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Full Length Research Paper

Modification of gravitational field equation and rational solution to cosmological puzzles

Ming Yang

Department of Physics, Zhoukou University 266700, China. E-mail: bps267890@qq.com.

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The present article has systematically solved the problem of galaxy formation and some significant cosmological puzzles. First a mistake with Einstein's equation of gravitational field is corrected, next space-time is proved to be infinite, cosmic expansion and contraction are proved to be in circles, the singular point of big bang is eliminated naturally, celestial bodies and galaxies are proved growing up with cosmic expansion, for example Earth's mass and radius at present increase 1.2 trillion tons and 0.45 mm respectively in a year, in response to which geostationary satellites rise 2.7 mm.

Key words: Background coordinates, standard coordinates, geodesic. PACS: 04.20.Jb, *02.40.−k*, 04.20.Cv.

INTRODUCTION

Though general relativity obtains considerable success, some significant problems such as the problem of singular point, the problem of horizon, the problem of distribution and existence of dark matter and dark energy, the problem of the formation of celestial bodies and galaxies, as well as why celestial bodies burst and earth-quakes take place, always are not solved reasonably and satisfactorily. These problems remaining today imply strongly that the fundamental of general relativity has flaw and needs perfection further. For this purpose, the present paper begins with the definite solution of field equation in the background coordinate system, then by correcting rationally Einstein's field equation get these problems solved radically.

BACKGROUND COORDINATE SYSTEM THE STATIC METRIC OF SPHERICAL SYMMETRY

According to general relativity, when gravitational source (celestial body) is static and spherically symmetric, in the standard coordinate system (Weinberg, 1972; Peng, 1998), the correct form of invariant interval outside gravitational source reads

$$ds^2 \equiv -d\tau^2 = -\left(1-\frac{2GM}{l}\right)dt^2 + \left(1-\frac{2GM}{l}\right)^{-1} dl^2 + l^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (1)$$

Here τ is proper time, M is the total mass of gravitational source; l called standard radial coordinate, doesn't have clear physical significance and only in the far field is approximately viewed as true radius. In order to describe clearly motion of particle and enable general relativity to link up with other theories of physics and to compare results, it is necessary to transform (1) into the form expressed in background coordinates. Hence we take $l = l(r)$. Here r is defined as background coordinates (Hou, 1982; Zhou, 1983; Fock, 1964) and refers to true radius which are said and used usually. t, θ, φ are standard coordinates and can also be viewed as background coordinates, which represent true time and angle. In the following we try to determine $l = l(r)$ by the introduction of an additional transformation equation, and such operation is allowed because metric tensor satisfies Bianchi identity and if a metric is a solution of field equation in one coordinate system it is also a solution under arbitrary coordinate transformation.

According to general relativity the dynamical equation of particle outside source is geodesic

$$\frac{d^2x^\mu}{dt^2} + \Gamma^\mu_{\nu\lambda}\frac{dx^\nu}{dt}.\frac{dx^\lambda}{dt} - \Gamma^0_{\nu\lambda}\frac{dx^\nu}{dt}.\frac{dx^\lambda}{dt}.\frac{dx^\mu}{dt} = 0\ , \quad (2)$$

Where; $x^0 = t$ and indexes $\lambda, \nu, \mu, \sigma, \alpha, \beta = 0,1,2,3$.
Proof: according to;

$\frac{d^2x^\mu}{ds^2}+\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}=0$, we have

$\frac{d^2x^\mu}{ds^2}=\frac{dt}{ds}\frac{d}{dt}(\frac{dt}{ds}\frac{dx^\mu}{dt})=(\frac{dt}{ds})^2\frac{d^2x^\mu}{dt^2}+\frac{d^2t}{ds^2}\frac{dx^\mu}{dt}=$
$(\frac{dt}{ds})^2(\frac{d^2x^\mu}{dt^2}-\Gamma^0_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}\frac{dx^\mu}{dt})$, in addition,

$\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}=(\frac{dt}{ds})^2\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}$, and adding them together yields immediately Equation (2). When a particle of mass m is moving along radius in the static gravitational field of spherical symmetry, giving consideration to the speed, in the far field (weak field) the radial component of Equation (2) should reduce to relativistic dynamic equation

$$\frac{d}{dt}\left[\left(\frac{dr}{dt}\right)m\right]=-\frac{mGM}{r^2} \quad (3),$$

Where; m refers to relativistic dynamic mass, namely $m=m_0\big/\sqrt{1-v^2}$. Why the radial component should reduce to (3) is that (3) stands for the equality of gravitational mass and inertial mass and also stands for the speed of light is the limit one. In order to enable it to reduce to (3) l and r should satisfy

$$\frac{dl}{dr}=\sqrt{1-\frac{2GM}{l}}\exp(-\frac{GM}{r}) \quad (4)$$

The correctness of Equation (4) will be seen later, and it is the transformation equation which is introduced to determine $l=l(r)$ and is equivalent to a coordinate transformation of $l\rightarrow r$. The solution of Equation (4) is given by

$$\sqrt{l(l-2GM)}+4GM\ln\left(\sqrt{l}+\sqrt{l-2GM}\right)=C_1+r-GM\ln r-\frac{1}{2r}G^2M^2+\frac{1}{12r^2}G^3M^3+\cdots$$

Here constant C_1 is decided by the continuity of $l(r)$ on boundary of source, and (23) can give out the boundary value $l(r_e)$, r_e denotes source's radius. Now from (4) we see $l=r$ for $r\rightarrow\infty$. Under transformation of Equation (4), (1) becomes (5) which is an exact external solution expressed in background coordinates r,t,θ,φ.

$$ds^2=-\left(1-\frac{2GM}{l}\right)dt^2+\exp(-\frac{2GM}{r})dr^2+l^2\left(d\theta^2+\sin^2\theta d\varphi^2\right) \quad (5)$$

Note that now $l=l(r)$ is already a specific function of r, which is determined by the solution of Equation (4).

In the far field, line element (5) gives out

$g_{00}=-1+\frac{2GM}{l(r)}\approx-1+\frac{2GM}{r}$,

$g_{11}=\exp(-\frac{2GM}{r})\approx1-\frac{2GM}{r}$,

$g_{22}=l^2(r)\approx r^2$,

$g_{33}=l^2(r)\sin^2\theta\approx r^2\sin^2\theta$,

$\Gamma^1_{00}\approx\frac{GM}{r^2}$,

$\Gamma^1_{11}\approx\frac{GM}{r^2}$,

$\Gamma^0_{01}\approx\frac{GM}{r^2}$,

$\Gamma^1_{01}\approx0$,

$\Gamma^0_{00}=0$,

and introducing them into (2) and putting $\mu=1$, $d\theta=d\varphi=0$, $v=\frac{dr}{dt}$, we obtain

$$\frac{d^2r}{dt^2}+(1-v^2)\frac{GM}{r^2}=0 \quad (6)$$

which is just Equation (3). Proof: assume $d\theta=d\varphi=0$, $m=\frac{m_0}{\sqrt{1-v^2}}$, then

$$\frac{d}{dt}\left[\left(\frac{dr}{dt}\right)m\right]=m_0[v\frac{d(1-v^2)^{-1/2}}{dt}+$$

$$(1-v^2)^{-1/2}\frac{dv}{dt}]=(1-v^2)^{-3/2}\frac{d^2r}{dt^2}m_0=m(1-v^2)^{-1}\frac{d^2r}{dt^2},$$

compare with (3), we see

$$\frac{d^2r}{dt^2}+(1-v^2)\frac{GM}{r^2}=0.$$

Consequently, we conclude that (5) is the right line element which satisfies requirements completely.

In addition, as an emphasis, we must point out that using directly $l=r$ in (1) gives another exact solution, namely the following (7), which is often used in practice,

$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{7}$$

However, in accordance with (7) the corresponding geodesic can't reduce to (3) in weak field, instead it reduces to

$$\frac{d^2r}{dt^2} + (1-3v^2)\frac{GM}{r^2} = 0 \tag{8}$$

Proof: from (7) we have $g_{00} = -1 + \frac{2GM}{r}$,

$g_{11} = \left(1-\frac{2GM}{r}\right)^{-1}$,

$g_{22} = r^2$,

$g_{33} = r^2 \sin^2\theta$,

$g^{\mu\mu} = \frac{1}{g_{\mu\mu}}$,

$\Gamma^1_{11} = \frac{1}{2} g^{1\rho}(\frac{\partial g_{\rho 1}}{\partial x^1} + \frac{\partial g_{\rho 1}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^\rho}) = -\frac{GM}{(1-2GM/r)r^2}$,

$\Gamma^0_{01} = \frac{GM}{(1-2GM/r)r^2}$,

$\Gamma^1_{00} = \frac{(1-2GM/r)GM}{r^2}$

$\Gamma^1_{01} = 0,$

And substituting them into (2) and taking $\mu = 1$ and $d\varphi = d\theta = 0$ yield immediately

$\frac{d^2r}{dt^2} = -\Gamma^1_{00} - \Gamma^1_{11}v^2 +$

$2v^2\Gamma^0_{01} = -(1-\frac{2GM}{r})\frac{GM}{r^2} + \frac{3GM}{(1-2GM/r)r^2}v^2$, and

for $\frac{2GM}{r} << 1$.

This equation obviously reduces to Equation (8), which is not Equation (3). It is easily found that Equation (8) not only goes against the elementary principle of equality of gravitational mass and inertial mass but also leads to incorrect conclusion that gravitational field becomes repulsive one for a particle whose speed exceeds 0.58c. Hence Equation (8) must be wrong, and implies (7) can not describe high speed and has shortcoming as compared with (5).

Note that the angle of orbital precession of Mercury described by (5) is still the same as that described by line element (7) (Peng, 1998), it does not change under the transformation of radial coordinates. In a word, (5) is the correct line element expressed in background coordinate system

CORRECTION TO GRAVITATIONAL FIELD EQUATION

It is seen from the above discussions that in the case of weak field approximation, $g_{00} = -1 + \frac{2GM}{r}$ and $g_{11} = 1 - \frac{2GM}{r}$ but not $g_{11} = 1 + \frac{2GM}{r}$, which imply that the constant γ in field equation $R_{\mu\nu} = \gamma(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu})$ needs modification. And now we set out to reconfirm the coefficient γ.

Here, $T_{\mu\nu} \equiv (\rho + p)U_\mu U_\nu + pg_{\mu\nu}$,

$U^\mu \equiv \frac{dx^\mu}{d\tau}$, $U_\mu \equiv g_{\mu\nu}U^\nu$.

And according to

$ds^2 = -d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu$, we conclude $U_u U^u = -1$

Hence. it follows that the scalar

$$T = g^{\mu\nu}T_{\mu\nu} = g^{\mu\nu}(\rho + p)U_u U_v + pg^{\mu\nu}g_{uv} = (\rho + p)U_u U^\mu + 4p = 3p - \rho$$

Note that the following discussions are carried out in right-angle coordinate system. Therefore, for weak field we may assume $g_{\mu\nu} = h_{\mu\nu} + \eta_{\mu\nu}$ and $|h_{uv}| << 1$, here $\eta_{00} = -1$, $\eta_{11} = \eta_{22} = \eta_{33} = 1$, $\eta_{\mu\nu} = 0$ ($\mu \neq \nu$).

Then $\Gamma^\mu_{\alpha\beta} = \frac{1}{2}\eta^{\mu\rho}(\frac{\partial g_{\rho\alpha}}{\partial x^\beta} + \frac{\partial g_{\rho\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\rho})$,

$h^\mu_\beta = \eta^{\mu\rho}h_{\rho\beta}$

$h = h^\mu_\mu = \eta^{\mu\rho}h_{\mu\rho}$.

Omitting the terms $o(h^2)$ we can infer that;

$R_{\mu\nu} = \Gamma^\sigma_{\mu\sigma},_\nu - \Gamma^\sigma_{\mu\nu},_\sigma = \frac{1}{2}\eta^{\alpha\lambda}h_{\mu\nu,\lambda,\sigma} +$

$\frac{1}{2}(h,_\mu,_\nu - h^\lambda_\mu,_\lambda,_\nu - h^\sigma_\nu,_\sigma,_\mu)$.

May as well assume

$$h^{\sigma}_{\mu},_{\sigma} = \frac{1}{2} h,_{\mu}. \qquad (9)$$

Differentiating Equation (9) with respect to x^{ν} yields $h^{\sigma}_{\mu},_{\sigma},_{\nu} = \frac{1}{2} h,_{\mu},_{\nu}$. Likewise, $h^{\sigma}_{\nu},_{\sigma},_{\mu} = \frac{1}{2} h,_{\nu},_{\mu}$.

Using $h,_{\nu},_{\mu} = h,_{\mu},_{\nu}$ and adding up above two equations yield directly $h,_{\mu},_{\nu} - h^{\lambda}_{\mu},_{\lambda},_{\nu} - h^{\sigma}_{\nu},_{\sigma},_{\mu} = 0$.

Hence, we have;

$$\nabla^2 h_{\mu\nu} - \frac{\partial^2 h_{\mu\nu}}{\partial t^2} = 2\gamma(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}) = 2\gamma[(\rho + p)U_{\mu}U_{\nu} + \frac{\rho - p}{2}\eta_{\mu\nu}]$$

It follows that;

$$h_{\lambda\lambda} = -\frac{\gamma}{4\pi}\int \frac{2(\rho + p)U^2_{\lambda} + (\rho - p)\eta_{\lambda\lambda}}{\xi} dx'dy'dz'.$$

Here, $\xi = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$, $i, j, k = 1, 2, 3$, the terms in the integral sign take the values of $t' = t - \xi$. Note that the above retarded solutions can be used in general cases of motion of source. Hence, in order to get the external metrics $g_{00} = -1 + \frac{2GM}{r}$ and $g_{jj} = 1 - \frac{2GM}{r}$ in the case of static spherical symmetry, ($U_0 = \eta_{0\mu}U^{\mu} = -1$, $U_j = 0$), it must be required that in the region of $r = \sqrt{x^2 + y^2 + z^2} \geq r_e$ there exist $\int \frac{p}{\xi} dx'dy'dz' = -\int \frac{\rho}{\xi} dx'dy'dz' = -\frac{M}{r}$ and simultaneously constant $\gamma = 4\pi G$. Since p, ρ and g_{uv} depend only on r, we easily infer

$$\int pdxdydz = -\int \rho dxdydz = -M. \qquad (10)$$

For static field it holds that $h_{0j} = 0$ in view of (3), and substituting h_{0j} and $h_{\lambda\lambda}$ into Equation (9) gives

$$h_{ji} = \frac{1}{4}\int_{\infty}^{x^j}\int_{\infty}^{x^i} [(\frac{\partial^2}{\partial(x^i)^2} + \frac{\partial^2}{\partial(x^j)^2} - \frac{\partial^2}{\partial(x^k)^2})(h_{11} - h_{00})]\, dx^j dx^i$$

Where; $i \neq j, i \neq k, j \neq k, x^1 = x, x^2 = y, x^3 = z$. And again, Bianchi identity can give out in weak field the conserved law (Weinberg, 1972; Peng, 1998; Zhou, 1982) $T^{\mu}_{\nu\ ,\mu} = 0$.

Proof:

$$R^{\mu}_{\nu\ ;\mu} = R^{\mu}_{\nu\ ,\mu} + \Gamma^{\mu}_{\lambda\mu}R^{\lambda}_{\nu} - \Gamma^{\mu}_{\lambda\nu}R^{\lambda}_{\mu} = R^{\mu}_{\nu\ ,\mu} + o(h^2) = R^{\mu}_{\nu\ ,\mu}$$

Hence,

$$0 = (R^{\mu}_{\nu} - \frac{1}{2}R\delta^{\mu}_{\nu})_{;\mu} = R^{\mu}_{\nu\ ;\mu} - \frac{1}{2}R_{\ ;\nu} = R^{\mu}_{\nu\ ,\mu} - \frac{1}{2}R_{,\nu},$$

moreover field equation gives $R = -\gamma T$ and $R^{\mu}_{\nu\ ,\mu} = \gamma(T^{\mu}_{\nu} - \frac{1}{2}T\delta^{\mu}_{\nu})_{,\mu} = \gamma(T^{\mu}_{\nu\ ,\mu} - \frac{1}{2}T_{,\nu}) = \gamma T^{\mu}_{\nu\ ,\mu} + \frac{1}{2}R_{\ ,\nu}$. Hence $T^{\mu}_{\nu\ ,\mu} = 0$, and for static case, from $T^{\mu}_{\nu\ ,\mu} = [(\rho + p)U_{\nu}U^{\mu}]_{,\mu} + (p\delta^{\mu}_{\nu})_{,\mu} = 0$ we infer $\frac{\partial p}{\partial x^{\nu}} = 0$. Considering $\nabla^2(h_{00} - h_{jj}) = 16\pi Gp$, we see

$$\nabla^2 h_{ji} = \frac{1}{4}\int_{\infty}^{x^j}\int_{\infty}^{x^i} [(\frac{\partial^2}{\partial(x^i)^2} + \frac{\partial^2}{\partial(x^j)^2} - \frac{\partial^2}{\partial(x^k)^2})\nabla^2(h_{11} - h_{00})]\, dx^j dx^i = 0$$

That is to say, $h_{\nu\mu}$ here is indeed reasonable approximate solution of field equation which has $\gamma = 4\pi G$.

And again, as a special case of spherical symmetry, namely $\frac{\partial\rho}{\partial x^{\nu}} = 0$, since $\frac{\partial p}{\partial x^{\nu}} = 0$ we easily infer from (10) a very useful result $p = -\rho$ which can be regarded as the form of pressure in weak field if ρ is homogeneous. Obviously it is too subjective to take pressure for zero before, and in fact, by serious calculation we see that pressure is negative where matter exists, the place where matter exists turns out to be so-called pseudo-vacuum (Gondolo and Fresse, 2003; Guth, 1981). This is a new important result which isn't in agreement with traditional opinion.

To sum up, we can conclude that in any coordinate system gravitational field equation is revised as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 4\pi GT_{\mu\nu} \qquad (11)$$

Where; 4 replaces previous -8, obviously Equation (11) preserves general covariance. Of course, (1) and (5) satisfy Equation (11) because both p and ρ vanish outside gravitational source, Equation (11) becomes $R_{\mu\nu} = 0$.

APPLICATIONS AND TESTS OF EQUATION (11) IN COSMOLOGY

With l as standard radial coordinate, Friedmann-Robertson-Walker line element is given by (Weinberg, 1972; Rong-Gen, 2005)

$$ds^2 = -dt^2 + a^2(t)\left[\frac{1}{1-kl^2}dl^2 + l^2 d\theta^2 + l^2 \sin^2\theta d\varphi^2\right]$$

Substituting the metrics into (11) and using the co-moving coordinates yield

$$\left(\frac{da(t)}{dt}\right)^2 + k = -\frac{4\pi G}{3}\rho a^2(t) \qquad (12)$$

Consequently k must be negative, cosmos is so far proved infinite or open. And again, because of having (Weinberg, 1972; Peng, 1998)

$$T^{\alpha\beta}{}_{;\beta} = (nU^{\nu})_{;\nu} = (U_{\mu}U^{\mu})_{;\beta} = 2U_{\mu}(U^{\mu})_{;\beta} = 2U^{\mu}(U_{\mu})_{;\beta} = 0$$

It follows that $d(\rho a^3) + p da^3 = 0$ and

$$pd\left(\frac{1}{n}\right) + d\left(\frac{\rho}{n}\right) = 0 \qquad (13)$$

Here, n represents the density of particle (galaxy) number. Since ρ is assumed homogeneous, we may use the weak field condition $p = -\rho$, and substituting it into Equation (13) yields $d\rho = 0$, that is to say,

$$\dot{p} = -\dot{\rho} = 0 \text{ or } p = -\rho = const, \qquad (14)$$

which implies cosmic density and pressure don't change all along and the singular point of big bang did not exist. In addition, (13) indicates the mass of galaxy increasing with cosmic expansion since ρ/n stands for per particle mass. And further, the solution Equation (12) is given by

$$a(t) = A\sin\left(2t\sqrt{\frac{\pi G\rho}{3}}\right). \qquad (15)$$

where A is a positive constant. So far cosmic expansion and contraction are proved to be in circles. Now we compute the relation between distance and red-shift. Taking $R(t_0) = 1$, the light from a galaxy to us satisfies (Weinberg, 1972; Rong-Gen, 2005)

$$1 + z = \frac{1}{a(t)} \text{ and } dz = -\frac{da}{a^2(t)}.$$

Here z denotes red-shift. And writing $\frac{4\pi G\rho}{3H_0^2} \equiv q_0$, we infer from Equation (12);

$$H \equiv \dot{a}/a = H_0\sqrt{(1+q_0)(1+z)^2 - q_0} \text{ and } k = -H_0^2(1+q_0)$$

The subscript "0" refers to present-day values. for light $ds^2 = 0$, then

$$\frac{dt}{a(t)} = -\frac{dz}{H} = -\frac{dl}{\sqrt{(1-kl^2)}}, \int_0^Z \frac{dz}{H} = \int_0^{l_e} \frac{dl}{\sqrt{1-kl^2}}.$$

l_e denotes the galaxy's invariant radial coordinate. In view of luminosity-distance (Weinberg, 1972; Peng and Xu, 1998) $d_L = (1+z)\int_0^{l_e} \frac{dl}{\sqrt{1-kl^2}}$, one can figure out

$$H_0 d_L = \frac{z+1}{\sqrt{q_0+1}} In \frac{(z+1)\sqrt{q_0+1} + \sqrt{(q_0+1)(z+1)^2 - q_0}}{1+\sqrt{q_0+1}} \qquad (16)$$

As $z \to 0$, expanding the nature logarithm into power series of z, (16) becomes $H_0 d_L = z + \frac{(1+q_0)z^2}{2} + \cdots$, which is the same result as that obtained via pure kinematics. The curved line in Figure 1 (Dai et al., 2005) is function image of (16) with $q_0 = 0.14$ and $H_0 = 70 km \cdot s^{-1} \cdot Mpc^{-1}$. The situation described by the curved line agrees well with the recent observations, and strongly indicates Equation (11) is correct and the modification is quite successful. Note that current observations show (Linder, 2003; Hamuy, 2003; Alcaniz, 2004) $q_0 = \frac{4\pi G\rho}{3H_0^2} \equiv \frac{\Omega_0}{2} = 0.1 \pm 0.05$ The spots in the Figure 1 represent galaxies (Dai et al., 2005), Distance-Modulus is equal to $5 \lg d_L + 25$ (d_L is in Mpc).

Next we calculate "our" cosmic age, namely the time from last $a(t) = 0$ (t may as well take 0) to today. Writing $H(t_0) = H_0$, from $H = \frac{\dot{a}}{a} = 2\sqrt{\frac{\pi G\rho}{3}} ctg\left(2t\sqrt{\frac{\pi G\rho}{3}}\right)$, q_0 takes

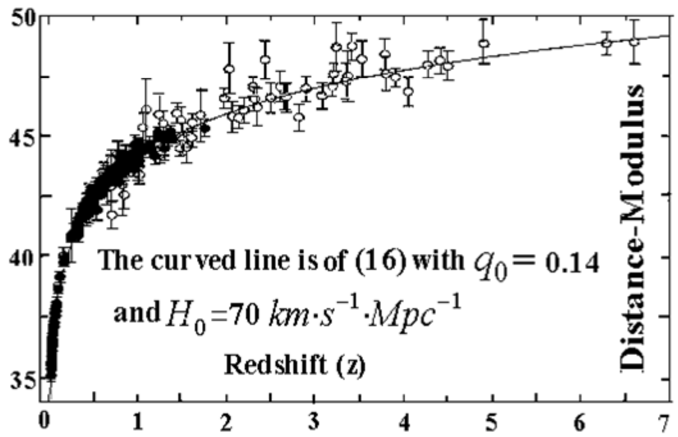

Figure 1. Recent Hubble figure

0.14, "our" cosmic age is given by;

$$t_0 = \frac{tg^{-1}\sqrt{q_0}}{H_0\sqrt{q_0}} = 1.37\times 10^{10}\, a\,, \qquad (17)$$

Which agrees with observations (Cayrel et al., 2001). Besides, we can also solve how a galaxy's mass changes with time. Writing a galaxy' mass $m(t)$, from $\rho = const = Nm(t)/a^3(t)$, in which N is a proportional coefficient, we conclude

$$\frac{m(t_1)}{a^3(t_1)} = \frac{m(t_2)}{a^3(t_2)}\,, \qquad (18)$$

which shows that galaxies can grow up without mergers, and conforms recent observations (Genzel et al., 2006). When (18) is applied to the earth of today, the increase of the earth's mass m_0 in a year is calculated as;

$$\Delta m_0 = [\frac{a^3(t_0+1)}{a^3(t_0)} - 1]m(t_0) \approx 3H_0 m_0 = 12.46\times 10^{14}\ \text{kg},$$

Correspondingly its radius will expand $H_0 r_\oplus = 0.45$ mm.

As for the motion in a central field, from Newton's law one has $\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$, where r is the radius of orbit, T is the round period, M is the mass of central body. And hence, due to the change of centre body mass M, from t to $t+\Delta t$ the round radius will change

$$\Delta r = [\frac{a(t+\Delta t)}{a(t)} - 1]r + \frac{2}{3}\frac{a(t+\Delta t)}{Ta(t)} r\Delta T\,. \qquad (19)$$

For geostationary satellite round Earth of today, neglecting ΔT_0 its radius will increase $\Delta r_0 = 2.7$ mm in a year from (19). Observations show Moon's orbit radius now increases 0.38 cm a year, thus from (19) we infer that the round period T_0 of Moon will slow 0.0001s a year today.

To sum up, it is seen that (18) and (19) determine the formation of celestial bodies and galaxies, and of course, some details need further complements.

Exact interior solution of Equation (11) and mechanism of celestial body's expansion

In the case of static spherical symmetry, inside a celestial body (gravitational source), with l as standard radial coordinate the exact interior solution of Equation (11) is easily given by (Weinberg, 1972; Peng, 1998).

$$ds^2 = -\exp\left[C_2 + \int_l^{l(r_e)} f(l)\left(1+\frac{\omega(l)}{l}\right)^{-1} dl\right] dt^2$$
$$+\left(1+\frac{G\omega(l)}{l}\right)^{-1} dl^2 + l^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (20)$$

Where; $\omega(l) \equiv 4\pi\int_0^l \rho(l) l^2 dl$, $f(l) \equiv \frac{G}{l^2}\left[4\pi l^3 p(l) + \omega(l)\right]$.

Constant $C_2 = In[1 - \frac{2GM}{l(r_e)}]$, it makes sure g_{00} is continual on the boundary of the celestial body. Note that as scalar $\rho = \rho(l) = \tilde{\rho}(r)$, $p = p(l) = \tilde{p}(r)$, and outside gravitational source both p and ρ vanish, namely $\tilde{\rho}(r) = \tilde{p}(r) = 0$ for $r > r_e$. In order to determine the interior form of (20) in background coordinates, Equation (4) is naturally extended as inside gravitational source

$$\frac{dl}{dr} = \sqrt{1+\frac{G\omega(l)}{l}}\exp\left(-G\int\frac{\rho}{\xi}dx'dy'dz'\right). \qquad (21)$$

Obvious under the transformation of Equation (21), line element (20) turns into

$$ds^2 = -\exp\left[C_2 + \int_l^{l(r_e)} f(l)\left(1+\frac{\omega(l)}{l}\right)^{-1} dl\right] dt^2 +$$

$$\exp\left(-2G\int\frac{\rho}{\xi}dx'dy'dz'\right) dr^2 + l^2(d\theta^2 + \sin^2\theta d\varphi^2)\,. \qquad (22)$$

Here $l = l(r)$ is a specific function of r, which is determined by Equation (21). Line element (22) is just the exact solution looked for, expressed in background coordinate r, t, θ, φ. Note that the solution of Equation

(21) need meet the initial condition $l(0)=0$. Because there is no gravity acceleration at the coordinate origin, dg_{00}/dr must be zero, from (22) one has

$$0=\frac{dg_{00}}{dr}=\frac{dl}{dr}\frac{dg_{00}}{dl}$$
$$=\frac{dl}{dr}f(l)\left(1+\frac{\omega(l)}{l}\right)^{-1}\exp\left[C_2+\int_l^{l(r_e)}f(l)\left(1+\frac{\omega(l)}{l}\right)^{-1}dl\right],$$

which indicates $f(l)=0$ at coordinate origin, that is to say, $l=l(0)=0$. And if $\rho=const=\frac{3M}{4\pi r_e^3}$, then

$$\int\frac{\rho}{\xi}dx'dy'dz'=\frac{3M}{2r_e}-\frac{M}{2r_e^3}r^2,\ \omega(l)=4\pi\int_0^l\rho(l)l^2dl=\frac{M}{r_e^3}l^3,$$

The solution of Equation (21) is easily given by

$$\sqrt{\frac{r_e^3}{GM}}In\left(\sqrt{\frac{GM}{r_e^3}}l+\sqrt{1+\frac{GM}{r_e^3}l^2}\right)=$$
$$\left[r+\frac{GM}{6r_e^3}r^3+\frac{1}{40}\left(\frac{GM}{r_e^3}\right)^2r^5+\cdots\right]\exp\left(-\frac{3GM}{2r_e}\right) \qquad (23)$$

Though ρ, generally speaking, is not constant, we may take its average value in practice for the convenience of calculation. For example, on the surface of Sun $r=r_e=6.96\times10^8$ m, $M=1.99\times10^{30}$ kg, using (23) we can work out the surface's $l=l(r_e)=6.96\times10^8\text{ m}-1720\text{ m}$, which is highly equal to Sun's radius. And likewise, we can work out $l=6371km-0.00038km$ on Earth's surface, and this almost equals Earth's radius (6371 km).

So far, using the continuity of $l=l(r)$ not only we can determine the constant C_1 but also can calculate the deflected angle of light line on the surface of Sun. For photon's propagation outside Sun from (5) we have

$$0=ds^2=-\left(1-\frac{2GM}{l}\right)dt^2+\exp\left(-\frac{2GM}{r}\right)dr^2+l^2\left(d\theta^2+\sin^2\theta d\varphi^2\right)$$

$$=-\left(1-\frac{2GM}{l}\right)dt^2+\left(1-\frac{2GM}{l}\right)^{-1}dl^2+\left(d\theta^2+\sin^2\theta d\varphi^2\right)l^2\cdot$$

Similar to former calculation, the deflected angle is given by $\alpha=\frac{4MG}{l}=\frac{4MG}{l(r_e)}=1.78''$, which is more consistent with observational result $(1.89'')$ than former theoretical value $\alpha=\frac{4MG}{r}=\frac{4MG}{r_e}=1.75''$.

On the other hand, the conserved law gives out

$$\frac{dp}{dl}=G(p+\rho)\left(2\pi l^3p+\frac{\omega}{2}\right)\left(l^2+lG\omega(l)\right)^{-1}\cdot \qquad (24)$$

On the boundary the gravity acceleration should be continual, according to (2), using (4), (5), (21), (22) we have

$$(\Gamma^1_{00})_{r=r_e^+}=(\Gamma^1_{00})_{r=r_e^-},\text{ that is,}$$

$$\left(g^{11}\frac{dg_{00}}{dr}\right)_{r=r_e^+}=\left(g^{11}\frac{dg_{00}}{dr}\right)_{r=r_e^-},$$

It follows that; $\left[\frac{dl}{dr}\frac{d}{dl}\left(1-\frac{2GM}{l}\right)\right]_{r=r_e^+}=\left\{\frac{dl}{dr}\frac{d}{dl}\exp\left[C_2+\int_l^{l(r_e)}f(l)\left(1+\frac{\omega(l)}{l}\right)^{-1}dl\right]\right\}_{r=r_e^-}$

And after simplifying further, this becomes

$$[4\pi l^3p+\omega(l)]\sqrt{l-2GM}=-2M\sqrt{l+G\omega(l)}, \qquad (25)$$

which is the boundary condition that p must satisfy, the condition determines p negative within celestial body.

As an emphasis, we must point out that when (1) or (5) is applied to a mass point of the surface of the static source, it holds that $0\ge ds^2=-(1-\frac{2GM}{l})dt^2$, which indicates that $1-\frac{2GM}{l}$ of static source is nonnegative.

Next let us consider a small volume V_i of mass m_i inside source, dV_i denotes V_i's change caused from the expansion of space-time, in view of Equation (12) we have $dm_i=-p_idV_i$, hence

$$d\rho_i=d\left(\frac{m_i}{V_i}\right)=-(\rho_i+p_i)\frac{dV_i}{V_i}=-(\rho_i+p_i)\frac{da^3(t)}{a^3(t)} \qquad (26)$$

which determines how matter density changes locally. It is seen from (26) that when celestial bodies expand with cosmic expansion its density may be unchanging if $\rho_i+p_i=0$.

So far, we deduce that bursts of celestial bodies and earthquakes originate from the unceasing accumulation of inside matter and the change of its distribution; and it is the negative that leads to production of matter.

NATURAL SOLUTION TO THE PROBLEM OF DARK MATTER

The invisible negative pressure is an important part of gravitational source, and it is the negative pressure that appears as the form of dark matter and leads to the phenomenon of missing mass, this is easily proven as follows.

Speaking generally, within a galaxy the metric field is weak field, and when a galaxy is treated as a celestial body of spherical symmetry, according to the discussion in section 3, within the galaxy ($0 \le r \le r_e$) pressure $p = const \neq 0$. And from (10), we infer $p = const = -\frac{3M}{4\pi r_e^3}$, further we have

$$h_{00} = -G\int \frac{\rho + 3p}{\xi} dx'dy'dz' =$$
$$-4\pi G\left(r^{-1}\int_0^r \rho r^2 dr + \int_0^{r_e} \rho r dr - \int_0^r \rho r dr\right) - 6G\pi p r_e^2 + 2G\pi p r^2$$

According to (2) the gravity acceleration (or gravitational field strength) within the galaxy is given by;

$$g = -\Gamma^1_{00} = \frac{1}{2}\frac{dh_{00}}{dr} = 2\pi G p r + \frac{2\pi G}{r^2}\int_0^r \rho r^2 dr = 2\pi G p r + \frac{Gm(r)}{2r^2}$$

Where; $m(r) \equiv 4\pi\int_0^r \rho r^2 dr$, and g may be positive or negative since pressure is negative, the negative g indicates the direction of acceleration is towards centre. And according to (2) the corresponding round orbital speed v satisfies

$$v^2 = -gr = -2\pi G p r^2 - \frac{Gm(r)}{2r} \tag{27}$$

From (27) it is seen that when $m(r)$ looks even on the verge of zero near the centre of the galaxy the speed v can become high, too, and this explains so-called missing mass. Again, from (27) we get $2rv^2 = -4\pi G p r^3 - Gm(r)$, and if v is a constant between r_1 and r_2, differentiating this equation and simultaneously using $v^2(r_1 \le r \le r_2) = -2\pi G p r^2 - \frac{Gm(r)}{2r} = \frac{3MG}{2r_e^3}r_1^2 - \frac{Gm(r_1)}{2r_1}$ yield

$$\rho(r_1 \le r \le r_2) = -3p - \frac{v^2}{2\pi G r^2} = \frac{9M}{4\pi r_e^3} - \frac{3M}{4\pi r_e^3 r^2}r_1^2 + \frac{m(r_1)}{4\pi r_1 r^2} \tag{28}$$

which is the condition that a typical spiral galaxy with a halo satisfies. May as well set $r_1 = nr_2$ $(0 < n < 1)$, then $m(r_2) = m(r_1) + 4\pi\int_{r_1}^{r_2} \rho r^2 dr = \frac{3M}{r_e^3}(1-n^2)r_2^3 + \frac{m(r_1)}{n}$, and in view of $0 \le m(r_2) \le M$ we infer

$$0 \le r_2^3 \le \frac{nM - m(r_1)}{3nM(1-n^2)} r_e^3 \tag{29}$$

which indicates it is impossible for r_2 to arrive at the galaxy's edge r_e in the case of $n < \sqrt{2/3}$. Obviously, as ρ begins to decrease continuously from r_2 to r_e both v and $|g|$ begin to increase. Of course, it isn't easy to observe the speed between r_2 and r_e because near the edge r_e matter becomes virtually very thin. The curve in the right Figure 2 describes the situation predicted according to the (27) and (29), and it is in conformity with recent observational results from gravitational lens and other experiments (Hamuy, 2003; Alcaniz, 2004; Cayrel et al., 2001; Genzel et al., 2006), and implies the negative pressure is a profound concept.

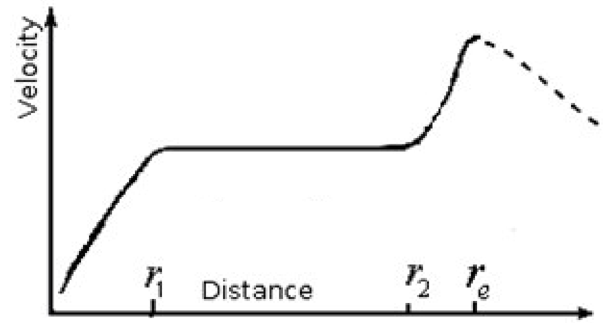

Figure 2. Velocity distribution figure

So far, we deduce that so-called dark matter is just the effect of the negative pressure, and the dark matter (Brownstein and Moffat; Baojiu et al. 2008, Stacy, 2008) puzzle has naturally been solved (Figure 2).

Of course, so-called dark energy's problem is now solved since cosmological constant becomes zero again, and the concept of dark energy is unnecessary in present amendment.

Conclusions

Celestial bodies and galaxies are growing up gradually with cosmic expansion; Background Radiation is the result early celestial body's mass approached zero, which implies early cosmos was filled with radiation, and only the temperature of early cosmos was higher than that of today but its density and pressure are unchanged all along; and it is the invisible negative pressure that acts as the role of dark matter and dark energy and leads to the phenomenon of missing mass and the expansions of celestial bodies and galaxies as well as cosmos. We expect that deepening research of the negative pressure will realize the unification of four interactions.

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