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Full Length Research Paper

Shell model calculations on even nuclei near ^{208}Pb

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Binding energy of the ground state, energy levels and the B(E2) values of both positive and negative parities for ^{202,204}Au, $^{202-206}$Hg, $^{202-206}$Tl and $^{202-206}$Pb isotopes have been calculated through shell model calculations using the shell model code OXBASH for Windows employing the Modified Kuo-Herling interaction (khhe) for neutron and proton hole orbits in ^{208}Pb. The binding energy calculations were in good agreement with experimental data. The predicted low-lying levels (energies, spins and parities) and B(E2) values results were reasonably consistent with the available experimental data. Truncation model space was applied on the ^{202}Au isotope, where *πg*7/2 and *νh9/2* kept filling as well as *νh9/2* for ^{202}Hg and ^{202}Tl.

Key words: Shell model, B(E2) value, binding energy, OXBASH.

INTRODUCTION

The study of low-lying excited states of closed shell and near-closed shell provide information about specific nuclear orbital nucleus (Jensen, 1957). This is because a few nuclear orbits dominate the contribution to their wave function. In the case of the ^{208}Pb region, the experimental and theoretical information available on neutron-rich species is relatively limited. For example, previous studies of low-lying states for nuclei near the ^{208}Pb region were scattered and incomplete, although ^{206}Tl was studied by Ma and True (1973), and even-mass Hg isotopes in (Covello and Sartoris, 1967). The nucleon-pair approximations (NPA) were applied to the low-lying states of Ir, Pt, Au, Hg and Tl isotope (Hui and Yu Min, 2011; Jiang et al., 2011). Experimental studies on this region have been limited to measured excited states in ^{206}Hg (Becker et al., 1982; Maier et al., 1984; Fornal et al., 2001). The new levels and lifetime were measured in ^{204}Tl (Fotiades et al., 2008). The isomeric states were observed in heavy neutron-rich nuclei populated in the fragmentation of a ^{208}Pb beam (Steer et al., 2011).

The purpose of this study is to apply the shell model and to use Modified Kuo-Herling (khhe) interaction for neutron and proton hole orbits in ^{208}Pb energy levels and the B(E2) values of even A (Au, Hg, Tl and Pb) isotopes with protons ranging from 79 to 82 and neutron numbers from 120 to 126, a total of eleven nuclei. Concerning the valence proton holes and neutron holes with respect to ^{208}Pb, a double closed shell nucleus is used in this study to construct the shell model configurations of the nuclei of interest. The purpose of this paper is to concentrate on the extent to which these nuclei describe binding energies, low-lying levels schemes and B(E2) values for even A; Au, Hg, Tl and Pb isotopes.

THEORETICAL SURVEY

Calculations have been carried out using the OXBASH code for Windows (http://www.nscl.msu.edu/~brown/) on the nuclei near of ^{208}Pb. The code uses an m-scheme Slater determinant. Using projection techniques, wave functions with good angular momentum J and isospin T are constructed. The jj56pn model space is comprised of (1g7/2, 2d5/2, 2d3/2, 3s1/2 and 1h11/2) below the Z = 82 closed shell for proton holes and (1h9/2, 2f7/2,

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Table 1. Experimental and calculated binding energies of even A isotopes.

Z	*N*	Nucleus	*B*(exp.) MeV*	σMeV	*B*(Cal.) MeV	σ*B* MeV
82	120	^{202}Pb	1592.188	0.008	1592.319	-0.131
	122	^{204}Pb	1607.507	0.001	1607.564	-0.057
	124	^{206}Pb	1622.325	0.001	1622.322	0.003
81	121	^{202}Tl	1593.020	0.015	1590.349	2.671
	123	^{204}Tl	1607.525	0.001	1605.977	1.547
	125	^{206}Tl	1621.575	0.001	1621.572	0.003
80	122	^{202}Hg	1595.165	0.001	1593.868	1.297
	124	^{204}Hg	1608.652	0.000	1608.622	0.030
	126	^{206}Hg	1621.050	0.020	1621.049	0.001
79	123	^{202}Au	1593.001	0.166	1591.461	1.539
	125	^{204}Au	1605.494	-0.200	1604.459	1.035

*Wapstra et al. (2003); Audi et al. (2003) http://www.nndc.bnl.gov/masses/mass.mas03).

2f5/2, 3p3/2, 3p1/2 and 1i13/2) below the closed N = 126 shell for neutron holes. Truncation model space is applied on the 202Au nuclei, where πg7/2 and νh9/2 kept filling as well as νh9/2 for ^{202}Hg and ^{202}Tl nuclei. Based upon the energy levels observed near the ^{208}Pb region (Rydstrom et al., 1990) the proton single-particle energies are +11.483, +9.696, +8.364, +8.013 and +9.361 for the 1g7/2, 2d5/2, 2d3/2, 3s1/2 and 1h11/2, respectively. The neutron single-particle energies are +10.781, +9.708, +7.938, +8.266, +7.368 and +9.001for the 1h9/2, 2f7/2, 2f5/2, 3p3/2, 3p1/2and 1i13/2 orbitals, respectively. The two-body interaction matrix elements (TBMEs) are formed (Rydstrom et al., 1990). Extended modifications of the Kuo-Herling interaction were applied to all nuclei near the ^{208}Pb region, and these modifications were explained by Steer et al. (2011). In addition we used the harmonic oscillator potential (HO, x), x<0.

RESULTS AND DISCUSSION

This study presented the calculated results of low-lying states of the even A Au, Hg, Tl and Pb isotopes, as well as protons ranging from 79 to 82, with neutron numbers ranging from 120 to 126. The results include binding energies with respect to ^{208}Pb, energy levels and the B(E2) values.

Binding energy

Binding energies are important to nuclear astrophysicists when determining Q-values of proton capture reactions and beta decays (Herndl and Brown, 1997). The binding energies of nuclei near the ^{208}Pb region using the effective interaction KHHE have been calculated by using the shell model OXBASH code. Binding energy *B* is defined by:

$$B=B(^{208}\mathrm{Pb}) - \langle H \rangle$$

The experimental binding energy of ^{208}Pb ($B(^{208}$Pb)) was taken to be 1636.430 (0.001) MeV (Wapstra et al., 2003; Audi et al., 2003; http://www.nndc.bnl.gov/masses/mass.mas03). The experimental and theoretical binding energies with errors, $\delta B=B$(exp.)-B(Cal.), are presented in Table 1. It can be seen that the experimental binding energies were reproduced satisfactorily. The root mean square deviation of eleven masses was 1.55 MeV. The shell model calculations of binding energies for these nuclei were not fitted with experimental data.

Energy levels

The objective of this study is to calculate the nuclei that lie near ^{208}Pb as these nuclei are of great importance in recent applications in astrophysics. The calculated energy levels and experimental results of low-lying states presented in Figures 1 and 2 correspond to even-even and odd-odd nuclei, respectively. Our calculations were plotted on the left and experimental data on the right for any band. Levels with '( )' correspond to cases for which the spin and/or parity of the corresponding states are not well established experimentally. The experimental data was taken from (http://www.nndc.bnl.gov/ensdf/) for all nuclei, as well as ^{204}Au nuclei which was taken from Lopez (2011). Figure 1 shows the comparison of the experimental energy levels of even-even nuclei with the calculated values from Modified Kuo-Herling interaction. This interaction gives good agreement with the experimental values. In Figure 1, it can be seen that the calculated energy levels reproduce the energies of the

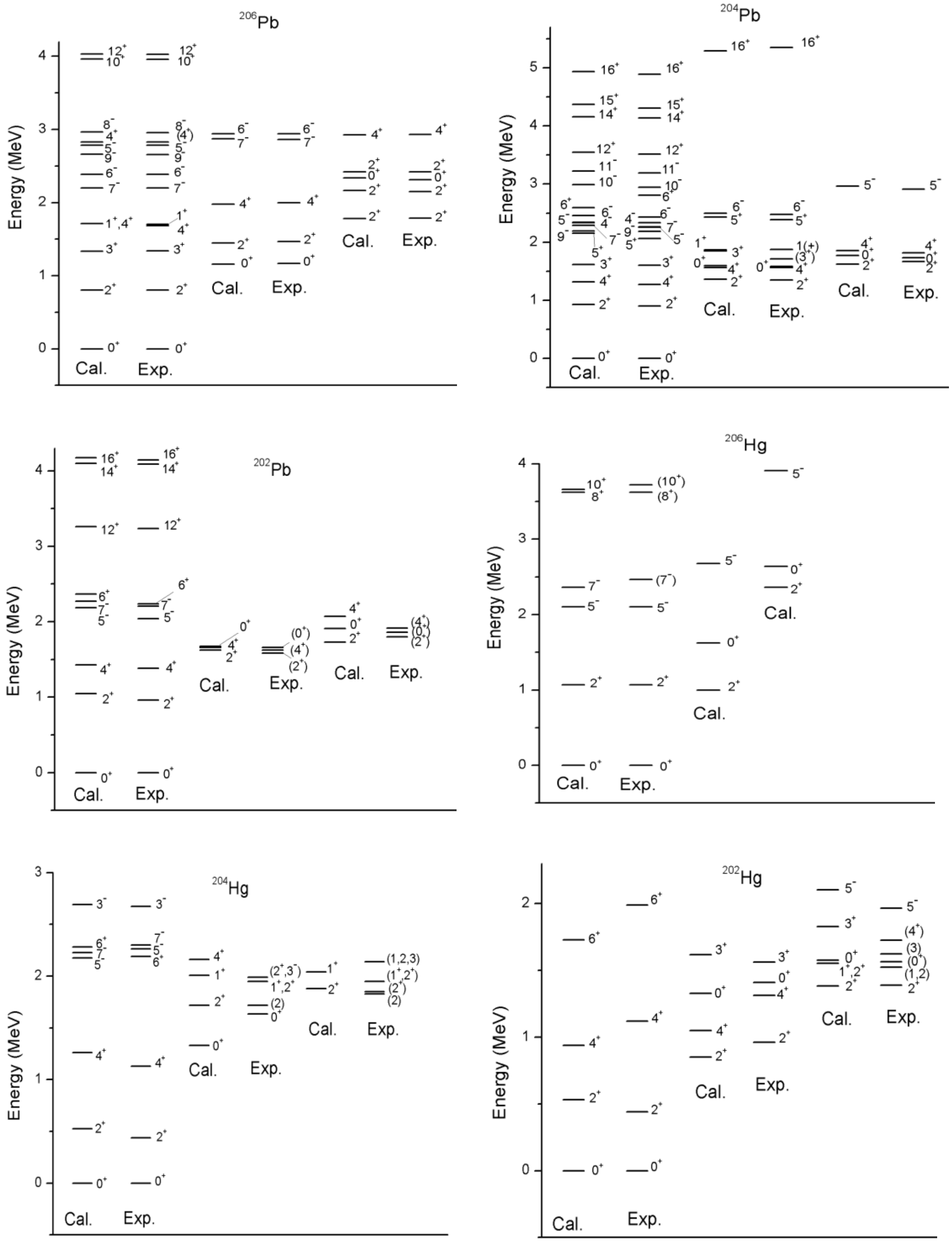

Figure 1. Comparison of calculated spectra with experimental ones for even-even nuclei.

yeast levels for all nuclei very well except 4_1^+ and 6_1^+ states in the ^{202}Hg nuclei where the calculated levels were higher than the experimental data because the truncation model space was used in this calculation. For non-yeast levels, the calculated energy levels of all even-even nuclei were in good agreement with the experimental results. In Figure 2, comparisons were made using the effective interaction for odd-odd (even-A mass number) nuclei. From this figure the same conclusion was drawn as that in Figure 1 for yeast and non-yeast levels. We can notice from Figure 2 that the calculated energies of up to the state 2_1^- for ^{204}Tl, ^{202}Tl and ^{204}Au isotopes were considerably lower than the experimental data. For non-yeast levels, the calculated energy levels of all even-A

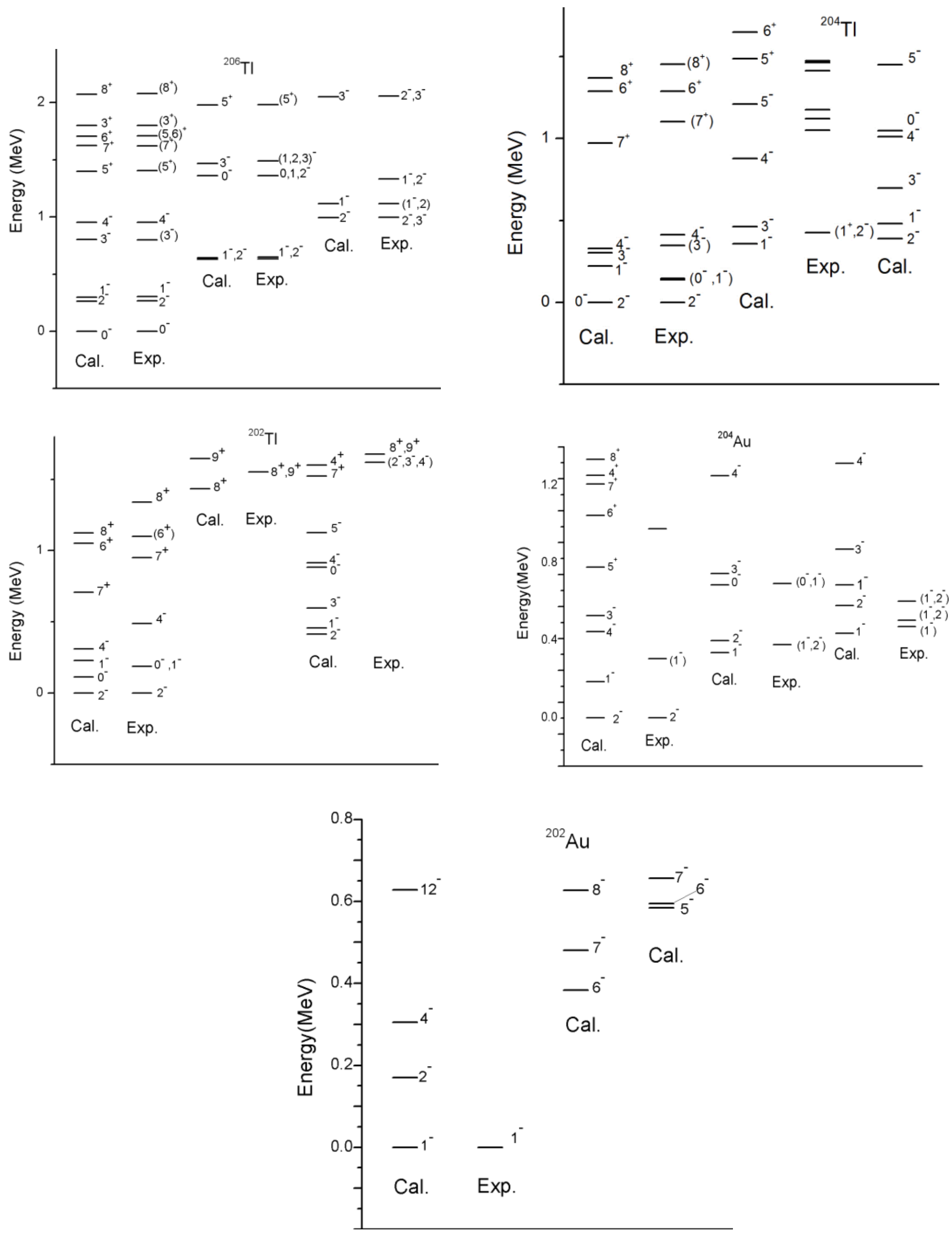

Figure 2. Comparison of calculated spectra with experimental ones for odd-odd nuclei.

nuclei were consistent with experimental results.

The B(E2) values

Transition rates represent a sensitive test for most modern effective interactions that have been developed to describe the region near ^{208}Pb. Transition strengths were calculated in this study using the harmonic oscillator potential (HO, x), where x<0 for each in-band transition by assuming pure *E2* transition. In this section, the calculated results of the B(E2) have been presented and the calculation of B(E2) values have been compared with experimental data (Zhu and Kondev, 2008; Chiara and Kondev, 2010; Kondev, 2008) as shown in Tables 2 and 3 for the (proton number-neutron number) even-even and odd-odd nuclei, respectively. In general, with the exception of a small number of nuclei, most of the

Table 2. *B*(*E*2) values for even-even nuclei (in W.u.).

Values	Calculated	Experiment *	Calculated	Experiment *	Calculated	Experiment*
	^{206}Pb		^{204}Pb		^{202}Pb	
$2_1^+ \rightarrow 0_1^+$	4.76	2.80(9)	7.70	4.69(5)	1.060	> 0.098
$4_1^+ \rightarrow 2_1^+$	0.459		0.0018	0.00382(9)	0.034	
$6_1^+ \rightarrow 4_1^+$	0.263		0.142		0.170	
	^{206}Hg		^{204}Hg		^{202}Hg	
$2_1^+ \rightarrow 0_1^+$	4.392		8.344	11.95(8)	5.383	17.3(2)
$4_1^+ \rightarrow 2_1^+$	4.434		10.498	17.0(12)	8.310	26.7(9)
$6_1^+ \rightarrow 4_1^+$	0.215		11.301	19(3)	5.407	25(2)

* (Zhu and Kondev, 2008; Chiara and Kondev, 2010; Kondev, 2008).

Table 3. *B*(*E*2) values for odd-odd nuclei (in W.u.)

Values	Calculated	Experiment*	Calculated	Experiment*	Calculated	Experiment
	^{206}Tl		^{204}Tl		^{202}Tl	
$2_1^- \rightarrow 0_1^-$	0.915	2.21(20)	0.123		0.820	
$2_2^- \rightarrow 0_1^-$	0.120	0.13(4)	0.623		0.220	
$4_1^- \rightarrow 2_1^-$	1.540	1.2(3)	2.429		2.565	
	^{204}Au		^{202}Au			
$2_1^- \rightarrow 0_1^-$	0.599		0.75			
$2_2^- \rightarrow 0_1^-$	1.749		0.346			
$4_1^- \rightarrow 2_1^-$	2.573		2.951			

*****(Zhu and Kondev, 2008; Chiara and Kondev, 2010; Kondev, 2008).

calculated results were consistent with available experimental data. In Table 1, the calculated values of B(E2; $2_1^+ \rightarrow 0_1^+$, $4_1^+ \rightarrow 2_1^+$ and $6_1^+ \rightarrow 4_1^+$) for ^{202}Hg isotope were much lower than experimental data. The deviation in the case of ^{202}Hg isotope is because the truncation model space was used.

Conclusion

The present study illustrates the binding energy of the ground state, low excited energy levels and the reduced probability for *E*2 transitions, B(E2) values, with positive and negative parities for Au, Hg, Tl and Pb even isotopes. Good agreement was obtained by comparing these calculations with the recently available experimental data for binding energy, the level spectra and transition probabilities for nuclei near ^{208}Pb.

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