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Full Length Research Paper

Anti-synchronization of chaotic system using adaptive modified function projective method with unknown parameters

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In this paper, a new type of anti-synchronization called adaptive modified function projective anti-synchronization was presented. In this study, state variables of drive system would be anti-synchronized with state variables of response system up to desired scale function matrix. The adaptive control law and the parameter updates were determined to make the states of two Lorenz systems modified function projective anti-synchronized by using Lyapunov stability theory. Numerical simulations were presented to verify the effectiveness of this control method.

Key words: Anti-synchronization, chaotic system, adaptive control, modified projective synchronization.

INTRODUCTION

Chaos an interesting phenomenon of nonlinear systems has been developed more and studied extensively in the last few years. Since Pecora and Carroll (1990) established a chaos synchronization scheme for two identical chaotic systems with different initial conditions, a variety of approaches have been proposed for the synchronization of chaotic systems which include complete synchronization (Zhou and Chen, 2008), phase synchronization (Chavez et al., 2006), lag synchronization (Yu and Cao, 2007), generalized synchronization (Zhang and Jiang, 2011), projective synchronization (Hu and Xu, 2008) and modified projective synchronization (Cai, 2010; Tang and Fang, 2008; Sudheer and Sabir, 2010; Park, 2008). Among all kinds of chaos synchronization, projective synchronization has been extensively investigated in recent years and provides greater security in secure communication. First, Mainieri and Rehacek (1999) reported projective synchronization in partially linear systems. After that, Xu et al. (2001) and Xu (2002) introduced several control schemes based on Lyapunov stability theory to conduct the scaling factor onto a desired value, and derived a general condition (Li and Xu, 2001; Xu et al., 2002) for projective synchronization.

Recently, function projective synchronization was investigated (Du et al., 2008), where the master and slave systems could be synchronized up to a scaling function. That is, master's and slave's state variables in pair correlate each other by a scaling function. Anti-synchronization (AS) can also be interpreted as anti-synchronization which is a phenomenon whereby the state vectors of the synchronized systems have the same amplitude but opposite signs as those of the driving system. Therefore, the sum of two signals is expected to converge to zero. In this paper, a new type of anti-synchronization phenomenon called adaptive modified function projective anti-synchronization (AMFPAS), is proposed, where the responses of the anti-synchronized dynamical states anti-synchronize up to a desired scaling function matrix. The organization of this paper is as follows: the definition of AMFPAS is given, after which we take Lorenz system as an example to illustrate the AMFPAS phenomenon. Numerical simulations are given to demonstrate the effectiveness of the proposed method and conclusion is finally drawn.

METHOD OVERVIEW

Here, adaptive modified projective scheme is presented (Aebastian and Sabir, 2009). Consider the following master

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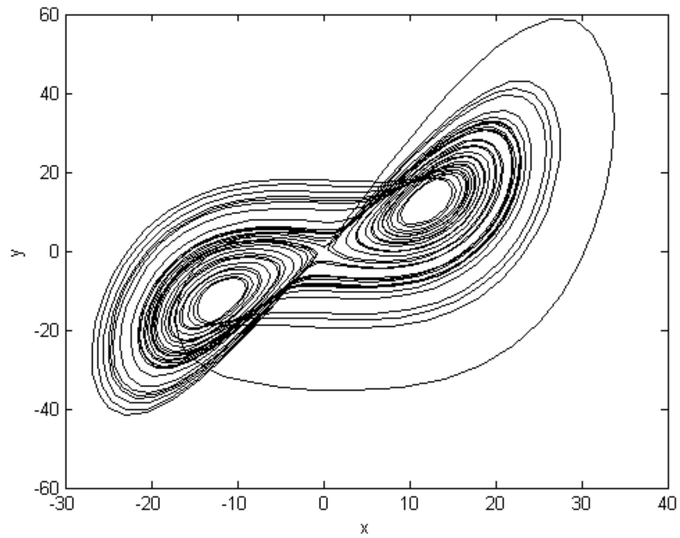

Figure 1. Chaotic behavior of Lorenz system.

master and slave system:

$$\dot{x} = f(t,x) \,, \tag{1}$$

$$\dot{y} = g(t,y) + u(t,x,y) \,, \tag{2}$$

where $x, y \in R^n$ are the state vectors, $f, g : R^n \to R^n$ are continuous nonlinear vector functions and $u(x,y)$ is the vector controller. By defining the error system as:

$$e(t) = x + M\mu(t)y \,,$$

where M is a diagonal matrix with constant arrays, $M = diag(m_1, m_2, ..., m_n) \in R^{n \times n}$ and $\mu(t)$ is an n-order diagonal matrix, $\mu(t) = diag(\alpha_1(t), \alpha_2(t), ..., \alpha_n(t))$ and $\alpha_i(t)$ is a continuously differentiable function with bounded, $\alpha_i(t) \neq 0$ for all t.

SYSTEM DESCRIPTION

Consider the chaotic Lorenz system as follows (Du et al., 2009):

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = (b - z)x - y \\ \dot{z} = xy - cz \end{cases} \tag{3}$$

where x, y and z are state variables. When three real parameters take the values, $a = 10$, $b = 60$ and $c = 8/3$, the system shows chaotic behaviour. Figure 1 shows the chaotic behavior of Lorenz system.

ADAPTIVE MODIFIED FUNCTION PROJECTIVE ANTI-SYNCHRONIZATION (AMFPAS) BETWEEN TWO LORENZ SYSTEMS

To anti-synchronize two Lorenz systems, we assume that the Lorenz system denoted by subscript d is the drive system and the Lorenz system denoted by subscript r is the response system. Following equations show the drive system (Equation 4) and response system (Equation 5):

$$\begin{cases} \dot{x}_d = a(y_d - x_d) \\ \dot{y}_d = (b - z_d)x_d - y_d \\ \dot{z}_d = x_d y_d - cz_d \end{cases} , \tag{4}$$

$$\begin{cases} \dot{x}_r = a(y_r - x_r) + u_1 \\ \dot{y}_r = (b - z_r)x_r - y_r + u_2 \\ \dot{z}_r = x_r y_r - cz_r + u_3 \end{cases} , \tag{5}$$

where $U = [u_1, u_2, u_3]^T$ is the control law that drive and response systems can be anti-synchronized such as defining AMFPAS errors as follow:

$$\begin{cases} e_1 = x_d + m_1\alpha_1(t)x_r \\ e_2 = y_d + m_2\alpha_2(t)y_r \\ e_3 = z_d + m_3\alpha_3(t)z_r \end{cases} , \tag{6}$$

for all $e_i (i = 1,2,3)$, we have:

$$\lim_{t \to \infty} \|e_i\| = 0 \,.$$

By differentiating Equation 6, we have:

$$\begin{cases} \dot{e}_1 = \dot{x}_d + m_1\dot{\alpha}_1(t)x_r + m_1\alpha_1(t)\dot{x}_r \\ \dot{e}_2 = \dot{y}_d + m_2\dot{\alpha}_2(t)y_r + m_2\alpha_2(t)\dot{y}_r \\ \dot{e}_3 = \dot{z}_d + m_3\dot{\alpha}_3(t)z_r + m_3\alpha_3(t)\dot{z}_r \end{cases} \tag{7}$$

Then we have the error dynamics as follows by substituting Equations 4 and 5 in 7:

$$\begin{cases} \dot{e}_1 = a(y_d - x_d) + m_1\dot{\alpha}_1(t)x_r + m_1\alpha_1(t)[a(y_r - x_r) + u_1] \\ \dot{e}_2 = (b - z_d)x_d - y_d + m_2\dot{\alpha}_2(t)y_r \\ \qquad + m_2\alpha_2(t)[(b - z_r)x_r - y_r + u_2] \\ \dot{e}_3 = x_d y_d - cz_d + m_3\dot{\alpha}_3(t)z_r + m_3\alpha_3(t)[x_r y_r - cz_r + u_3] \end{cases} , \tag{8}$$

The goal is to find control law $U = [u_1, u_2, u_3]^T$ to achieve stability. So, following control laws and update rules for the unknown parameters a_1, b_1 and c_1 are proposed:

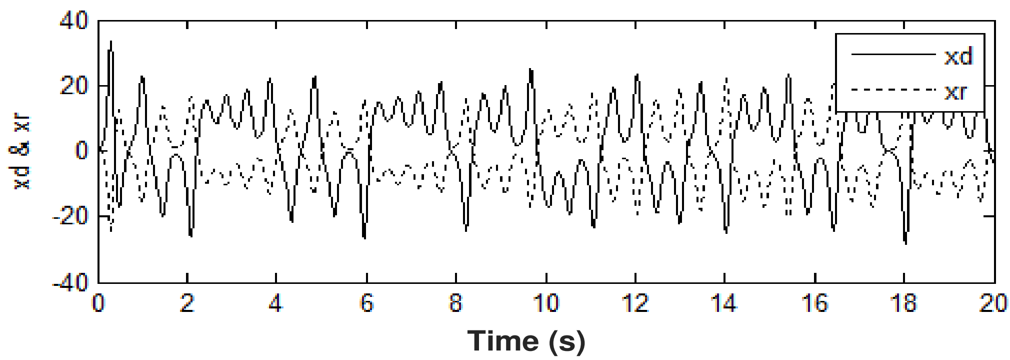

Figure 2. The state trajectories of the drive system and the response system for state "x".

$$u_1 = \frac{-1}{m_1\alpha_1(t)}[a_1(y_d - x_d) + m_1\dot{\alpha}_1(t)x_r + m_1\alpha_1(t)a_1(y_r - x_r) + k_1e_1],$$

$$u_2 = \frac{-1}{m_2\alpha_2(t)}[(b_1 - z_d)x_d - y_d + m_2\dot{\alpha}_2(t)y_r + m_2\alpha_2(t)((b_1 - z_r)x_r - y_r) + k_2e_2],$$

$$u_3 = \frac{-1}{m_3\alpha_3(t)}[x_d y_d - c_1z_d + m_3\dot{\alpha}_3(t)z_r + m_3\alpha_3(t)(x_r y_r - c_1z_r) + k_3e_3] \quad (9)$$

and

$$\dot{a}_1 = e_1[(y_d - x_d) + m_1\alpha_1(t)(y_r - x_r)] + k_4e_4,$$

$$\dot{b}_1 = e_2[x_d + m_2\alpha_2(t)x_r] + k_5e_5,$$

$$\dot{c}_1 = e_3[-z_d - m_3\alpha_3(t)z_r] + k_6e_6 \quad (10)$$

where $k_i > 0$ for $i = 1,...,6$.

We use Lyapunov stability theorem to show that the drive system (Equation 4) and the response system (Equation 5) will be anti-synchronized using adaptive modified function projective method. By defining $e_a = a - a_1$, $e_b = b - b_1$ and $e_c = c - c_1$, we have $\dot{e}_a = -\dot{a}_1$, $\dot{e}_b = -\dot{b}_1$ and $\dot{e}_c = -\dot{c}_1$.

Using the positive definite Lyapunov function as follows:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2), \quad (11)$$

The derivative of v is:

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_a\dot{e}_a + e_b\dot{e}_b + e_c\dot{e}_c$$

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 - e_a\dot{a}_1 - e_b\dot{b}_1 - e_c\dot{c}_1, \quad (12)$$

and by substituting Equations 8 and 10 in 12, we have:

$$\dot{V} = -e^{TKe}$$

where $e = [e_1, e_2, e_3, e_a, e_b, e_c]^T$ and $K = diag(k_1, k_2, k_3, k_4, k_5, k_6)$.

So $\dot{V}$ is negative definite and we have $e_1, e_2, e_3, e_a, e_b, e_c \to 0$ as $t \to \infty$.

NUMERICAL SIMULATIONS

Here, numerical simulations are presented to demonstrate the effectiveness of adaptive modified function projective method for anti-synchronization. The initial conditions of the drive systems were $x_d(0) = 0$, $y_d(0) = 1$ and $z_d(0) = 1$, and those of the response system were $x_r(0) = 0.6$, $y_r(0) = 0.8$ and $z_r(0) = 0.5$. The scaling functions were chosen as $\alpha_1(t) = 2 + 0.4\sin(2\pi t/20)$, $\alpha_2(t) = 1 + 0.25\sin(2\pi t/20)$ and $\alpha_3(t) = 5 + 0.15\sin(2\pi t/20)$, and the scaling factors were $m_1 = 0.7$, $m_2 = 1$ and $m_3 = 0.5$. The initial condition values of the estimated parameters were $a_1(0) = 8$, $b_1(0) = 50$ and $c_1(0) = 2$. Moreover, the control gains were chosen as $k_1 = 1$, $k_2 = 1$, $k_3 = 1$, $k_4 = 1$, $k_5 = 1$ and $k_6 = 1$. Figures 2 to 4 showed the state trajectories of the drive system and the response system. Figure 5 exhibits the evolution of the AMFPAS errors, it could be seen that by $e \to 0$ as $t \to \infty$, globally asymptotically stability of the error dynamical system between the drive and the response systems was proved. The time evolution of the control law is depicted in Figure 6 and in Figure 7 the time evolution of the estimated parameters is shown. These results showed that the required anti-synchronization has been achieved with proposed method for designing control law (Equation 9).

Conclusion

This paper investigated the adaptive modified function projective anti-synchronization between two chaotic

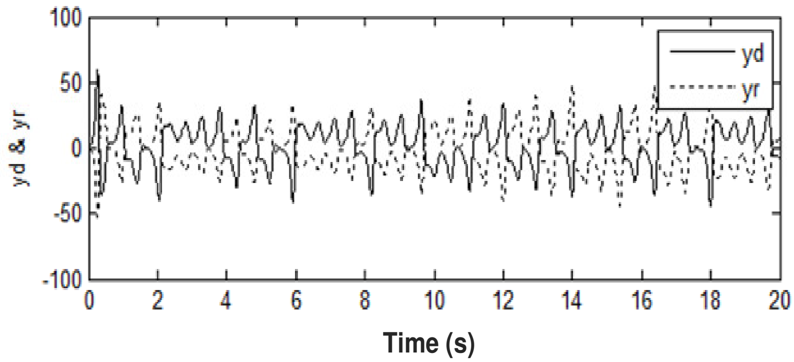

Figure 3. The state trajectories of the drive system and the response system for state “y”.

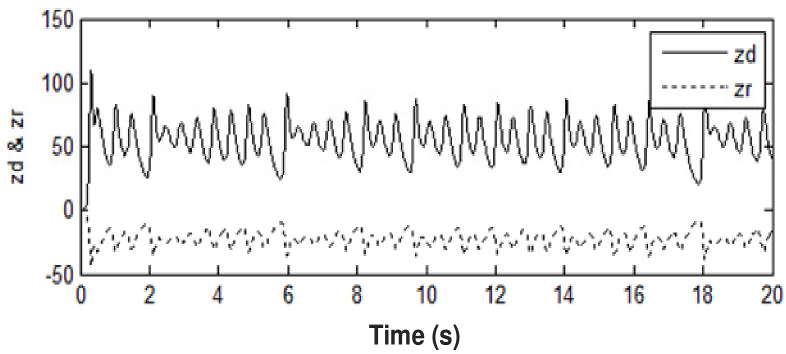

Figure 4. The state trajectories of the drive system and the response system for state “z”.

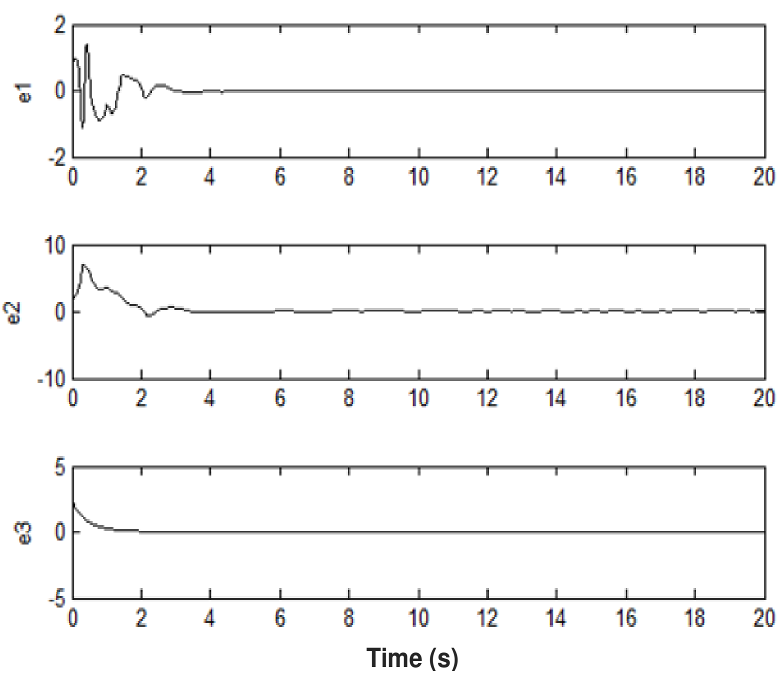

Figure 5. The evolution of the AMFPAS errors.

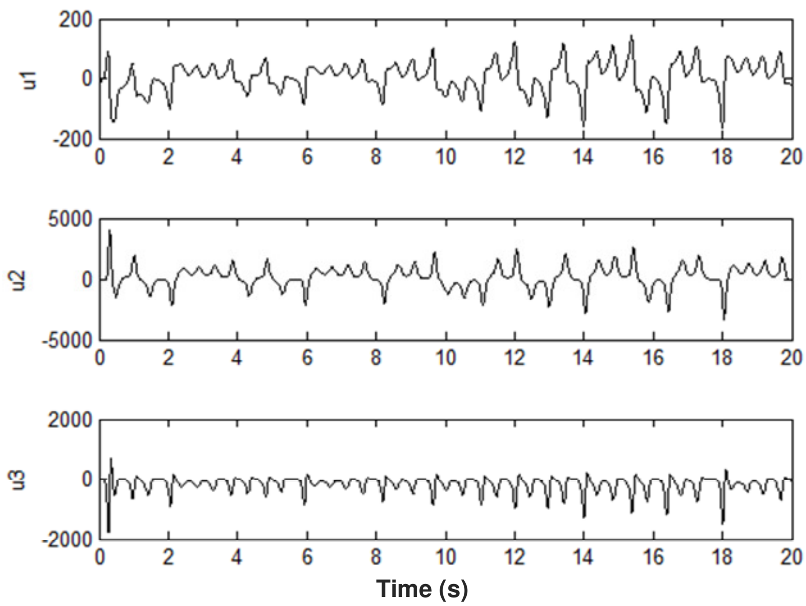

Figure 6. The time evolution of the control laws.

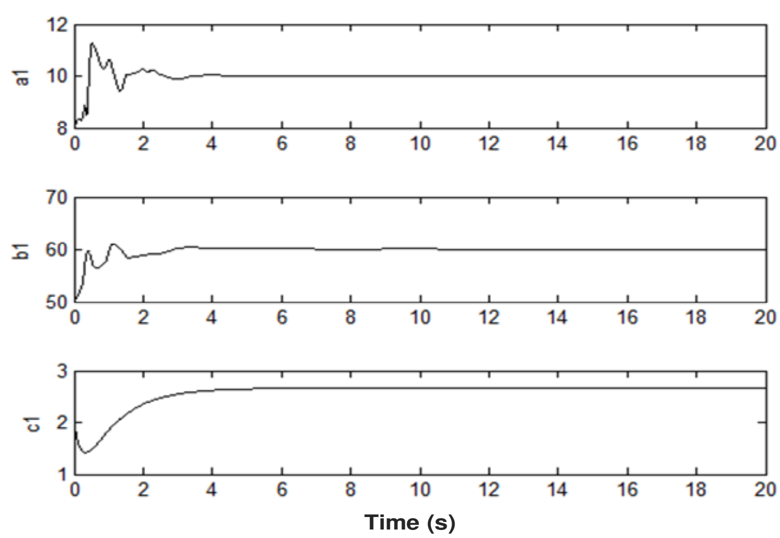

Figure 7. The time evolution of estimated unknown parameters a_1, b_1 and c_1.

Lorenz systems. On the basis of Lyapunov stability theory, we design anti-synchronization controllers with corresponding parameter update laws. All the theoretical results are verified by numerical simulations to demonstrate the effectiveness of the proposed anti-synchronization schemes.

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