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Full Length Research Paper

Nonlineaer analysis of high-ninlinearity structures

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Rapid progress in the analysis and construction of high-nonlinearity structures has been made over the last three decades. The use of a new method is necessary to accuracy and stability to improve performance of this structure. The Fletcher-Reeves method belongs to a group of methods called conjugate gradient methods which attempt to locate a local minimum function. The approach of the proposed method was based upon the principle of conservation of energy. All nonlinear effects due to material properties, large displacements or local failure can be incorporated in the nonlinear dynamic analysis. Fletcher-Reeves algorithm was applied to calculate the set of displacements to minimize the energy of structural system. In this paper, a theory for nonlinear dynamics response analysis of high nonlinearity structure was developed based on the minimization of the total potential dynamic work.

Key words: Fletcher-Reeves method, optimization of energy, finite element method, perturbation technique, analyzing modal, nonlinear dynamic response.

INTRODUCTION

The aim of this study is to develop a new method to for non-linear analysis of structures with high degree of freedom. The proposed theory for nonlinear analysis of 3D space structure is based on minimization of the total potential dynamic work. The minimization of the total potential dynamic work is indirect method which is based on principle of convergence of energy in structures. Conventional methods such as superposition methods are direct method (Dehghan, 2002). They are usually employed for the solution of equilibrium equations of structures. However, the conventional methods use for structural analysis of 3D nonlinear space structures overestimates the displacements when the structures is stiffening and underestimate when it is softening. For the conventional method, the number of iteration increases with increase in degree of freedom and these methods need large computer storage for solution of equation of motion (Roy and Dash, 2002). The material is homogeneous and isotropic. The stress–strain relationship of all material remains within the linear elastic range during the whole nonlinear response. The external loads are displacement independent. Large displacements and large rotations are allowed. For the initial shape analysis, a linear and a nonlinear computation procedure are set up. These structures have many advantages such as prefabrication, ease of transportation and erection, relatively low cost and provision for coverage of large clear spans and high strength, large flexibility and elasticity. The design process is a relatively complex problem. In the present study, we consider the effect of dynamic loads in tension structures and describe a Fletcher-Reeves method for the determination of free and forced vibration analysis of structures. The structure can be analyzed as discrete system or continuous membrane. A unified approach to the static analysis of both linear and nonlinear structures is to consider the determination of equilibrium as an iterative process of minimizing the total potential energy. Regarding this fact, the non-linear systems have no fixed sets of eigenvectors and eigenvalues. The new sets of eigenvectors and eigenvalues must be calculated at each time step and the stiffness matrix must be revaluated at end of each time step (Guo and Chen, 2007). This makes the use of conventional methods extensively time consuming and costly. In the dynamic problems, the differential equations arising from the equilibrium of the dynamic forces acting on the mass is solved by implicit or explicit methods. They assume the structural properties to remain constant during the interval, but revaluate them at the end of time step. This cannot be sufficient for highly non-linear

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structures. It is important to revaluate both the stiffness and damping during the time step. The revaluation process makes the methods more expensive to use. Also, the reduction of time consumed, cost and the high accurate result justify the used of indirect methods, such as, optimization theory (Celebi et al., 2009).

EQUATION OF MOTION FOR A SYSTEM

The equation of motion for a multi degree (MDOF) system can be written as (Argyris et al., 1979):

$$M \ddot{x} + C(t)\dot{x} + K(t)x = P(t) \quad (1)$$

Where: M = mass matrix, C (t) = Damping matrix, K (t) = stiffness matrix, x = Displacement vector, $\dot{x}$ = Velocity vector, $\ddot{x}$ = Acceleration vector and P (t) = Load vector

The assumption of a constant mass in the case of both MDOF systems is arbitrary as it could be represented as a time varying quantity. Since m is a non-zero constant value, both sides of Equation 1 can be divided by m, and for-

$$P = \frac{C(t)}{m},\ Q = \frac{K(t)}{m},\ F = \frac{p(t)}{m}$$

Equation 1 can be written as:

$$\ddot{X} + P\dot{x} + QX = F \quad (2)$$

The mathematical solution of Equation 2 depends on the values of P, Q and F. Equation 2 is a linear differential equation if P and Q are independent of x and remains so even if P and Q are functions of t (Bradford et al., 1999).

The method of Fletcher-Reeves

The method avoids explicit construction and inversion of the Hessian matrix k, by using the iterative formula (Fletcher, 2007):

$$X_{K+1} = X_k - H_k g_k \quad (3)$$

Where

$$H_k = I + \sum_{i}^{k-1} A_i \quad (4)$$

And

$$A_i = \frac{V_i V_i^T}{V_i^T x_i} - \frac{H_i \gamma_i \gamma_i^T H_i}{\gamma_i^T H_i \gamma_i} \quad (5)$$

And

$$\gamma_i = g_{i+1} - g_i \quad (6)$$

In the first iteration H_i = I, which is the identity matrix. Thus the first step is in the direction of steepest descent. The slow convergency of the steepest descent method is then overcome by choosing the sequence of H such that as i approach k, H_k becomes approximately equal to k^{-1}. For linear problem the method converges in n+1 steps in which case $H_{n+1} = k^{-1}$. It finds the solution to the second equation that is closest to the current estimate and satisfies the curvature condition. This update maintains the symmetry and positive definiteness of the Hessian matrix. The essential feature of the method is a recursion formula for updating an initial approximation to the Hessian matrix of second partial derivatives of the function to be minimized (Ademoyero et al., 2004). The iterative method applied ensures that each step in the procedure leads to a function decrease until a stationary point is reached. The function to be minimized is f(x) where x denotes the argument vector of the decision variables $x_1, x_2, \ldots, x_n$.

The expression for the total potential energy

The total potential energy is written as:

$$W = U + V \quad (7)$$

Where: W = the total potential energy, U = the strain energy of the system and V = the potential energy of the loading.

Taking the unloaded position of the assembly as datum, we have

$$W = \sum_{n=1}^{m} U_n + \sum_{j=1}^{j} \sum_{i=1}^{3} F_{ji} X_{ji} \quad (8)$$

Where: M = total number of members, J = total number of cable joints, F_{ji} = external applied load on joint j in direction i, and X_{ji}= displacement of joint j in direction i.

The condition for structural equilibrium is that the total potential energy of the system is at minimum, and is written as;

$$\partial W / \partial X_{ji} = 0 \quad (9)$$

Thus, at the solution, the gradient vector of the total potential energy function is zero.

The gradient of the total potential energy

Differentiating Equation 9 with respect to X_{ji} gives g_{ji}, which is the element of the gradient vector g as follows (Hashamdar et al., 2011):

$$g_{ji} = \partial W / \partial X_{ji} = \sum_{n=1}^{q} \partial U_n / \partial X_{ji} - F_{ji} \quad (10)$$

Where: $T_{\circ jn}$ = the initial tension in member jn, T_{jn} = the instantaneous tension in member jn, e_{jn} = elastic elongation of member jn, E = young Modulus of Elasticity, A = cross-sectional area of cable, L $_{jn}$= length of member jn, and Q = number of member meeting at joint j.

The expression for g $_{ji}$ can then be written as:

$$g_{ji} = \sum_{n=1}^{q} \frac{\partial U_n}{\partial e_{jn}} . \frac{\partial e_{jn}}{\partial X_{ji}} - F_{ji} \quad (11)$$

The strain energy of the member j_n is given as:

$$U_{jn} = T_{\circ jn} e_{jn} + \frac{EA}{2L_{jn}} e_{jn}^2 \quad (12)$$

Differentiating U_{jn} with respect to e_{jn} yields

$$\partial U_{jn} / \partial e_{jn} = T_{\circ jn} + \frac{EA}{L_{jn}} e_{jn} = T_{jn} \quad (13)$$

The initial and elongated length of member j_n may be expressed as:

$$L_{jn}^2 = \sum_{i=1}^{3} (X_{ni} - X_{ji})^2 \quad (14)$$

$$(L_{jn} + e_{jn})^2 = \sum_{i=1}^{3} (X_{ni} - X_{ji} + x_{ni} - x_{ji})^2 \quad (15)$$

Where X_{ji} is the coordinate of joint j in direction i. Simplifying Equation 15 and substituting for L $_{jn}$ in Equation 14, yields the following expression for e $_{jn}$:

$$e_{jn} = \frac{1}{2L_{jn} + e_{jn}} \sum_{i=1}^{3} \{ (X_{ni} - X_{ji}) (2X_{ni} - 2X_{ji} + X_{ni} - X_{ji}) \} \quad (16)$$

Differentiating Equation 10 with respect to X_{ji} yields

$$\partial e_{jn} / \partial X_{ji} = \frac{-1}{L_{jn} + e_{jn}} (X_{ni} - X_{ji} + X_{ni} - X_{ji}) \quad (17)$$

Substituting Equations 10 and 17 into Equation 18 yields the expression for the gradient as follows:

$$g_{ji} = -\sum_{n=1}^{q} t_{jn} (X_{ni} - X_{ji} + X_{ni} - X_{ji}) - F_{ji} \quad (18)$$

Where $t_{jn} = T_{jn} / (L_{jn} + e_{jn})$ is the tension coefficient of member j_n.

Total potential energy in the direction of descent

The correct value of X for which W is at minimum can be found by the iterative process;

$$X_{ji(k+1)} = X_{ji(k)} + S_{(k)} V_{ji(k)} \quad (19)$$

Where: the suffices (k) and (k+1) denote the (k)th and (k+1)th iterate respectively

V_{ji} = the element of the direction vector

S $_{(k)}$ = the steplength which defines the position along $V_{ji(k)}$ where the total potential energy is at minimum.

The expression for V_{ji} when the Fletcher-Reeves formulation method is used is given by:

$$V_{ji(k)} = -g_{ji(k)} + \frac{\sum_{j=1}^{J} \sum_{i=1}^{3} g_{ji(k)} g_{ji(k)}}{\sum_{j=1}^{J} \sum_{i=1}^{3} g_{ji(k-1)} g_{ji(k-1)}} V_{ji(k-1)} \quad (20)$$

The stationary point in the direction of descent can be found by expressing the total potential energy as a function of the step length along V_{ji}. Thus the required value of S $_{(k)}$ can be determined by the condition and is given as follows (Daston 1979);

$$\partial W_{(k)} / \partial S_{(k)} = 0 \quad (21)$$

NUMERICAL AND EXPERIMENTAL TESTING

The analytical method was used to experiment with mathematical model and experimental work.

Theoretical

Theoretical analysis (mathematical modelling) structural property matrices below for a pin jointed member with three degrees of freedom at each end as follows.

The lumped mass matrices for a pin jointed member:

$$\frac{\bar{m} L}{3} \bullet \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \quad (25)$$

Where $\bar{m}$ is the mass and L is the length of member.

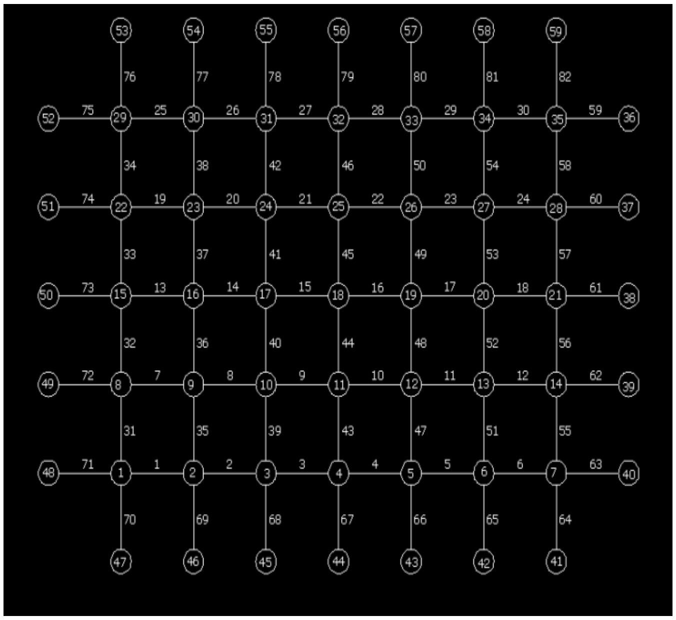

Figure 1. Grid lines of the flat net.

Table 1. The specifications of flat net and cables.

Description	Details
Overall dimensions	3000*4000
Spacing of the cables	500 mm
Number of free joints	35
Diameter (mm)	15.34
Section Area (mm^2)	142.90
Young's Modulus	192.60 KN/ mm^2

Figure 2. Construction of frame steel.

The stiffness matrix for a pin jointed member:

$$=\frac{EA}{L}\begin{vmatrix}\lambda_1^2 & \lambda_1\lambda_2 & \lambda_1\lambda_3\\ \lambda_2\lambda_1 & \lambda_2^2 & \lambda_2\lambda_3\\ \lambda_3\lambda_1 & \lambda_3\lambda_2 & \lambda_3^2\end{vmatrix}+\frac{T}{L}\begin{vmatrix}1-\lambda_1^2 & -\lambda_1\lambda_2 & -\lambda_1\lambda_3\\ -\lambda_2\lambda_1 & 1-\lambda_2^2 & -\lambda_2\lambda_3\\ -\lambda_3\lambda_1 & -\lambda_3\lambda_2 & 1-\lambda_3^2\end{vmatrix} \qquad (26)$$

Where T is the axial force in the axial force and λ_1, λ_2 and λ_3 are the corresponding direction cosines.

Experimental work

The mathematical model chosen is a 7*5 flat net with 105 degrees of freedom. The 7*5 net was built as an experimental model and tested in order to verify the static and dynamic nonlinear Fletcher-Reeves theory. The construction of the experimental model is shown in Figure 1.

The specifications of erected rectangular net and cables are given in Table 1. Each steel cable was initially tensioned to about 1 KN and then left for two weeks to permit the individual wires in the strands to bed in; then, the tension on the cables were readjusted to 11.5 KN. This tension was maintained throughout the test programme by checking at interval times. The wedge and barrel used on hollow cylindrical steel to provide endcaster degree of freedom for boundary condition of cables. Endcaster joints are used to fix boundary condition. General view of steel frame is shown in Figure 2 and 3. Specifications of steel frame made are given in Table 2.

RESULT AND DISCUSSION

Static test

Any deficiency in the model could influence the dynamic behavior and make subsequent comparison of experimental and theoretical values difficult. Hence, a static test was carried out to investigate the degree of symmetric behavior on the frame. The investigation consisted of checking the degree of symmetric behavior about the major and minor axes. The degree of symmetric behavior about the minor axis was investigated by first placing an increasing load on joint 11 and the resultant displacement was compared with those obtained by placing similar loads on joint 25. The degree of symmetric behavior about the major axis was similarly studied by loading first joint 16 and then joint 20. Table 3 shows the degree of symmetric behavior about the minor and major axis respectively, and also shows the percentage difference between the experimental and calculated displacements. The average lack in symmetric behavior about the minor and major axis over the load range as measured by the percentage difference in the movements of selected joints is approximately 3.4%.

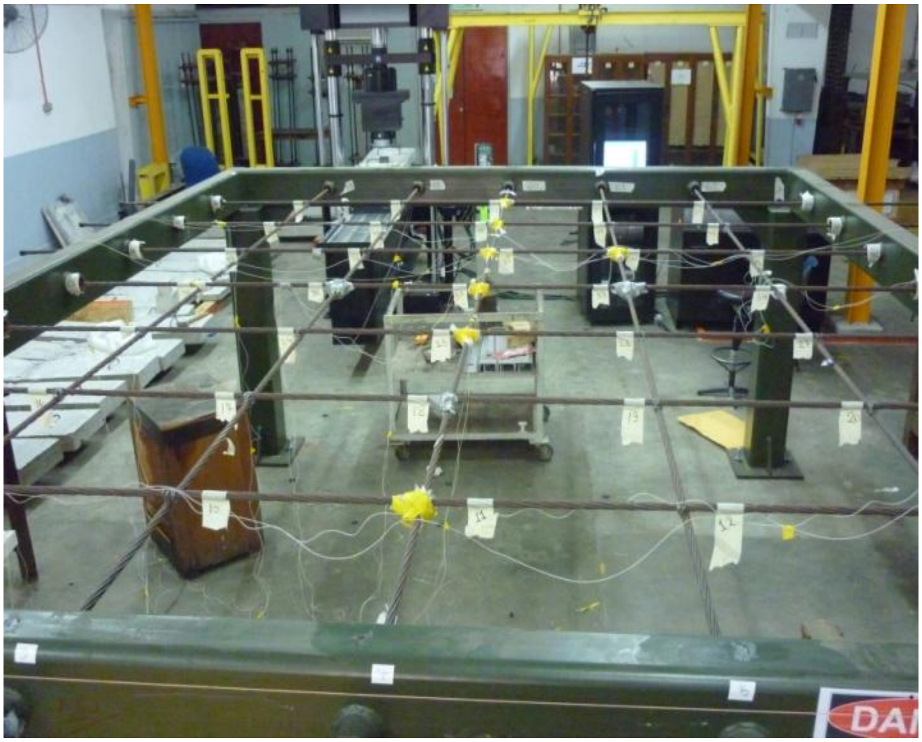

Figure 3. General view of steel frame.

Table 2. Features of steel frame made.

Frame supported specification	
Column	1400 mm (box) Height
Beam	300mm * 400 mm (box) Length
Beam Size	100*200*9 mm (hollow section)
Column Size	200*200*9 mm (hollow section)

Table 3. Deflections due to concentrated load on joint 11.

Load (N) = 2400	Theoretical (T) Z axis (m)	Experimental (E) Z axis (m)	(T – E)/T*100
Deflections node 18	127.9E-03	125.2E-03	2.11
Deflections node 11	142.3E-03	141.5E-03	0.56
Deflections node 4	67.78E-03	65.28E-03	3.69
Deflections node 25	78.68E-03	77.28E-03	1.78
Deflections node 32	29.54E-0	29.24E-0	1.02
Deflections node 15	20.70E-03	20.32E-03	1.84
Deflections node 16	54.46E-03	53.23E-03	2.26
Deflections node 17	104.2E-03	101.5E-03	2.59
Deflections node 19	104.2E-03	102.1E-03	2.02
Deflections node 20	59.30E-03	57.22E-03	3.51
Deflections node 21	20.70E-03	20.52E-03	0.87
Deflections node 1	7.726E-03	7.700E-03	0.34
Deflections node 7	7.726E-03	7.700E-03	0.34
Deflections node 29	5.590E-03	5.40E-03	3.4
Deflections node 35	5.590E-03	5.40E-03	3.4

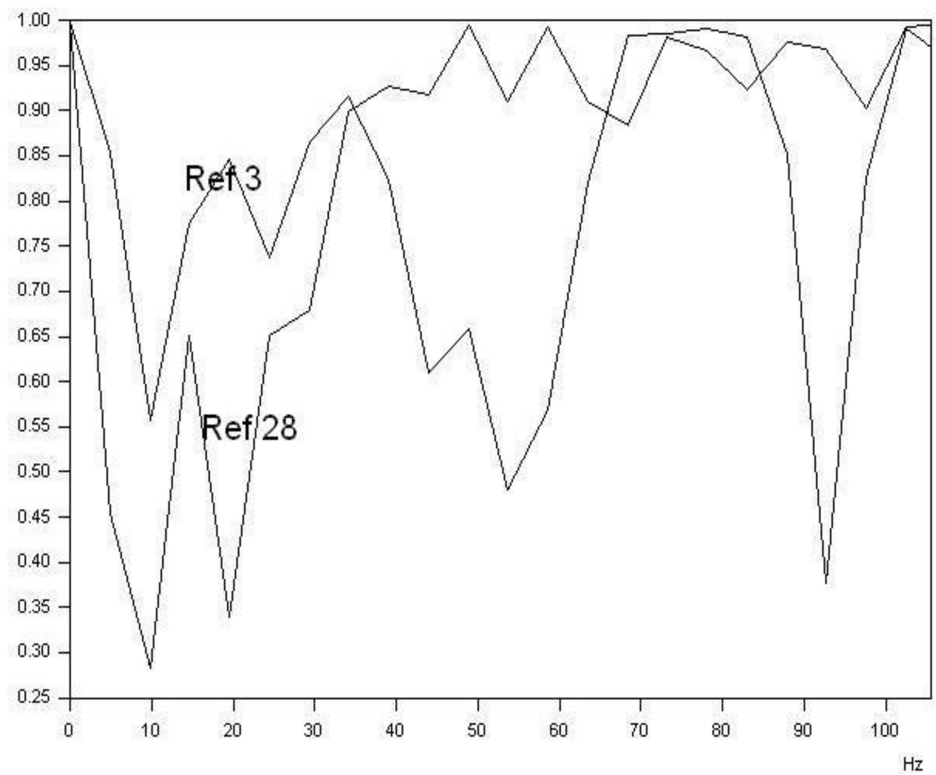

Figure 4. Coherence graph of channel 1 and 2 based on references 3 and 28.

The values between the calculated and measured static deflections are in the same value to each other. A static test checked the stiffness of the boundary and then shows the degree of error for any elastic deformation of the frame is probably zero. The result verifies the frame is symmetric. Test with different pattern and intensities of static loading in order to compare the experimental and theoretical values of the static deformation showed that the deflection calculated by the proposed nonlinear method gives reasonably accurate results.

Modal test

The objectives of the modal testing described are to verify the dynamic proposed theory. The modal analysis is defined as the process of characterizing the dynamics of a structure in terms of its modes of vibration. It turns out that the eigenvalues and eigenvectors which define the resonant frequencies and mode shapes of the modes of vibration of the structure. The structure is excited by impact hammer on nodes 3 and 28. Two nodes were selected for more evolution of received data. Figure 4 shows the visual comparison of coherence graph based on references 3 and 28. The signals output came out as reference 3, and were suitable because the ratios of amounts based on reference 3 were higher than the ratio amounts of reference 28 and close to the amount one. It means that, the received signals have linear behavior and it is sufficient to analyze modal.

Optimization of modal parameters

Scrutinizing of frequency response function is needed to apply a new technique. In present study, the newest method such as Nyquist plot for optimization modal parameter was used. Figure 5 shows sprawling frequencies on reference 28, which indicated the stability of the system is not enough and an unsuitable ratio amplitude frequency exist between output and input signal. Hence, node 3 was selected for reference. Figures 6 and 7 show that, the whole theoretical and experimental modes shapes were close to each other and verify the proposed theory. The net had to be excited five times for each setup to reach a suitable response for each of nodes.

The comparisons between theoretical and experimental natural frequencies are presented in Table 4.

Conclusion

The values between the calculated and measured static deflections are in good agreement. The comparison of experimental and theoretically predicted values of

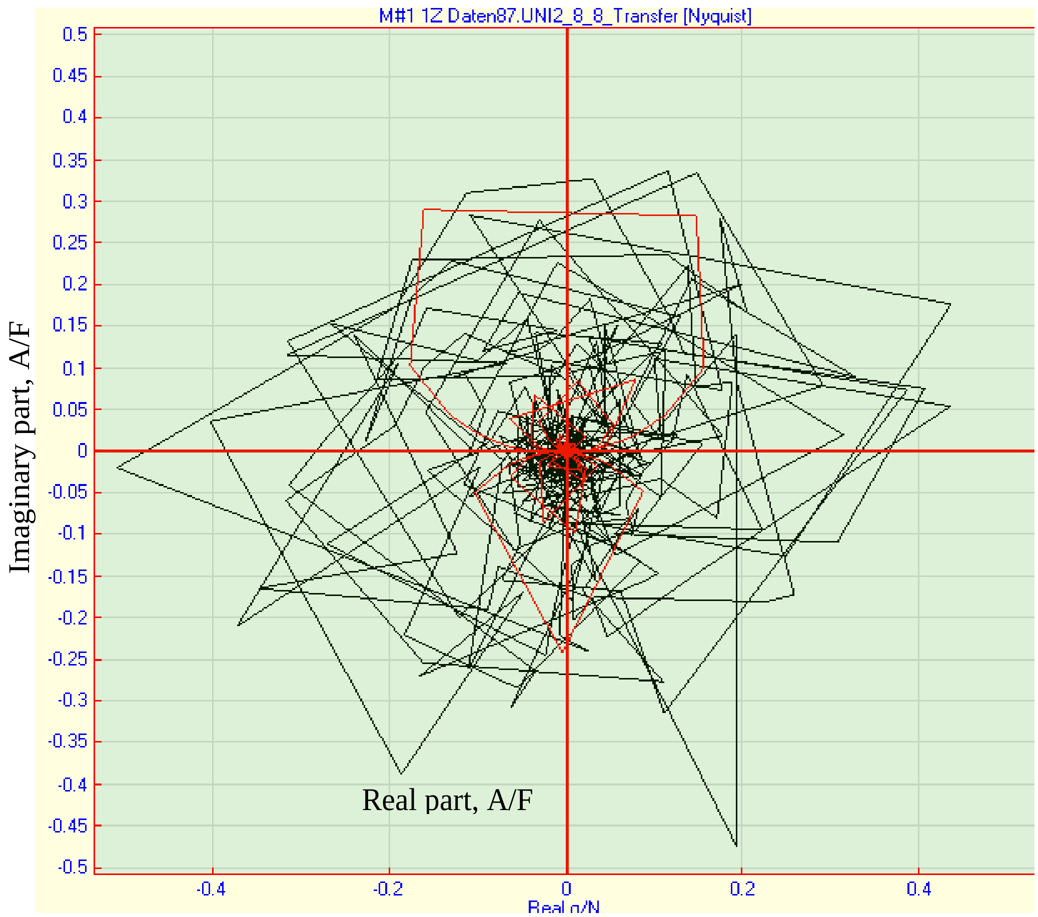

Figure 5. Nyquist diagram of an FRF for node 1 based on reference 28.

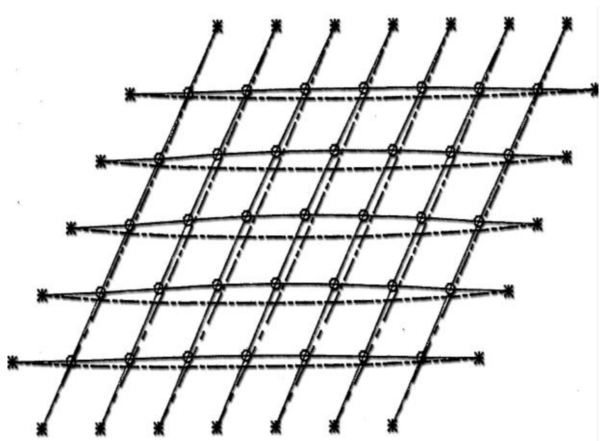

Figure 6. Mode shape 1 of the structure (theoretical).

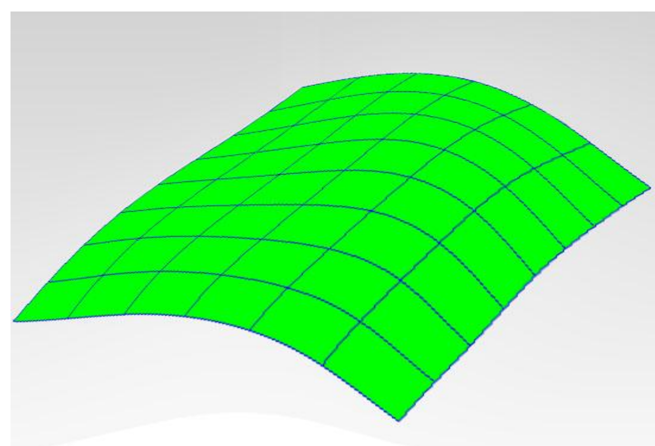

Figure 7. Mode shape 1 of the structure (experimental).

dynamic response shows that the response calculated by the proposed nonlinear r method gives reasonably accurate results. The proposed method was found to be stable for time steps equal to or less than half the smallest time period of the system. The experimental work carried out by static and dynamic testing of the flat net showed good agreement between the experimental result and theoretically predicted values. The percentage

Table 4. Theoretical and experimental natural frequencies.

Pretension load (N) = 11500	**Natural frequencies (Hz)**		
	References 3		
	ω_T **Theoretical**	ω_E **Experimental**	$\frac{\omega_E - \omega_T}{\omega_E}$ %
Mode 1	2.9708	2.9531	0.60
Mode 2	6.4601	6.4142	0.71
Mode 3	8.0102	7.8945	1.44
Mode 4	9.1213	9.1023	0.21
Mode 5	14.243	14.553	2.18
Mode 6	17.347	17.235	0.65
Mode 7	23.762	23.151	2.57

differences between the theoretical and experimental results did not in any case exceed 10%. This was thought to be acceptable. Finally, it can be concluded that, the Fletcher-Reeves algorithm was more efficient in terms of computing time and storage practically in high nonlinear structures.

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