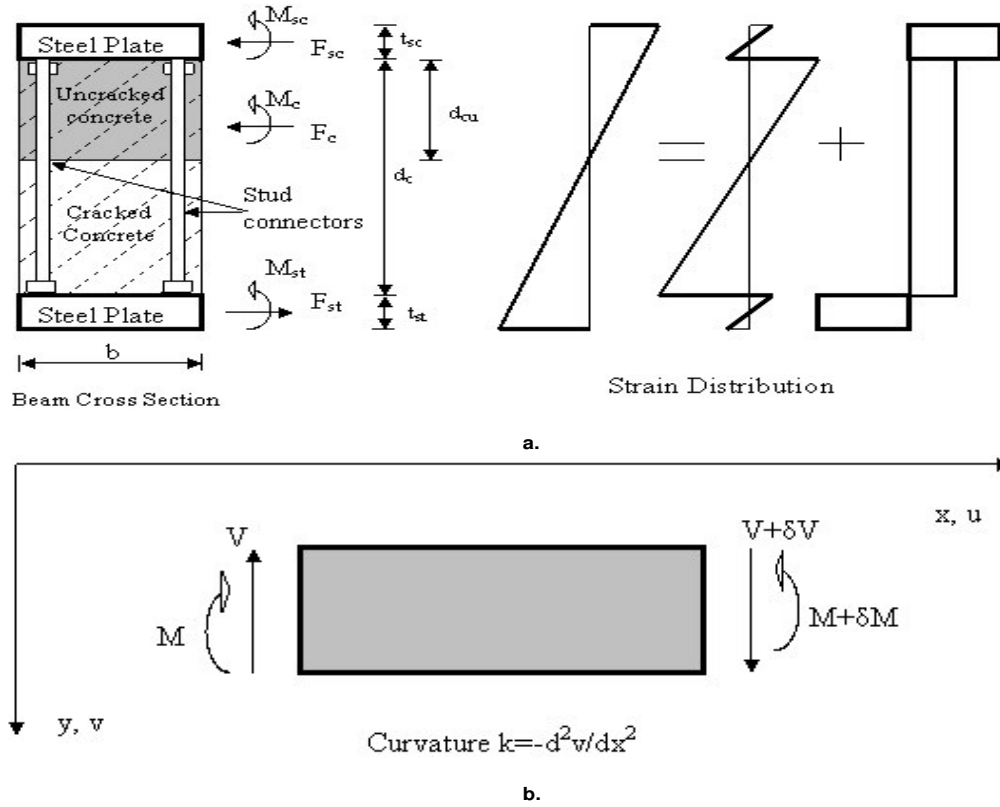
Figure 1. a. Internal forces and strain distribution over the depth of a DSC section for full interaction. b. The assumed positive sign conventions for displacements u and v in x and y directions.
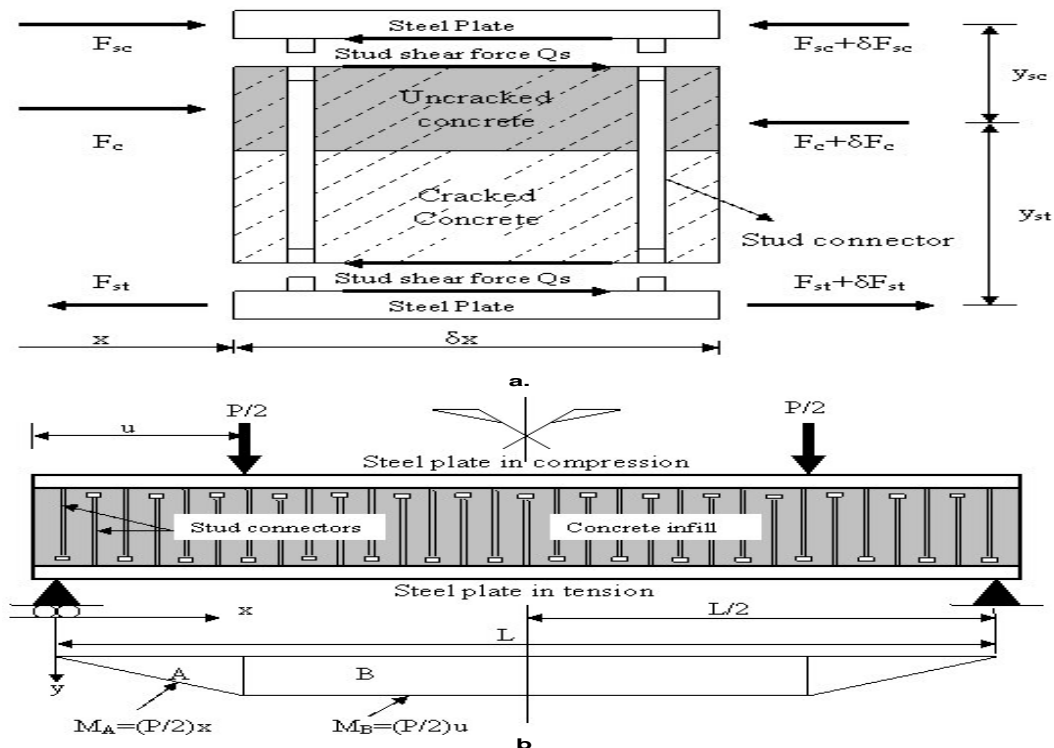

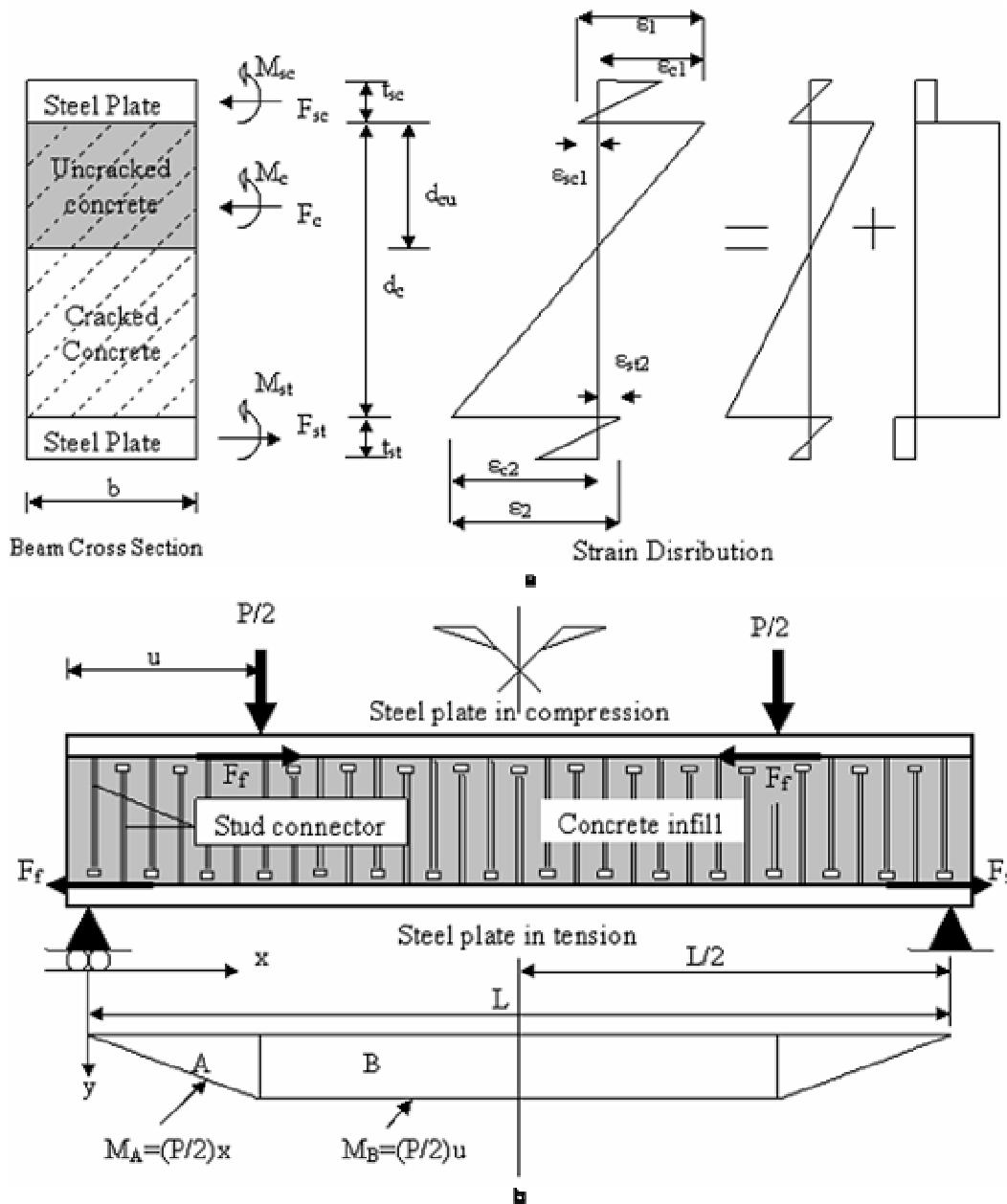**Figure 2.** a. Interface shearing forces of a DSC beam. b. Support, loading and bending moment diagram.

Figure 3 a. Internal forces and strain distribution over the depth of a DSC section for partial interaction.
b. Support, loading and frictional forces Ff at the supports and load points.

thicknesses ts = 8 mm and stud spacing on both plates st = 200 mm. The Young's modulus of the steel Es was assumed to be 210 kN/mm². Due to variation in concrete compressive strength the Young's modulus of concrete Ec, determined from the equation 67

$$E_c = 9.1(f_{cu})^{0.33}$$ (9)

where fcu is the concrete cube compressive strength in N/mm² and Ec is in kN/mm². Ec varied between 25.2 and 30.2 kN/mm². The test beams were therefore divided into four groups according to their estimated concrete Young's modulus (Group 1: B1 and B2

with Ec = 25.2 kN/mm², Group 2: B3 to B6 with Ec = 28.3 kN/mm², Group 3: B7 and B8 with Ec = 27.1 kN/mm² and Group 4: B9 and B10 with Ec = 30.2 kN/mm2).

RESULTS

General

The behaviour of DSC beams is extremely complex and therefore various assumptions are used in full and partial
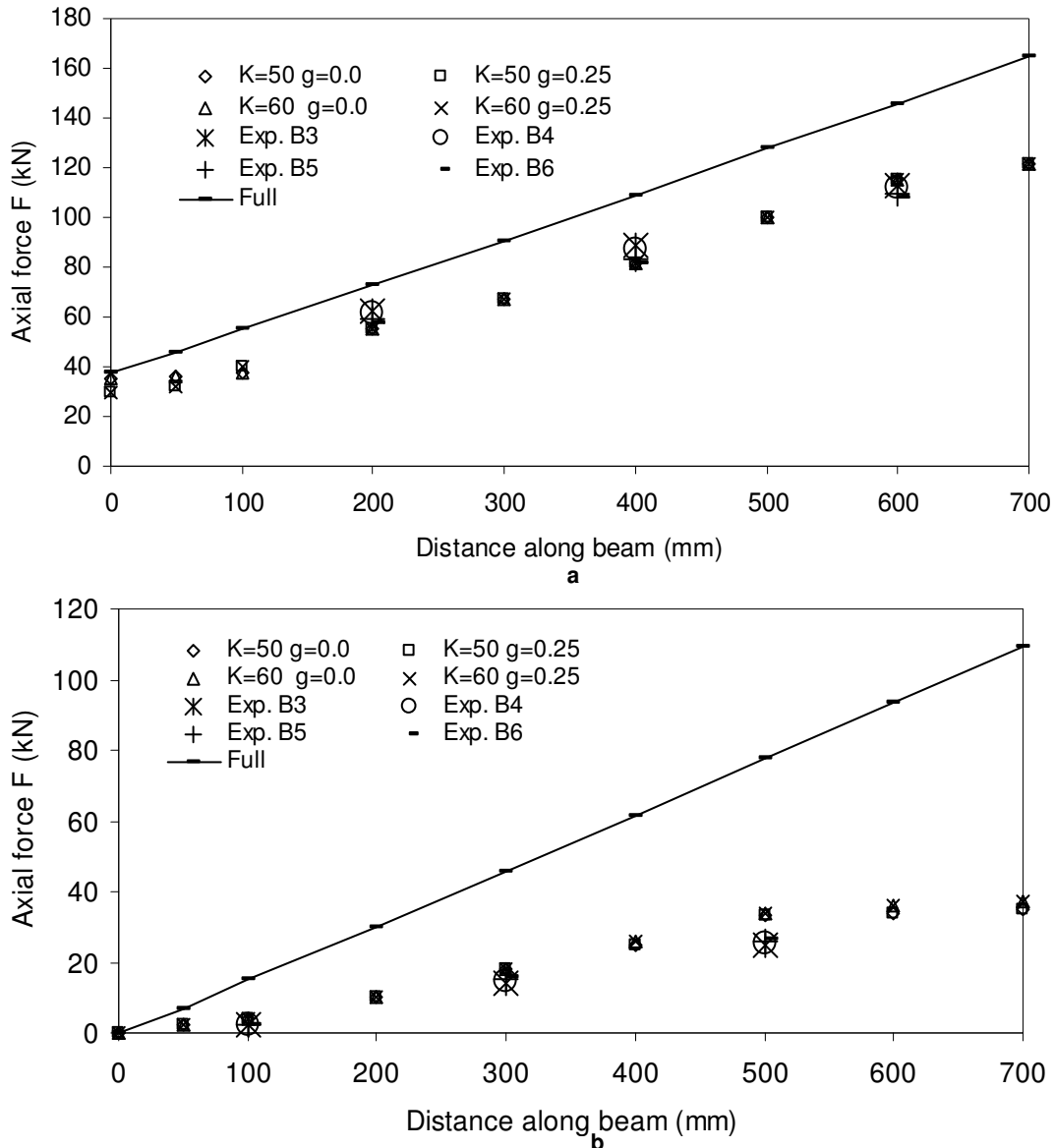
Figure 4. a. Comparison of experimental tension plate axial forces for the second group of beams B3-6 (P = 50 kN). b. Comparison of experimental compression plate axial forces for the second group of beams B3-6 (P = 50 kN)

interaction analysis in order to simplify the system as mentioned before. For comparison of theoretical solutions with the experimental test results, the system geometry and material properties assumed were the same as reported by Dogan (1997).

Here, full and partial interaction theories are compared, firstly neglecting friction between the layers at the supports and secondly taking frictional forces into consideration. Comparisons are also made with the test results at particular applied load levels. Results are presented for axial forces in the steel plates and shear forces in the studs.
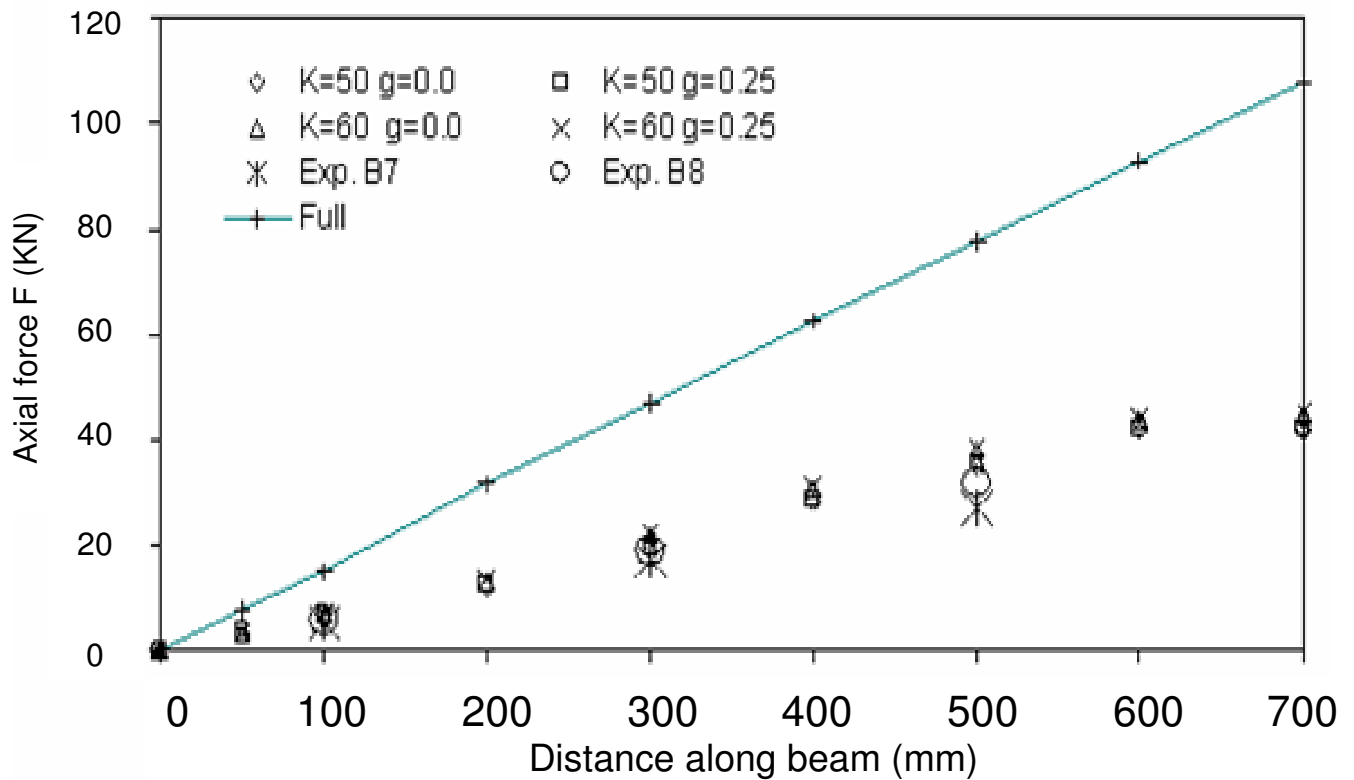
Steel plate axial forces

Figures 4 to 6 show a comparison of the axial forces in the tension and compression steel plates along beams B3-10, for values of the connection stiffness K = 50 and 60 kN/mm, with and without frictional forces between the layers at the supports. The axial forces in the plates increase with an increase of the shear connection stiffness, and tend to values based on full interaction theory as the shear connection stiffness tends to infinity.

Theoretical results based on partial interaction theory show close agreement with experimental results. Full
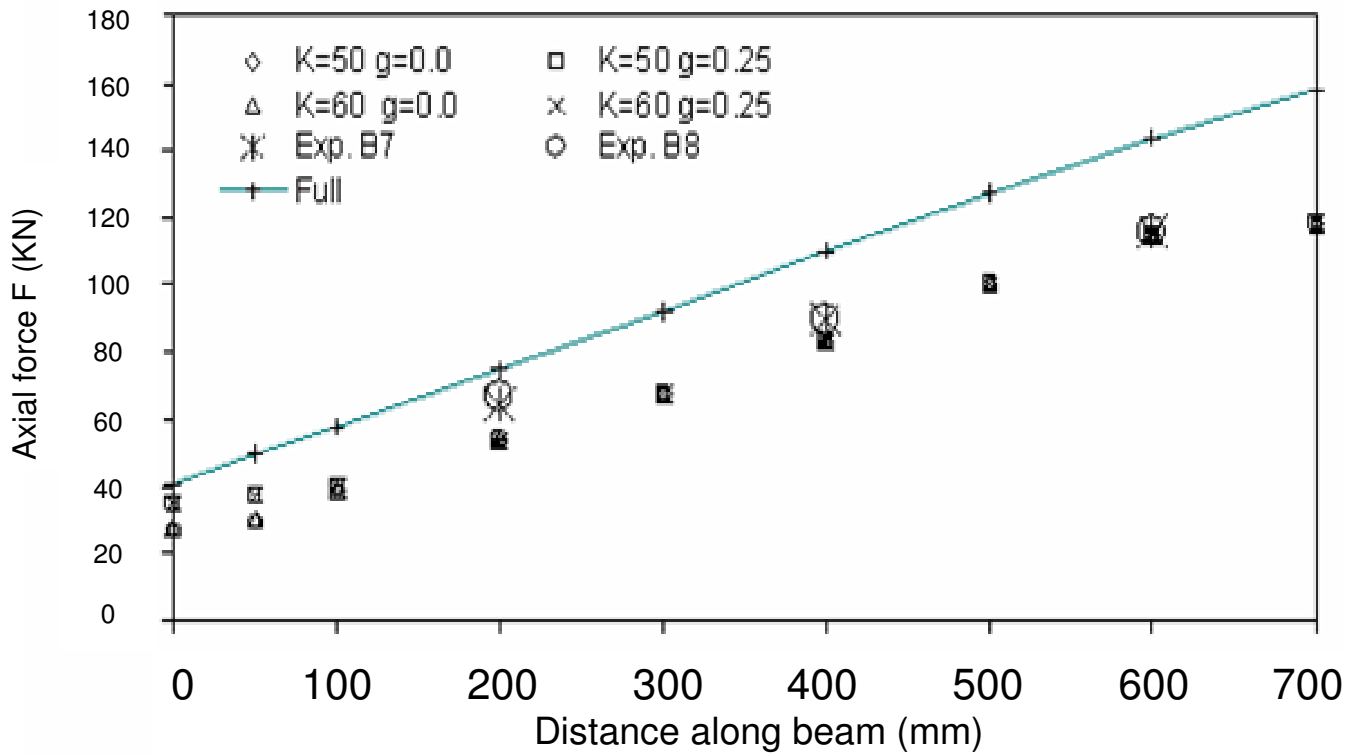
Figure 5. a. Comparison of experimental tension plate axial forces for the third group of beams B7-8 (P = 50 kN). b. Comparison of experimental compression plate axial forces for the third group of beams B7-8 (P = 50 kN).
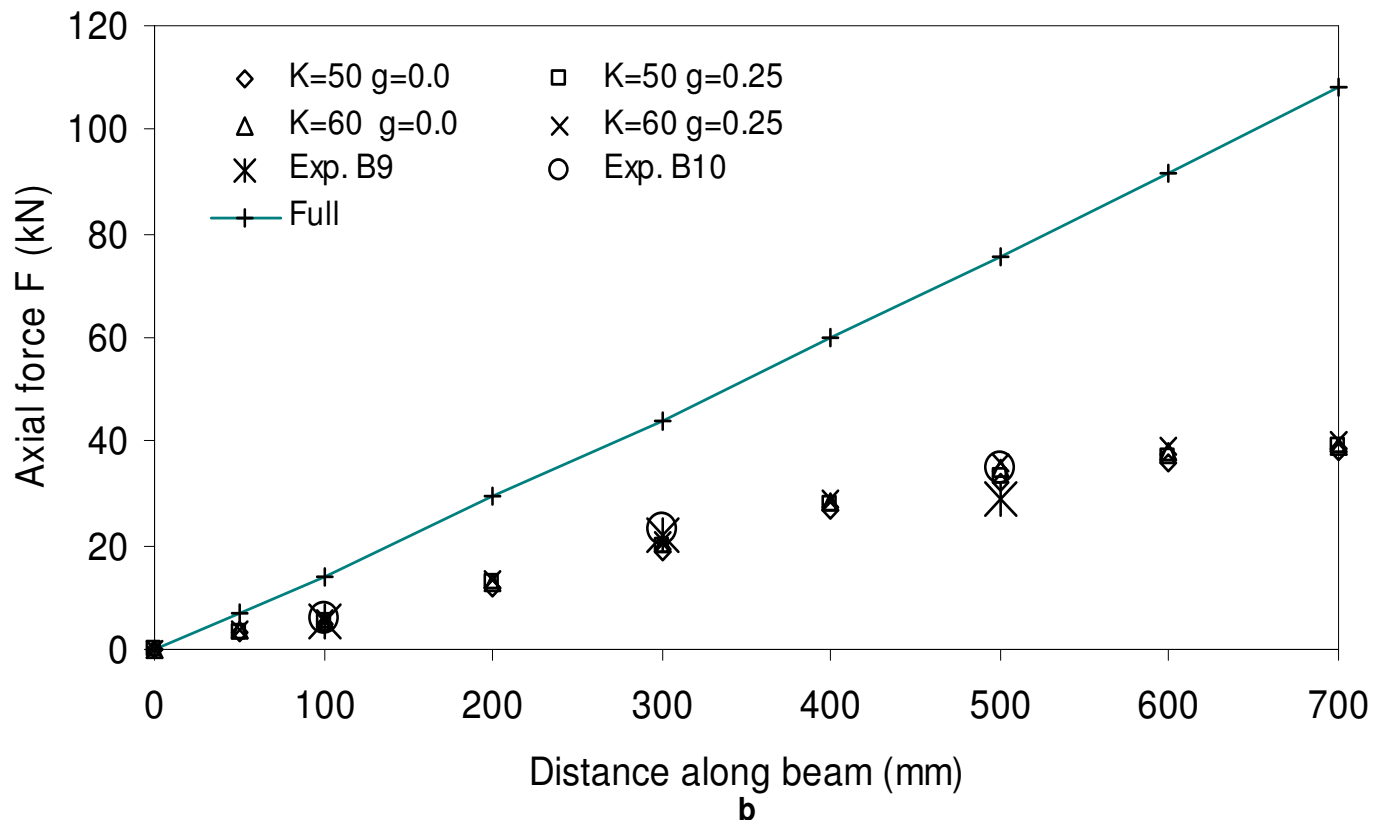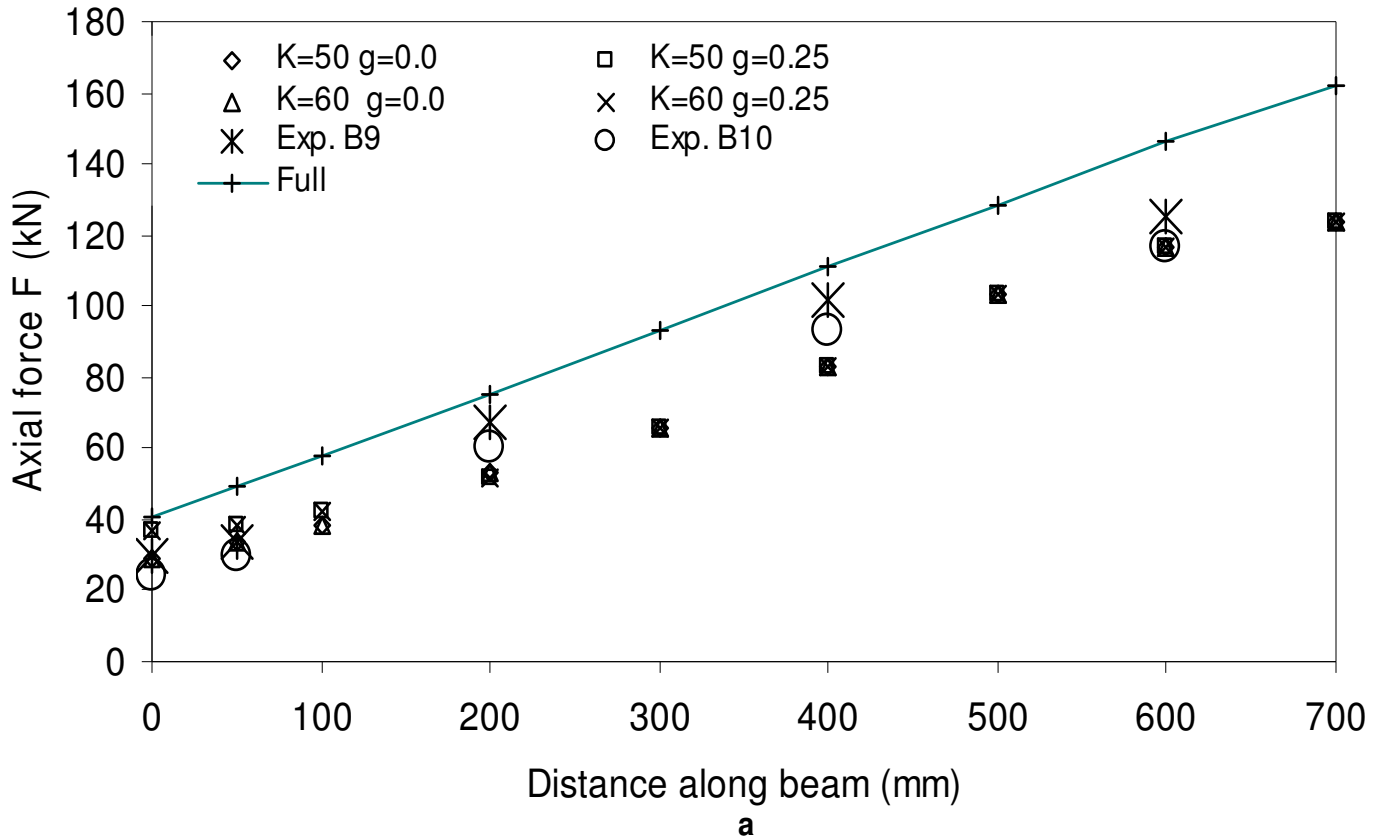
Figure 6. a. Comparison of experimental tension plate axial forces for the fourth group of beams B9-10 (P = 50 kN). b. Comparison of experimental compression plate axial forces for the fourth group of beams B9-10 (P = 50 kN).
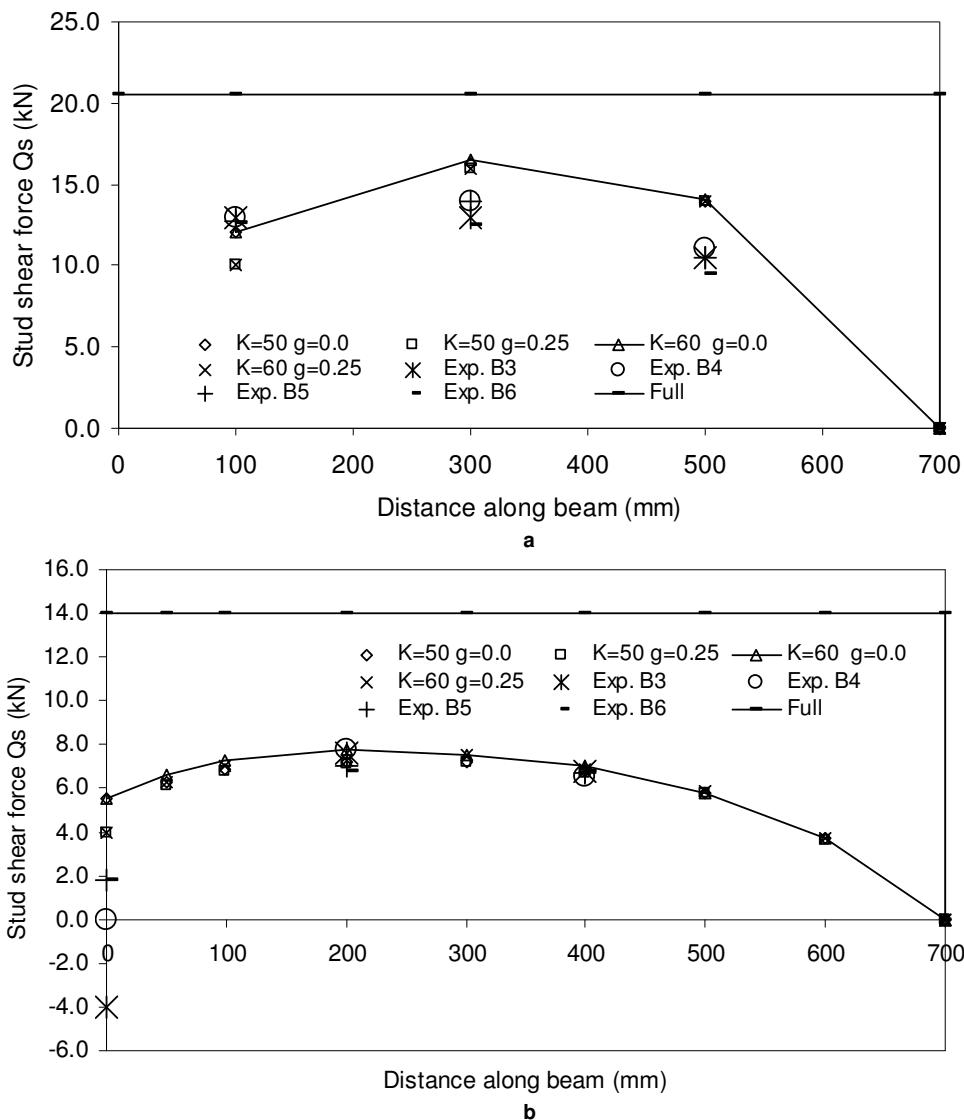
Figure 7. a. Comparison of experimental tension plate stud shear forces for the second group of beams B3-6 (P = 50 kN). b. Comparison of experimental compression plate stud shear forces for the second group of beams B3-6 (P = 50 kN).

interaction theory gives higher values of axial forces for both the tension and compression plates.

Stud shear forces

Figures 7 to 9 show a comparison of theoretical and experimental stud shear forces along beams B3-10, at a load level of 50 kN, for values of the connection stiffness K = 50 and 60 kN/mm, with and without frictional forces between the layers at the supports. Theoretical tension and compression plate stud shear forces increase with an increase of shear connection stiffness, and tend to values

based on full interaction theory. In general there is reasonable agreement between theoretical and experimental results.

DISCUSSION AND CONCLUSIONS

Comparisons have been made between experimental results and theoretical predications of the behaviour of DSC beams, based on full and partial interaction analysis. Because of the variation in concrete cube strength and elastic modulus of the test beams, they have been divided into four groups for which comparisons between experimental and theoretical axial forces and stud shear
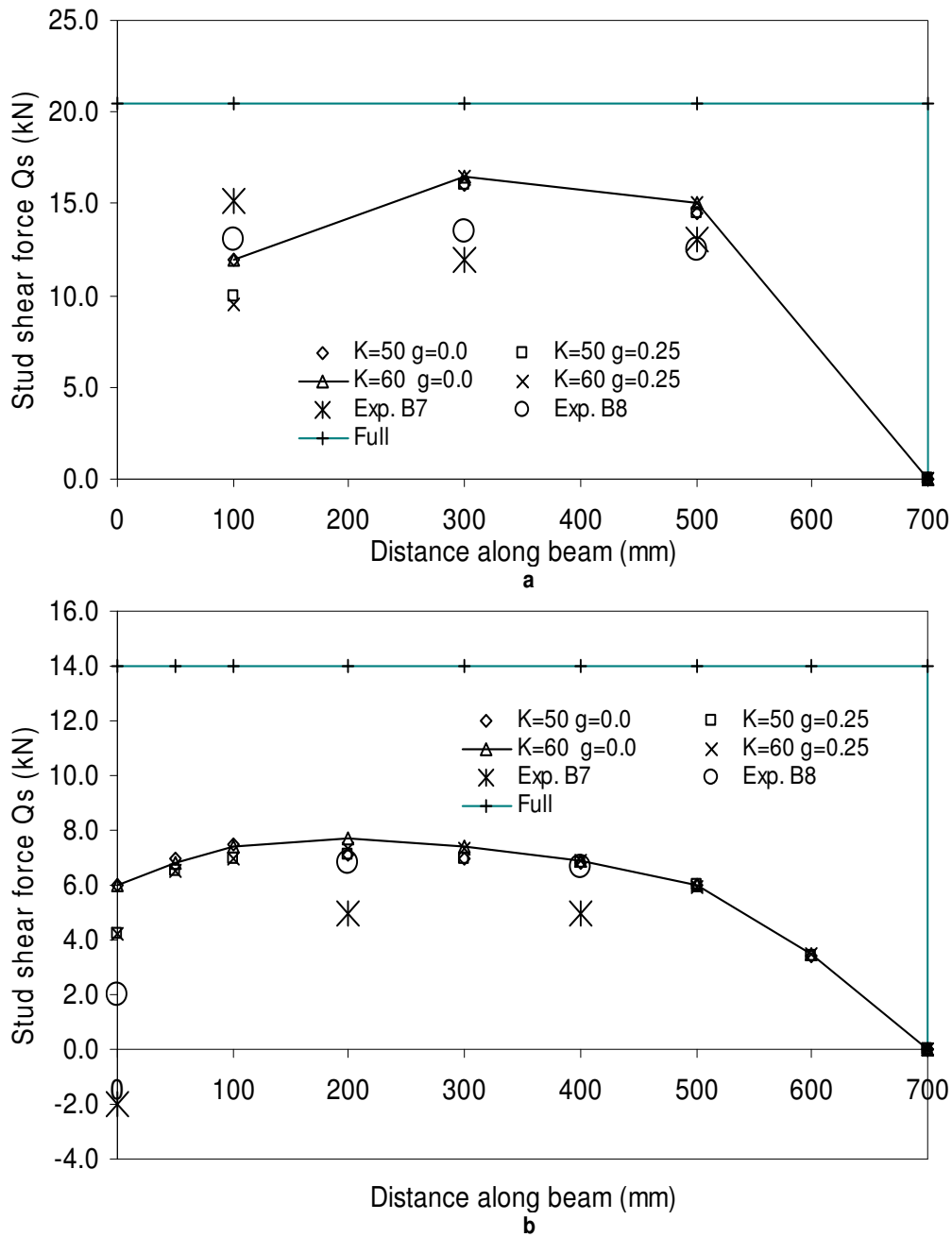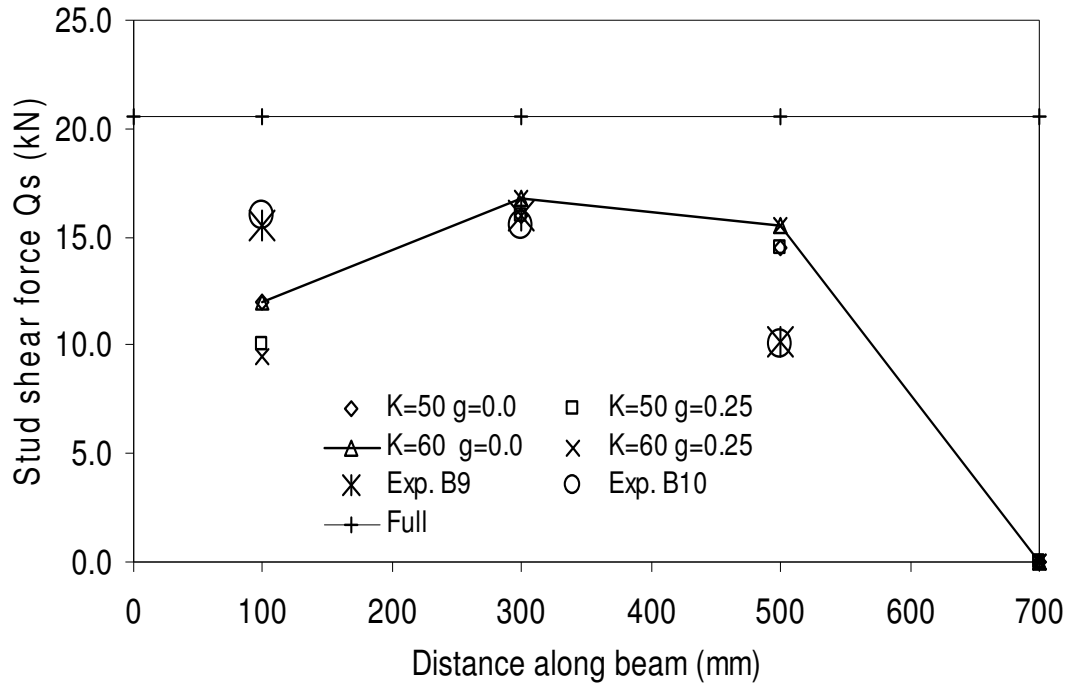
Figure 8. a. Comparison of experimental tension plate stud shear forces for the third group of beams B7-8 (P = 50 kN). b. Comparison of experimental tension plate stud shear forces for the third group of beams B7-8 (P = 50 kN).

forces are presented.

Due to the variation of concrete crack depths along the beams and separation between the tension steel plates and concrete infill, experimental results differed slightly from the theoretical results. Local concrete cracking at the fourth group of studs from the end of the beam resulted in a redistribution of stud shear forces and a discontinuity in

slip. The increase in crack depth increased the moment lever-arm and decreased the axial force in the steel plates.

The partial interaction analysis indicates that frictional forces at the supports and studs outside of the supports have a significant influence on the behaviour of DSC beams.
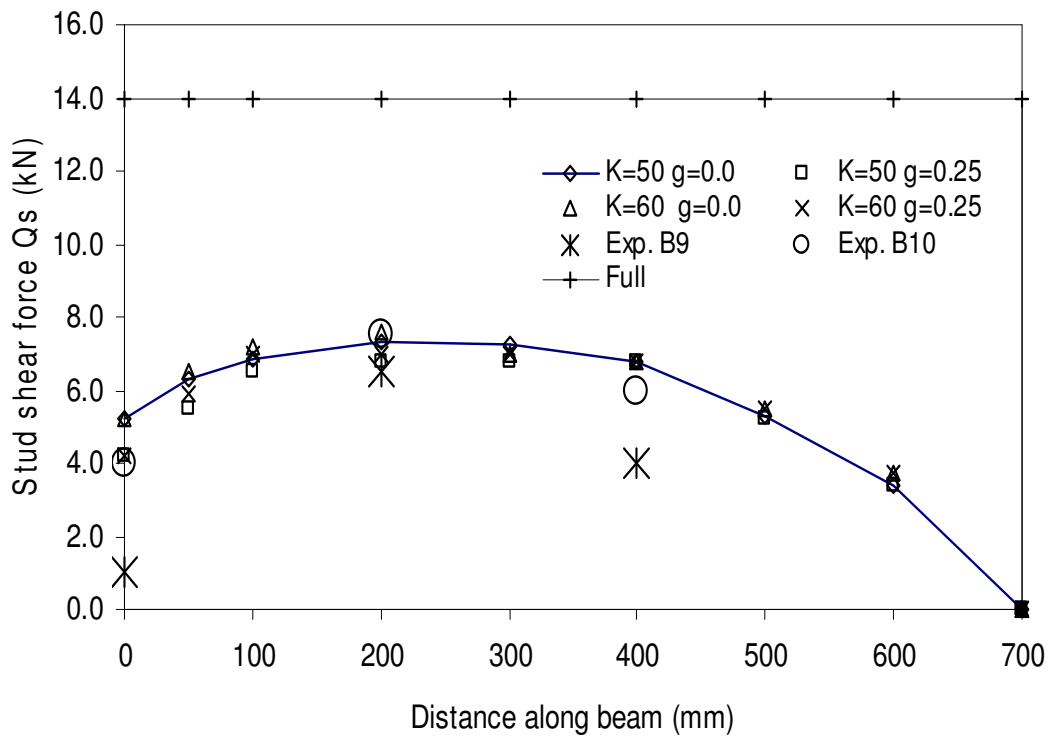
Figure 9. a. Comparison of experimental tension plate stud shear forces for the fourth group of beams B9-10 (P = 50 kN) b. Comparison of experimental compression plate stud shear forces for the fourth group of beams B9-10 (P = 50 kN).

The theoretical results based on partial interaction theory, assuming realistic material and shear connector properties and incorporating the influence of interface frictional forces, show satisfactory correlation with test

results.

ACKNOWLEDGEMENTS

NOTATION

A	cross-section area
b	width of beam section
d	depth of concrete
e	strain difference at steel-concrete interface
E	Young's modulus
EA	axial rigidity
EI	flexural rigidity
F	axial force in steel plates
f	ultimate strength of concrete
g	coefficient of friction at steel-concrete interface
I	second moment of area
k	curvature
K	stiffness of shear connector
L	span of beam
M	bending moment
n	number of connectors across the beam
P	applied point load on beam
p	longitudinal pitch of connectors
q	shear force (shear flow) per unit length between concrete infill and steel plate
Q	shear force on one connector
s	stud spacing
t	thickness of steel plate
u	distance of point load from support
V	transverse shear force
x, y	co-ordinate axes
x	distance along beam from support
y	moment lever arm
v	deflection
α	composite stiffness factor
ε	strain

Subscripts

A	cross-section area of steel plate
c	concrete core
cu	uncracked concrete core
f	frictional force
p	partially interactive section
s	fully interactive section
sc	steel plates in compression
st	steel plates in tension

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