International Journal of the Physical Sciences Vol. 6(29), pp. 6741-6750, 16 November, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.338
ISSN 1992 - 1950 ©2011 Academic Journals

Full Length Research Paper

Adaptive feature extraction and mapping for mobile robots in unknown environment

Seyed Mohammad Mavaei

Young Researchers Club, Kermanshah Branch, Islamic Azad University, Kermanshah, Iran.
E-mail: mohammad.mavaei@gmail.com.

Accepted 12 October, 2011

This paper proposes a new approach for feature and line extraction use in mobile robot mapping with high accuracy, efficient speed and low complexity to determine line boundaries. This algorithm called modified split and merge (MSAM) is robust against measurement noises and demonstrates satisfactory results on different surfaces. The method is based on the least square method to fit a line to a series of uncertain points. Different least square criterion is investigated to choose the best one for line extraction. A novel approach is proposed here to adopt threshold base split and merge method on different surfaces. The approach is applied to NAJI2 rescue robot for simultaneously localizing and mapping the observation of its performance. The results of this study proved that it has a good real-time capability to integrate the information of laser scanner into the navigation algorithm of the mobile robot; however, the extracted lines are suitable for object base mapping approaches.

Key words: Line extraction, mapping, modified split and merge, feature extraction, fitting.

INTRODUCTION

One of the most important problems in robotics is mapping and localization. A precise and stable self localization is a key feature to act successfully in an unknown environment. Dead reckoning such as Odometry (wheel rotation count or IMU) may conventionally be used to estimate a robot position. Due to unbounded position error generated by the Odometry, it does not suffice alone for localization. A large number of experiments using various kinds of sensors has shown that range sensor based SLAM techniques using laser (Lu and Milios, 1994), sonar (Diosi and Kleeman, 2004; Diosi et al., 2005), and vision (Yeon et al., 2006) work well in a real environment for both indoor (Cox and Blanche, 1991) and outdoor applications (Lingemann et al., 2004). A possible way to enhance localization is to use laser scan matching. Compared to other sensors, laser scanners have unique advantages such as: dense and accurate range measurement, high sampling rate, excessive angular resolution, as well as good range and distance resolution. In laser scan matching, the position and orientation or pose of the current scan is sought with respect to a reference laser scan. The pose of the current scan is adjusted until the best overlap with the reference scan is achieved. Laser scan matching methods are categorized based on their association: point to point and feature to feature. The point to point matching approach (Lu and Milios, 1994; Diosi and Kleeman, 2005; Bevington and Robinson, 1992), is to approximate the alignment of two consecutive scans, and then iteratively improve the alignment by defining and minimizing a distance between the scans. Moreover, it does not require the environment to be structured or contain predefined features. In the feature to feature matching approach, instead of working directly with raw scan points, the raw scans are transformed into geometric features. These extracted features are used in matching at the next step. Such approaches interpret laser scans and require the presence of chosen features in the environment. Features such as line segments (Pfister et al., 2003; Mavaei et al., 2011), corners (Altermatt et al., 2004) or range extrema (Lingemann et al., 2004) are extracted from laser scans, and matched. Features require less memory space while providing rich and accurate information. Algorithms based on parameterized geometric features are expected to be more efficient compared to the point-based algorithms. Several algorithms have been proposed for extracting line segments from 2D range data. Since the algorithms do not incorporate noises of the range data, the fitted lines do not have a sound statistical interpretation. Nguyen et al. (2005) presents an experimental evaluation of different line extraction algorithms on 2D laser scans for indoor environment. Diosi et al. (2003) consider line fitting systematic errors as they mainly depend on a specific hardware and testing environment. Pfister et al. (2003) suggest a line extraction algorithm using weighted line fitting for line based map building. Pavlidis and Horowitz (1974) proposed a split and merge algorithm for the line extraction which is extracted from computer vision. This method is very popular and has been used by others. Split-and-Merge is clearly the best choice for real-time applications, due to its superior speed. It is also the first choice for localization problems with a priori map, where *FalsePos* is not very important. However, the quality of the split and merge method is not guaranteed in all applications. For example, in line based SLAM, bad feature extraction may lead to the system divergence. This paper introduces an Adaptive Line Fitting Algorithm (ALFA) to create line-based maps using a series of range data collected from multiple poses. ALFA is a modified version of the split and merge method with increased quality and robustness in application where Split-and-Merge fails to function.

Sensor noise model

Range sensors are subjected to both random noises and bias (Diosi and Kleeman, 2003). Equation (1) describes the polar representation of scanned data. Let the range measurement, *d* be comprised of the "true" range, *D,* and an additive noise term, ε_d Equation (2):

$$u_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix} = d_i \begin{bmatrix} \cos\varphi_i \\ \sin\varphi_i \end{bmatrix} \quad (1)$$

$$d_i = D_i + \varepsilon_{di} \quad (2)$$

ε_d is assumed to be a zero-mean Gaussian random variable with variance σ_d^2. In a similar way, (3) represents the measurement error of angleφ_i.

$$\varphi_i = \emptyset_i + \varepsilon_{\varphi i} \quad (3)$$

where $\emptyset_i$ is the "true" angle of the *ith* direction, and ε_φ is again a zero-mean Gaussian random variable with variance σ_d^2 . Hence:

$$d_i = (D_i + \varepsilon_{di}) \begin{bmatrix} \cos(\varphi_i + \varepsilon_{\varphi i}) \\ \sin(\varphi_i + \varepsilon_{\varphi i}) \end{bmatrix} \quad (4)$$

Generally, one can think of the scan point u_i as the sum of the true component, U_i, and the uncertain component, δu_i:

$$u_i = U_i + \delta u_i \quad (5)$$

if max{ε_φ , ε_d }$\ll$ 1 , which is a valid for most laser scanners, by replacing the values of u_i and U_i form (4) into (5), it can be written in the form of (6):

$$\delta u_i = D_i \varepsilon_{\varphi i} \begin{bmatrix} -\sin(\emptyset_i) \\ \cos(\emptyset_i) \end{bmatrix} + \varepsilon_{di} \begin{bmatrix} \cos(\emptyset_i) \\ \sin(\emptyset_i) \end{bmatrix} \quad (6)$$

Assuming ε_φ and ε_d are independent, the covariance of the range measurement data is:

$$Q_i \triangleq E[\,\delta u_i (\delta u_i)^T] = \frac{(D_i)^2 \sigma_\varphi^2}{2} \begin{bmatrix} 2\sin^2(\emptyset_i) & -\sin 2\emptyset_i \\ -\sin 2\emptyset_i & 2\cos^2(\emptyset_i) \end{bmatrix} \frac{\sigma_d^2}{2} \begin{bmatrix} 2\cos^2(\emptyset_i) & \sin 2\emptyset_i \\ \sin 2\emptyset_i & 2\sin^2(\emptyset_i) \end{bmatrix} \quad (7)$$

For practical purposes, φ_i and d_i are good estimates of the Quantities $\emptyset_i$and D_i (Pfister et al., 2003). Equation 7 describes the impact of noise on data distortion.

Adaptive line extracting

Smoothing data to increase the algorithm efficiency

To increase the algorithm, efficiency data are split into segments. The segmentation is based on the continuity of the distance data acquired from laser. Each segment is smoothed and fed into ALFA. If laser scanner data contain outliers, the smoothed values might be distorted, and lose to reflect the behavior of the bulk of the neighboring data points. To overcome this problem, the data can be smoothed using a robust procedure which is not influenced by a small number of outliers (Figure 1). "Lowess" and "loess" methods are two good candidates to handle this type of smoothing. The terms "lowess" and "loess" are derived from "locally weighted scatter plot smooth." Since each smoothed value is determined by adjacent data points and their assigned regression weight function, in the defined span, the methods are considered to be both local and weighted. In addition, it is possible to use a robust weighted function to make the smoothing process resistant to the outliers. These two methods differ in their regression type: Lowess utilizes a linear polynomial, while loess employs a quadratic polynomial. ALFA takes advantage of lowess smoothing method. The robust "lowess" smoothing process follows these steps for each data point:

1. Compute the regression weights for each data point in the span. The weights are given by the tri-cube function represented by (8).

$$w_{i1} = \left[1 - \left|\frac{x - x_i}{d(x)}\right|^3\right]^3 \quad (8)$$

x is the predictor value associated with the response value to be smoothed, *xi* is the nearest neighbors of *x* as defined by the span, and *d*(*x*) is the distance along the

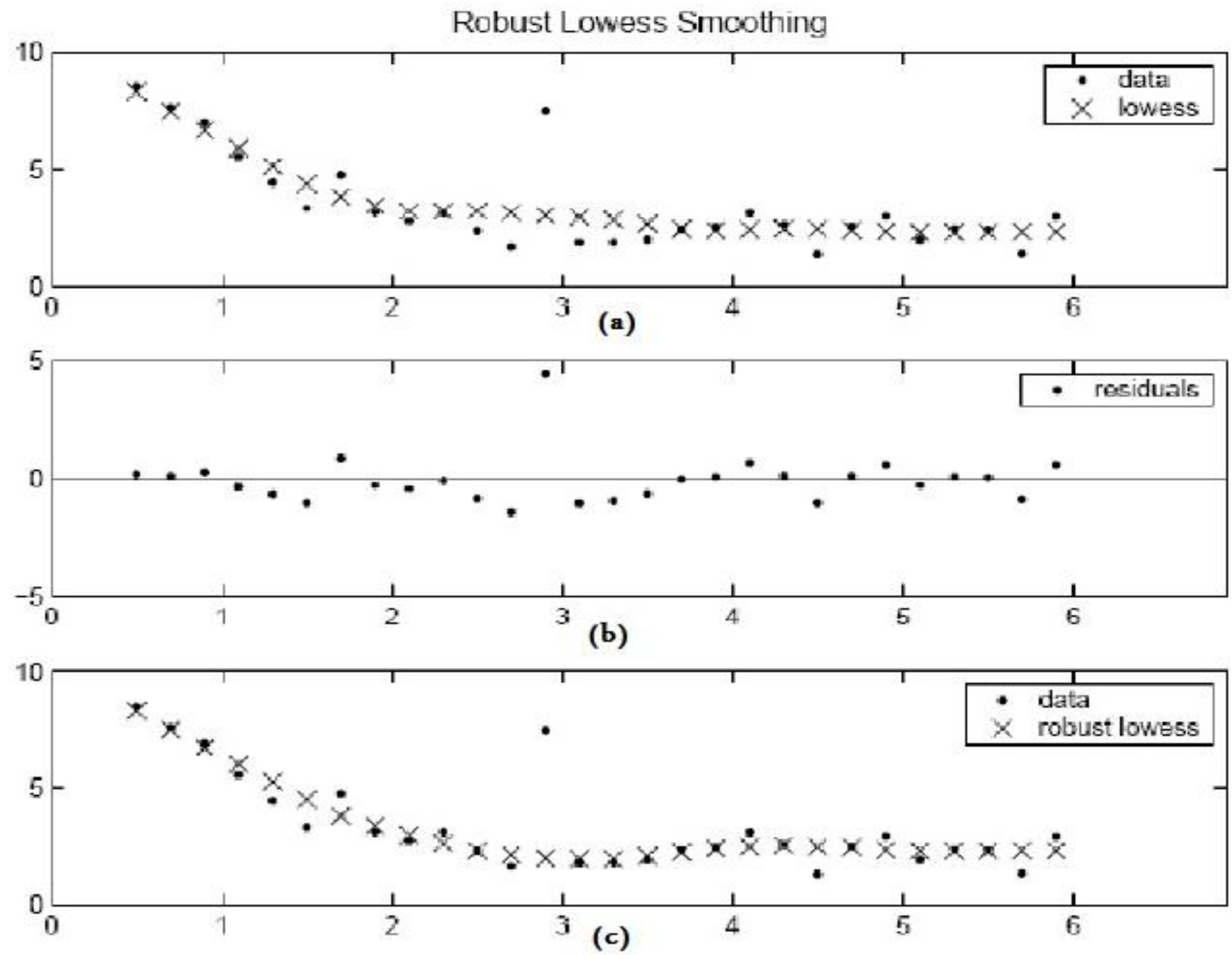


Figure 1. (a) Plot of the outlier influencing the smoothed value for several nearest neighbors. (b) Plot suggesting that the residual of the outlier is greater than six median absolute deviations. Therefore, the robust weight is zero for this data point. (c) Plot showing the smoothed values neighboring.

abscissa from *x* to the most distant predictor value within the span.

2. Calculate the robust weights for each data point in the span. The weights are given by the bisquare function.

$$w_{i2} = \begin{Bmatrix} \left(1-\left(\frac{r_i}{6M}\right)^2\right)^2 & |r_i < 6M| \\ 0 & |r_i \geq 6M| \end{Bmatrix} \quad (9)$$

r_i is the residual of the *ith* data point produced by the regression smoothing procedure, and *M* is the median absolute deviation of the residuals.

3. The final smoothed value is calculated using both the local regression weight and the robust weight.

$$w_i = w_{i1} + w_{i2} \quad (10)$$

A weighted linear least squares regression is performed. The regression employs a first degree polynomial for lowness. For more information about this regression and its notation, you can refer to MATLAB Curve Fitting Toolbox user guide.

Split and merge with binary search

Split and merge method has a better performance from speed point of view (Nguyen et al., 2005) and therefore a suitable choice for real-time localization or SLAM applications. The novel approach (ALFA), proposed in this paper, is based on the split and merges procedure with a higher accuracy. Furthermore, the least square criterion is used instead of maximum distance between point data

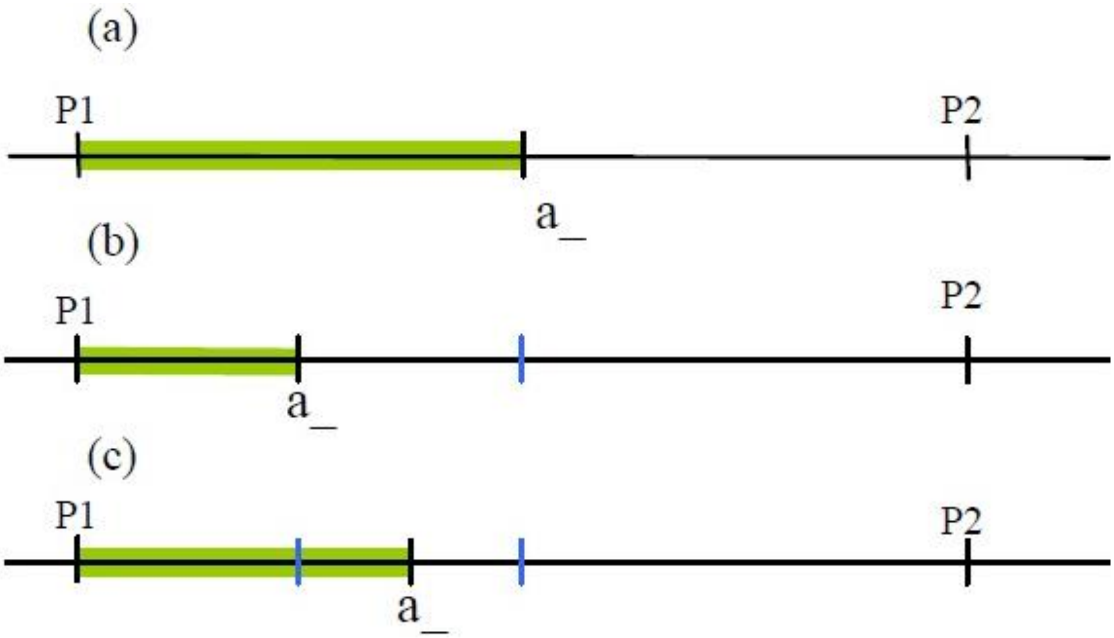


Figure 2. (a) Fitting is not satisfactory, split. (b) Fitting is satisfactory, add a split. (c) Check if fitting is satisfactory.

by fitted line (Borges and Aldon, 2004) to evaluate fitting. When a line is fitted, the only decision the algorithm makes is whether the fitting is proper or not. Hence the idea of binary search is used to obtain the estimated line with the maximum length and precision. Figure 2 illustrates three states of the proposed approach. This leads to the fact that split procedures and merges procedures are simultaneous which are in contradiction with the usual approach. Consequently, the line boundaries can be identified with more precision.

Least square line fitting criterion

To obtain the coefficients' estimates, the least squares method minimizes the summed square of residuals (12). The residual for the *ith* data point r_i is defined as the difference between the observed response value y_i and the fitted response value $\hat{y}_i$, and is identified as the error associated with the data. In order to evaluate parameter estimation performance, a different criterion might be required. Usually a threshold is needed when the least square method is used for line segmentation. The threshold assigns whether a line can be fitted above this number of points or not. To the best of our knowledge, the benefit of least square criterion to select the line segmentation threshold is not addressed in any research. Several least square criteria of a straight wall from different ranges and views (Figure 3) have been measured (Figure 4). The best criterion must have similar values for identical bodies in different ranges and views, since all measured criterions belong to the same 20 cm wall. Let (11) be the fitted line equation to x_i and y_i data:

$$y = p_1 x + p_2 \tag{11}$$

The least square method determines *p* parameter for minimizing SSE:

$$SSE = \sum_{i=1}^{n} (y_i - (p_1 x_i + p_2))^2 \tag{12}$$

By solving least square, the *p* parameter is obtained in Equation (15):

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \\ x_n & 1 \end{bmatrix} \times \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \tag{13}$$

$$A \triangleq \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \\ x_n & 1 \end{bmatrix} \tag{14}$$

$$p = (A^T A)^{-1} A^T y \tag{15}$$

$$\hat{y} = AP = Hy \tag{16}$$

$$H \triangleq A(A^T A)^{-1} A^T \tag{17}$$

Sum of square error (SSE), regression mean square error (RMSE), mean absolute error (MAE) and R-square criterion in the order, is defined by Equations (18), (21),

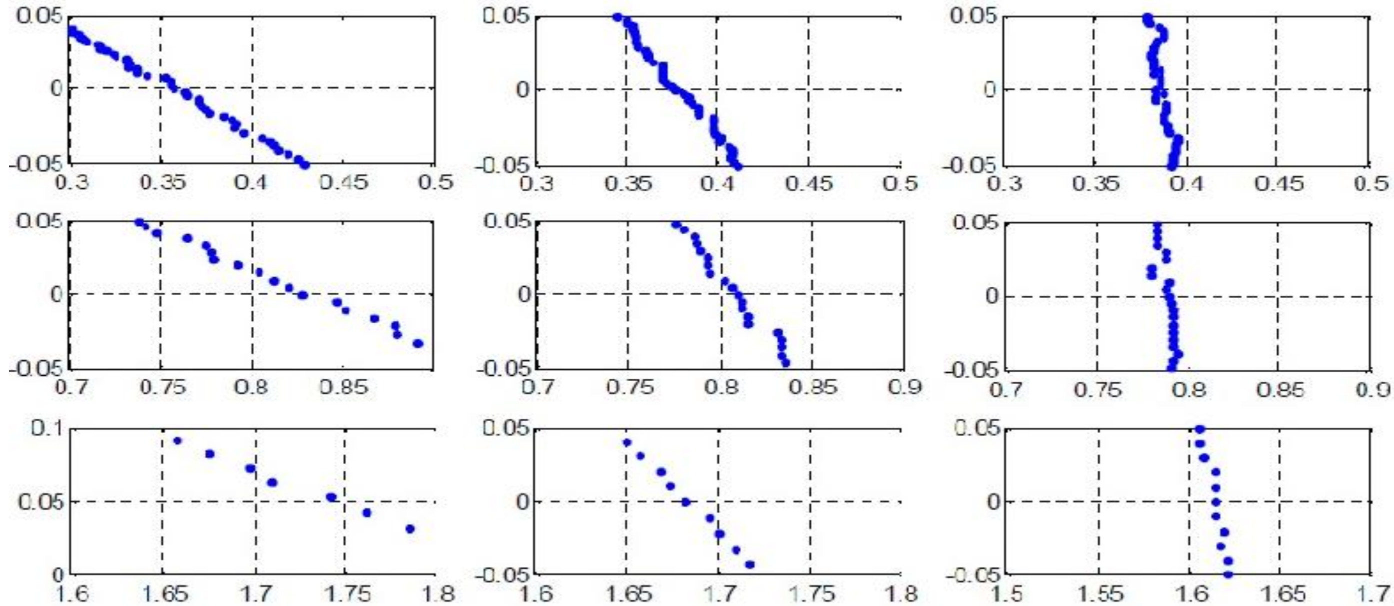


Figure 3. Sample of raw data in Cartesian coordinate for different ranges and views angles.

(22), and (24):

$$SSE = \sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \quad (18)$$

$$r = y - \hat{y} = (1 - H)y \quad (19)$$

$$MSE = \frac{SSE}{n} \quad (20)$$

where *MSE* is the mean square error or the residual mean square.

$$RMSE = s = \sqrt{MSE} \quad (21)$$

$$MAE = \frac{\sqrt{SSE}}{n} \quad (22)$$

$$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2 \quad (23)$$

$$R^2 = 1 - \frac{SSE}{SST} \quad (24)$$

If the fitted line perfectly matches the wall:

$$\mathrm{Y_i} = \mathrm{p_1 X_i} + \mathrm{p}_{2(} \quad (25)$$

$$\mathrm{u_i} = \begin{bmatrix} \mathrm{x_i} \\ \mathrm{y_i} \end{bmatrix} \cong \begin{bmatrix} \mathrm{X_i} \\ \mathrm{Y_i} \end{bmatrix} + \delta \mathrm{u_i} \quad (26)$$

As previously explained, the MSE can be estimated by sensor noise parameter for a straight line.

$$M\dot{S}E = E[(y_i - p_1 x_i - p_2)^2] = E[(\delta y_i - p_i \delta x_i)^2] \quad (27)$$
$$M\dot{S}E = E[(\delta y_i)^2] + {p_1}^2 E[(\delta x_i)^2] - 2p_1 E[\delta x_i \delta y_i]$$

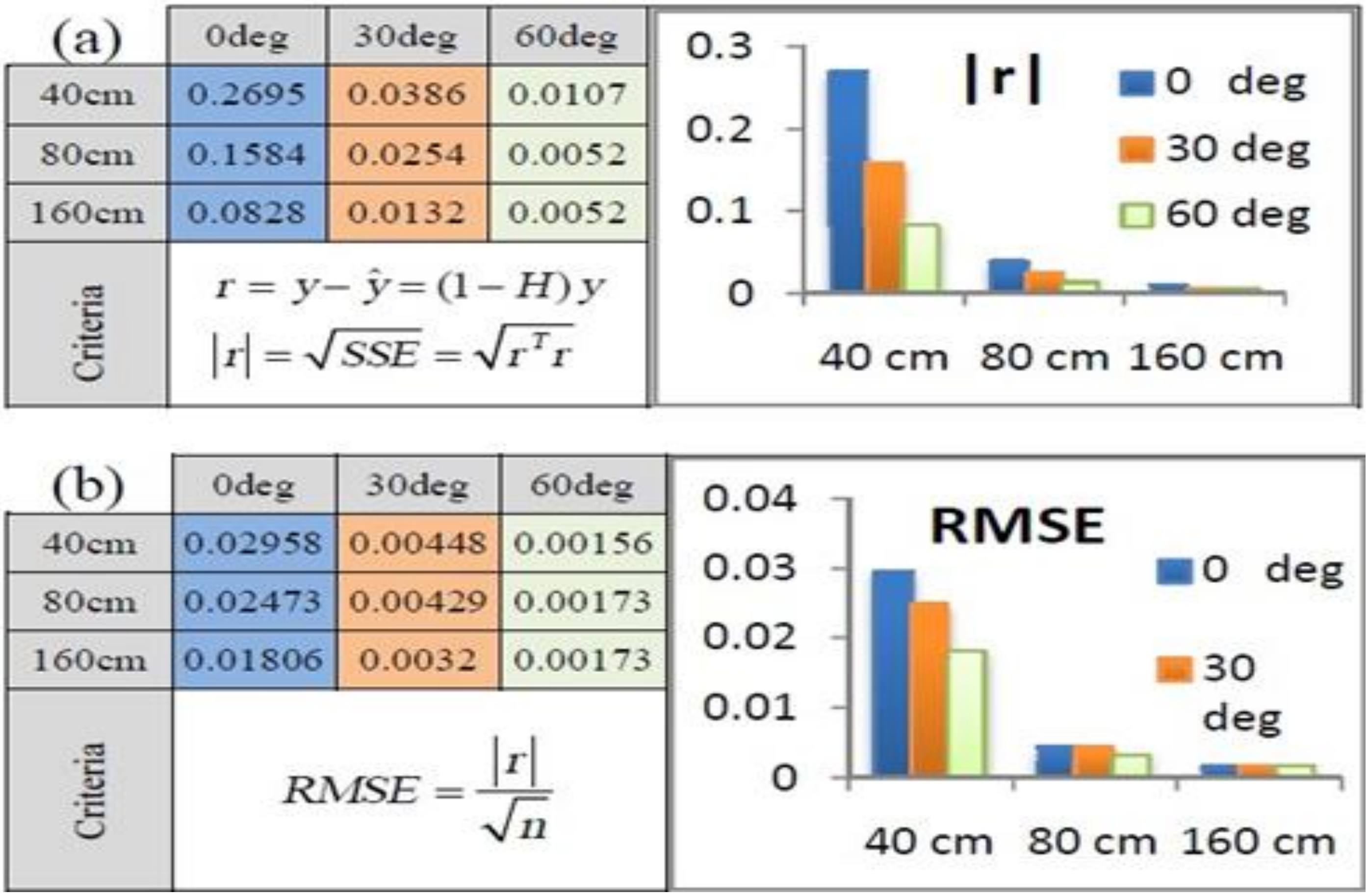
$$\begin{aligned} &= \left[(D_i)^2 \sigma_\varphi^2 \cos^2 \emptyset_i + \sigma_d^2 \sin^2 \emptyset_i\right] \\ &+ {p_1}^2 \left[(D_i)^2 \sigma_\varphi^2 \sin^2 \emptyset_i + \sigma_d^2 \cos^2 \emptyset_i\right] \\ &+ 2p_1 \left[(D_i)^2 \sigma_\varphi^2 \sin 2\,\emptyset_i - \sigma_d^2 \sin 2\,\emptyset_i\right] \end{aligned} \quad (28)$$

Using the relation between MSE and other least square criteria showed that it can be interpreted as stochastic variable, but the main goals are:

1. Quantity of criteria differs for several surfaces by different roughness.
2. Each of the criteria SSE, RMSE, MAE and R-square is a function of range and status of the wall relative to the laser and sensor noise model; furthermore all criteria are functions of the number of points except for RMSE.
3. Experimental results indicate comparative eminence for RMSE (Figure 4), since it has acceptable sensitivity and fast computation. Also, the difference in range and view for the same body has similar results.
4. According to the theoretical results, RMSE must increase when the range increases. However the experimental results are the other way around. The nonlinearity in sensor noise model could be the reason. Thereby the threshold cannot be determined via a static function of range or view angle.

Floating threshold

In split and merge methods, choosing threshold values is an important task since algorithm performances are very sensitive to the values used (Nguyen et al., 2005). A low threshold may break the line into two segment (Figure 5a),



(a)	0deg	30deg	60deg
40cm	0.2695	0.0386	0.0107
80cm	0.1584	0.0254	0.0052
160cm	0.0828	0.0132	0.0052
Criteria	$r = y - \hat{y} = (1-H)y$ $\lvert r\rvert = \sqrt{SSE} = \sqrt{r^T r}$		

(b)	0deg	30deg	60deg
40cm	0.02958	0.00448	0.00156
80cm	0.02473	0.00429	0.00173
160cm	0.01806	0.0032	0.00173
Criteria	$RMSE = \frac{\lvert r\rvert}{\sqrt{n}}$		

Figure 4. Absolute residual (a) RMSE (b) criterion for raw data which is depicted previously.

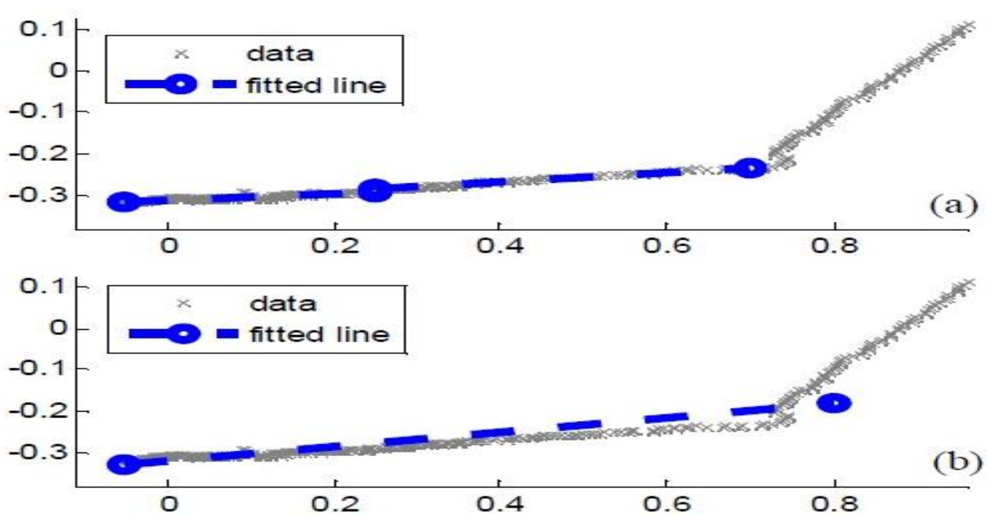


Figure 5. (a) Fit by low threshold; (b) Fit by high threshold.

and a high threshold could include the next line data (Figure 5b). It is expected for the RMSE to rise sharply at the refraction point of two lines. To capture this point, both the gradient and value of RMSE are used in ALFA.

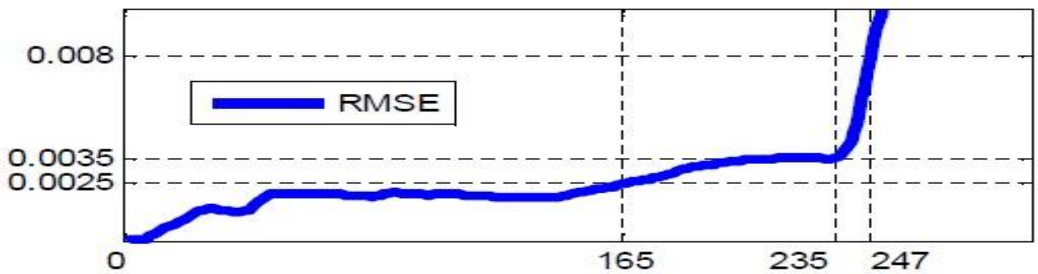


Figure 6. Plot of RMSE criterion change.

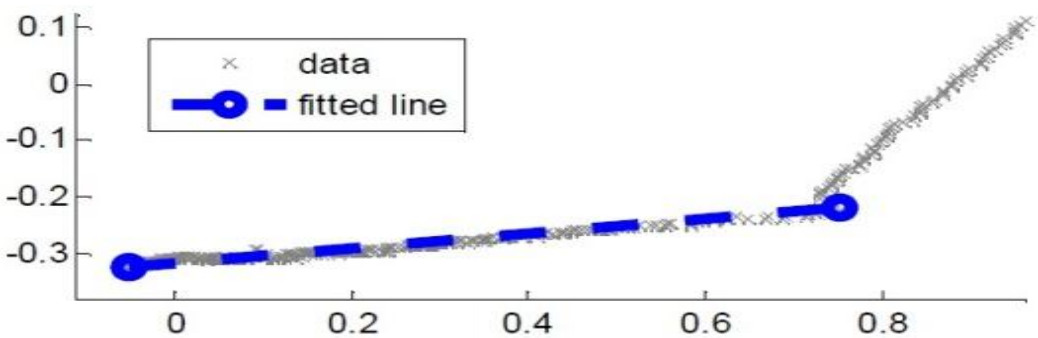


Figure 7. Extracted line by proposed approach in this paper.

Therefore a proposal is made to select threshold of least square criterion where the gradient of RMSE growth suddenly.

In this paper, to achieve this goal, suggest a floating method which dynamically changes threshold between a maximum and minimum value. This method which is based on binary search is described in following pseudo code. If the estimation of the RMSE gradient is low, the average of the previous and the current threshold would determine the next threshold. On the other hand, if the estimation of the RMSE gradient is high, it would decrease the threshold to the RMSE value in the current point.

It is clearly seen that by using the ALFA, the final threshold is 0.0037 and the fitted RMSE is 0.0035. The test results are obtained by defining the minimum threshold to 0.0025 (Figure 5a) and the maximum threshold to 0.008 (Figure 5b). Figures 6 and 7 illustrate the output.

Simultaneous localization and mapping (SLAM)

We are planning to perform simultaneous localization and mapping (SLAM) by fusing corners, edges and line segments which are measured by a laser range finder sensor. The scan matching algorithm computes a transformation Δd and a rotation Δ such that a set of features, extracted from the first scan, is mapped optimally to a feature set of the second scan. Human brain uses a simple method to adopt images. In this process, brain detects and compares corners between the two above images, and tries to find a proper match. By adopting one corner in each image, a rotation is used to increase the overlap between images. If the result is not satisfied, then it checks the next match in the same way. For example in Figure 8-C, the corner marked with a circle is similar to the corners marked with a square. Thus there are two matches for this corner. A comparison between the square marked flags shows that the right corner is a better match. A similar idea is used here to find the matches in different scans. In each laser scan, lines are extracted to identify the corners. The combination of these two features is used in a feature based SLAM. By comparing the poses and angles of a pair of features in two scans, the corresponding corner is identified. In the next step, a transformation is performed to find the maximum overlapping. At the end, the transformation is applied to the current image and is added to reference. Robot position is updated by the following formula.

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \\ \varphi_{n+1} \end{bmatrix} = \begin{bmatrix} x_n \\ y_n \\ \varphi_n \end{bmatrix} + \begin{bmatrix} \cos\varphi_n & \sin\varphi_n & 0 \\ -\sin\varphi_n & \cos\varphi_n & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\varphi \end{bmatrix} \tag{29}$$

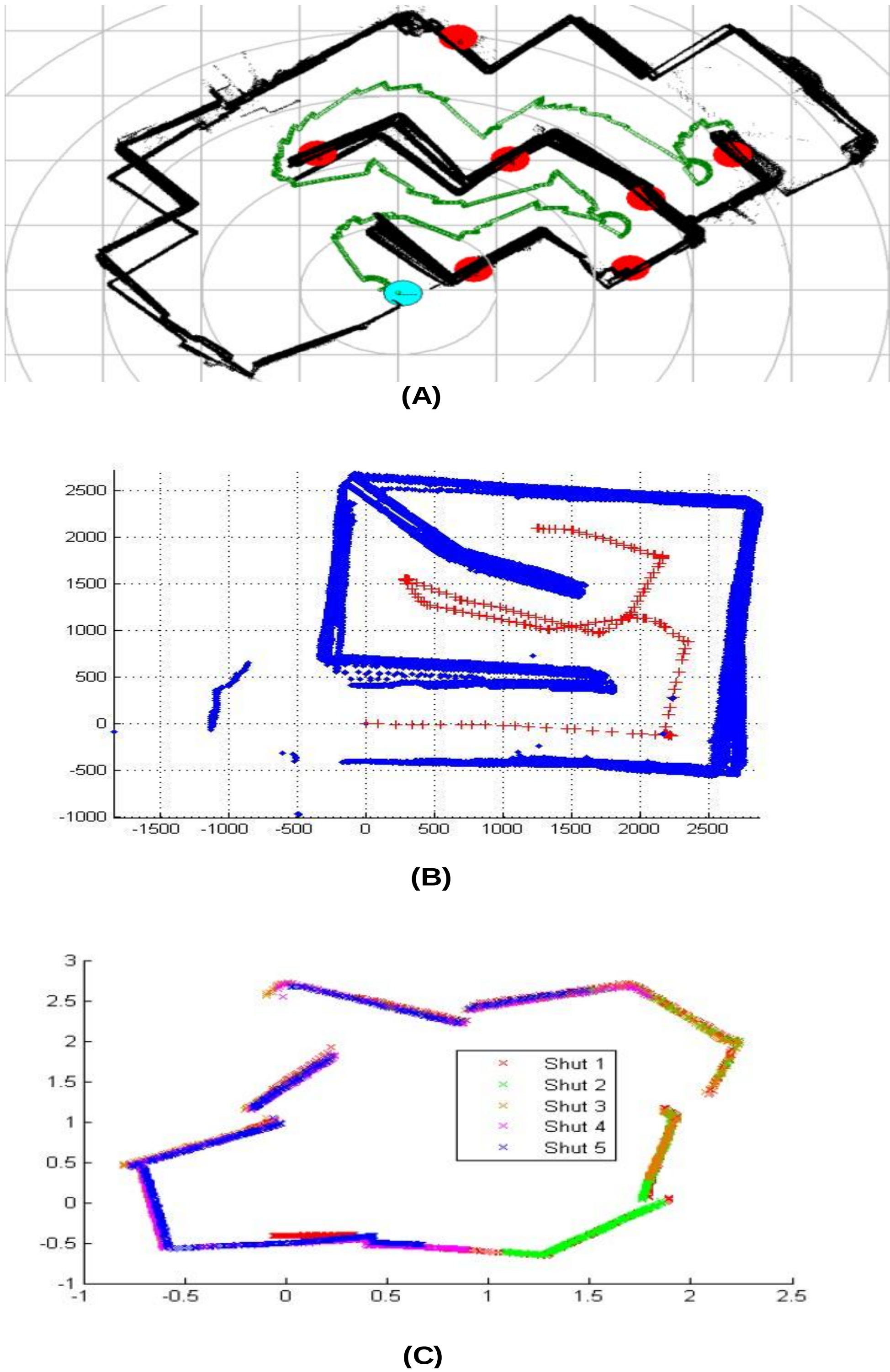


Figure 8. (a) Slam in real arena programming in Linux. (b) Maze in laboratory. (c) Feature corner extraction and Slam.

RESULTS

This algorithm is programmed in C++. The benchmarks are performed on a PC with PentiumIV-3.4GHz and 1GB of memory. Figure 8-A,B depicts a map which is obtained by this robot and corner feature extraction. The circular

Figure 9. NAJI III and URG-04LX.

Pseudo code

```
ALFA (r, θ, Rpos)
{
  seg ← Create Segments from polar data
  [r, θ ] ← Smooth each segment
  [X,Y] ← Change data from polar to Cartesian
  [K_Min, K_Max] ← First and last point for each seg
  {
   While (K_Max - K_Min) < least_point_in_line
   Split data & a_ is a pointer to middle of data
   b1 ← K_Min
   b2 ← K_Max
  While (b2 –b1) > 0
    {
     [Line_ param, RMSE] ← Fit a line from K_Min to a_
     RMSE_grad ← Estimate RMSE gradient
     If (RMSE > Max_ERR) or (RMSE_grad > Max_grad)
      // bad fit     Split recent data
       b2 ← a_ - 1
       a_ ← floor ((b1 + b2)/2)
       Max_ERR ← min {RMSE, Max_ERR}
    Else
     // satisfyè Add a split of remainder data
     b1 ← a_
     a_ ← round ((b1 + b2)/2)
    Max_RMSE ← mean { RMSE, Max_RMSE}
    }
   If (a_ - K_Min) < least_point_in_line
    // detect a line
   K_Min ← a_ + 1
  }
 }
}
```

and rectangular grid is 1 m. The black lines shows walls around the arena, green circle shows robot's initial point and red ones show probably victims placed in map. Green line shows the robot path. In the arena instruction, the floor has 10o roll and pitch ramps so the walls are duplicated somewhere especially in the left side, but the path is acceptable in maze arena. The extracted lines are suitable for object base mapping approaches as well.

Conclusion

This work was realised in the frame of NAJI III project (Autonomous Rescue Robot). It is constructed at the base of a four wheel differential system. It is equipped with an IBM T61 Laptop and a URG-04LX scanning range finder with 240° measuring area and 0.36° angular resolution (Figure 9).

The superior speed of the split and merge method, makes it the best option for most of the real-time line extracting applications. However, the threshold values affect the algorithm performance in split and merge method. The conducted experiments revealed that a static threshold does not demonstrate a desirable accuracy and leads to a bad feature extraction and system divergence in the line based SLAM. An adaptive line fitting algorithm (ALFA) for SLAM application is presented here. ALFA is a modified version of split and merge method for line based SLAM. It changes the threshold dynamically and finds the best line boundary. ALFA is composed of the following steps: Data smoothing to decrease the effect of noise, fitting a line to a data set using the least square method, applying RMSE criterion to evaluate the fitted line quality. The strength of ALFA is on the splitting method and the dynamic threshold. These two features enable ALFA to identify line boundary precisely. It is planned to develop a fast algorithm using DBN'S (Dynamic Bayesian Networks), smoothing to decrease the effect of noise, fitting a line to a data set using the least square method, and applying RMSE criterion to evaluate the fitted line quality. The strength of ALFA is on the splitting method and the dynamic threshold. These two features enable ALFA to identify line boundary precisely. It is planned to develop a fast algorithm using DBN'S (Dynamic Bayesian Networks).

REFERENCES

Altermatt M, Martinelli A, Tomatis N, Siegwart R (2004). SLAM with Corner Features Based on a Relative Map. In Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS.

Bevington PR, Robinson DK (1992). Data Reduction and Error Analysis for the Physical Sciences, 2nd Ed., WCB/McGraw-Hill, Boston. pp. 96-115,194-214.

Borges GA, Aldon J (2004). Line Extraction in 2D Range Images for Mobile Robotics. Journal of Intelligent and Robotic Systems, 40: 267–297.

Cox I, Blanche J (1991). An Experiment in Guidance and Navigation of an Autonomous Robot Vehicle. *IEEE Transations on Robotics and Automation,* 7(2): 193–204.

Diosi A, Kleeman L (2003). Uncertainty of Line Segments Extracted from Static SICK PLS Laser Scans. In Proc. of the Australasian Conference on Robotics and Automation.

Diosi AT, Kleeman L (2004). Advanced sonar and laser range finder fusion for simultaneous localization and mapping. In Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS.

Diosi AT, Kleeman L (2005). Laser Scan Matching in Polar Coordinates with Application to SLAM. In Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS.

Diosi AT, Taylor G, Kleeman L (2005). Interactive SLAM using laser and advanced sonar. In Proc. of the IEEE/ICRA International Conference on Robotics and Automation.

Lingemann K, Surmann H, Nüchter A, Hertzberg J (2004). Indoor and outdoor localization for fast mobile robots. In Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS.

Lu F, Milios E (1994). Robot Pose Estimation in Unknown Environments by Matching 2D Range Scans. In Proc. of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition, pp. 935-938.

Mavaei SM, Ebrahimian A, Sadeghi H (2011). Adaptive Line segmentation for Slam Line Based Applications. India, International Conference on Control, Robotics and Cybernetics, IEEE, V1-133.

Nguyen V, Martinelli A, Tomatis N, Siegwart A (2005). A Comparison of Line Extraction Algorithms using 2D Laser Rangefinder for Indoor Mobile Robotics. In Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS.

Pavlidis T, Horowitz SL (1974). Segmentation of Plane Curves. IEEE Transactions on Computers, C-23(8): 860–870.

Pfister ST, Roumeliotis SI, Burdick JW (2003). Weighted Line Fitting Algorithms for Mobile Robot Map Building and Efficient Data Representation. In Proc. of the IEEE/ICRA International Conference on Robotics and Automation.

Yeon W, Jeong K, Lee MU (2006). Visual SLAM with line and corner features. In Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS.