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Full Length Research Paper

Anti-synchronization of chaotic neural networks with time-varying delays via linear matrix inequality (LMI)

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In this paper, anti-synchronization problem of two identical chaotic neural networks with time-varying delays is proposed. By using time-delay feedback control technique, mean value theorem and the Leibniz-Newton formula, and by constructing appropriately Lyapunov-Krasovskii functional, sufficient condition is proposed to guarantee the asymptotically anti-synchronization of two identical chaotic neural networks. This condition, which is expressed in terms of linear matrix inequality, rely on the connection matrix in the drive and response networks as well as the suitable designed feedback gains in the response network. Finally, the anti-synchronization of two chaotic cellular neural network and Hopfield neural network with time-varying delays are considered to illustrate the effectiveness of the proposed control scheme, in which, when compared with the nonlinear feedback control method, the proposed method shows superior performance.

Key words: Lyapunov-Krasovskii functional, chaotic neural networks, anti-synchronization, time-varying delay, linear matrix inequality.

INTRODUCTION

Over the recent decades, existence of chaos has been discovered and reported in different aspects of science and technology, such as electrical circuits, chemical reactions, information processing, lasers, optics and neural networks (Chen and Dong, 1998; Wieczorek and Chow, 2009; Yang and Yuan, 2005; Gutzwiller, 1990). Since Pecora and Carroll (1990) established a chaos synchronization scheme for two identical chaotic systems with different initial conditions, chaos synchronization has attracted a great deal of attention (Sun and Cao, 2007; Sanjaya et al., 2010). Another interesting phenomenon discovered was the anti-synchronization (AS), which is noticeable in periodic oscillators. AS is a phenomenon that the state vectors of the synchronized systems have the same amplitude but opposite signs as those of the driving system. In this case, the sum of two signals is expected to converge to zero. So far, different techniques and methods have been proposed to achieve chaos anti-synchronization, such as, active control method (Ho et al., 2002), adaptive control (Li et al., 2009), H_∞ control (Ahn, 2009), nonlinear control (Al Sawalha and Noorani, 2009), sliding mode control (Chiang et al., 2008), backstepping control (Hu et al., 2005), adaptive modified function projective method (Adeli et al., 2011), etc.

Recently, the study of dynamical properties of neural networks appears more due to their extensive applications in differential fields, such as signal and image processing, pattern recognition, combinatorial optimization and other areas (Cohen and Grossberg, 1983; Carpenter and Grossberg, 1987; Chua and Yang, 1988). In the electronic implementation of the neural networks, time delay will occur in the interactions between the neurons inevitably, and will affect the dynamic behavior of the neural network models and may lead to instability and/or deteriorate the performance of the underlying neural networks. In some particular cases, it has been shown that these networks can exhibit some complicated dynamics and even chaotic behaviors if the network's parameters are appropriately chosen (Yuan, 2007; Lu, 2002).

An efficient tool for solving many optimization problems is linear matrix inequality approach which has been

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effectively applied in controller design for nonlinear process (Chen et al., 2010). Linear matrix inequalities (LMIs) have been playing an increasingly important role in the field of optimization and control theory, because a wide variety of problems (linear and convex quadratic inequalities, matrix norm inequalities, convex constraints, etc.) can be written as LMIs (Boyd et al., 1994; Guo et al., 2009; Hencey and Alleyne, 2009).

In addition, LMIs have found many applications in exploring properties of recurrent neural networks, since their stability conditions are often expressed with the aid of LMIs (Liu et al., 2005; Lu and Chen, 2006; Lou and Cui, 2006; Li et al., 2008). The objective of this paper is to prepare a control law based on the LMI approach for anti-synchronization of two identical chaotic neural networks with time varying delays, where the stability of the proposed method is guaranteed using Lyapunov stability theory. It will be shown that the performance of the proposed scheme is improved when compared with a recently published paper.

PROBLEM FORMULATION AND SOME PRELIMINARIES

The chaotic neural network with time-varying delay under consideration is described by:

$$\begin{aligned} \dot{x}(t) &= -Cx(t) + Df(x(t)) + Ef(x(t-\tau(t))) \\ x(t) &= \varphi(t) \quad t \in [-\kappa, 0] \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), ..., x_n(t)]$ is the state vector of the neural network with n neurons, $C = diag\{c_{11}, ..., c_{nn}\}$ is a diagonal matrix with $c_{ii} > 0, i = 1, ..., n$ and the matrices D and E are, respectively, the connection weight matrix and the delayed connection weight matrix. $f(x(t)) = [f_1(x_1(t)), ..., f_n(x_n(t))]$ denotes the neuron activation function, $\varphi(t)$ is the initial condition of state vector and $\tau(t)$ is time-varying delay and satisfying:

$$0 \le \tau(t) \le \lambda_1, \qquad \dot{\tau}(t) \le \lambda_2 \tag{2}$$

where $\lambda_1 > 0$ and λ_2 are known parameters. Suppose that the system (Equation 1) be the drive system. The response system is represented by:

$$\begin{aligned} \dot{y}(t) &= -Cy(t) + Df(y(t)) + Ef(y(t-\tau(t))) + B_u u(t) \\ y(t) &= 0 \quad t \in [-\kappa, 0] \end{aligned} \tag{3}$$

where $y(t) \in R^n$ is the state vector of the response system, $u(t)$ is the control input to be designed and $B_u \in R^{n \times n}$ is the input matrix. Let the anti-synchronous error be defined as $e = x + y$. The objective of the anti-synchronization is to control the behavior of the response system to follow the inverse behavior of the drive system such that $\lim_{t\to\infty} \|x(t) + y(t)\|_2 \to 0$, where $\|\cdot\|_2$ is the Euclidean norm. Then, the error dynamics, can be expressed by:

$$\dot{e}(t) = -Ce(t) + D\psi_1(e(t)) + E\psi_2(e(t-\tau(t))) + B_u u(t) \tag{4}$$

where $\psi_1(e(t)) = f(y(t)) + f(e(t) - y(t))$ and $\psi_2(e(t)) = f(y(t-\tau(t))) + f(e(t-\tau(t)) - y(t-\tau(t)))$.

Since the information on the size of $\tau(t)$ is available, the controller of the following form is considered:

$$u(t) = K_1 e(t) + K_2 e(t-\tau(t)) \tag{5}$$

where K_1 and K_2 are suitable feedback gains. Substituting Equation 5 into Equation 4, we have:

$$\dot{e}(t) = -(C - B_u K_1)e(t) + B_u K_2 e(t-\tau(t)) + D\psi_1(e(t)) + E\psi_2(e(t-\tau(t))) \tag{6}$$

Remark 1

From the mean value theorem (Leu, 2010) and the Leibniz-Newton formula, that is, $e(t) - e(t-\tau(t)) = \int_{t-\tau(t)}^{t} \dot{e}(s)ds$, it is easy to see that:

$$\psi_2(e(t)) - \psi_2(e(t-\tau(t))) = \dot{\psi}_2(\sigma)(e(t) - e(t-\tau(t))) = \dot{\psi}_2(\sigma)\int_{t-\tau(t)}^{t} \dot{e}(s)ds \tag{7}$$

where σ is a point on the straight line between $e(t)$ and $e(t-\tau(t))$.

Therefore, the error dynamic (Equation 6) can be represented as follows:

$$\dot{e}(t) = -(C - B_u K_1 - B_u K_2)e(t) - (EZ + B_u K_2)\int_{t-\tau(t)}^{t} \dot{e}(s)ds + D\psi_1(e(t)) + E\psi_2(e(t)) \tag{8}$$

where $Z = \dot{\psi}_2(\sigma)$.

Assumption 1

The neuron activation function $f(\cdot)$ is continuous and satisfy $f(0) = 0$ and the Lipschitz condition, that is,

$\left\| f_i(a) - f_i(b) \right\| \le \left\| U_i(a-b) \right\|$ for any a, b and U_i are known matrices. Thus, we have:

$$0 \le -\psi_1(e(t))^T \psi_1(e(t)) + e(t)^T U_1^T U_1 e(t) \quad \text{(9a)}$$

$$0 \le -\psi_2(e(t-\tau(t)))^T \psi_2(e(t-\tau(t))) + e(t-\tau(t))^T U_2^T U_2 e(t-\tau(t)) \quad \text{(9b)}$$

Lemma 1

Let $\alpha(\cdot) \in \Re^n$, $\beta(\cdot) \in \Re^m$ and $N(\cdot) \in \Re^{n\times m}$ be defined in the set Ω, then for any matrices $R \in \Re^{n\times m}$, $S \in \Re^{n\times m}$ and $W \in \Re^{n\times m}$, the following inequality holds (Park, 1999):

$$-2\int_\Omega \alpha(r)^T N\beta(r)dr \le \int_\Omega \begin{bmatrix} \alpha(r) \\ \beta(r) \end{bmatrix}^T \begin{bmatrix} R & W-N \\ * & S \end{bmatrix} \begin{bmatrix} \alpha(r) \\ \beta(r) \end{bmatrix} dr \quad (10)$$

where $\begin{bmatrix} R & W \\ * & S \end{bmatrix} \ge 0$.

MAIN RESULTS

The following inequality lemma is necessary to develop the main theorem in this paper.

Theorem 1

For any given scalars $\lambda_1 > 0$ and λ_2, the error dynamic system (Equation 8) is asymptotically stable, if there exist the matrices $U, T_1, \ldots T_5$ and the positive definite matrices $P, H_1, Q_1, \ldots, Q_3$ such that the following matrix inequalities hold:

$$\Sigma + \lambda_1 T Q_3^{-1} T^T = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} \\ * & \Sigma_{22} & -T_3^T & -T_4^T & -T_5^T \\ * & * & -I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & \Sigma_{55} \end{bmatrix} + \lambda_1 T Q_3^{-1} T^T < 0 \quad (11)$$

where

$$\Sigma_{11} = P^T A + AP + U + U^T - (P^T(B_u K_2) + (B_u K_2)P) - (T_1 + T_1^T) + \lambda_1 H_1 + Q_1 + U_1^T U_1,$$

$$\Sigma_{12} = -U + P^T (B_u K_2) - T_1 + T_2^T,$$

$$\Sigma_{13} = P^T D + T_3^T,$$

$$\Sigma_{14} = P^T E + T_4^T,$$

$$\Sigma_{15} = T_5^T,$$

$$\Sigma_{22} = -(1-\lambda_2)Q_1 + U_2^T U_2 - (T_2 + T_2^T),$$

$$\Sigma_{55} = \lambda_1 (Q_2 + Q_3).$$

Proof

Construct a Lyapunov-Krasovskii functional of the form:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (12)$$

where

$$V_1(t) = e(t)^T P e(t) \quad (13)$$

$$V_2(t) = \int_{t-\tau(t)}^{t} e(s)^T Q_1 e(s) ds \quad (14)$$

$$V_3(t) = \int_{t-\lambda_1}^{t} \int_s^t \dot{e}(\theta)^T (Q_2 + Q_3) \dot{e}(\theta) d\theta ds \quad (15)$$

Taking the time-derivative of $V_1(t)$ along the trajectories of error dynamic (Equation 8) yields:

$$\dot{V}_1(t) = 2e(t)^T P\dot{e}(t) = 2e(t)^T P(Ae(t) + D\psi_1(e(t)) + E\psi_2(e(t))) + \Psi(t) \quad (16)$$

where $A = -(C - B_u K_1 - B_u K_2)$,

$$\Psi(t) = -2e(t)^T P(EZ + B_u K_2) \int_{t-\tau(t)}^{t} \dot{e}(s) ds$$

Using Lemma 1 and Equation 7, it is clear that:

$$\begin{aligned} \Psi(t) &\le \int_{t-\tau(t)}^{t} \begin{bmatrix} e(t) \\ \dot{e}(s) \end{bmatrix}^T \begin{bmatrix} H_1 & U - P(EZ + B_u K_2) \\ * & Q_2 \end{bmatrix} \begin{bmatrix} e(t) \\ \dot{e}(s) \end{bmatrix} ds \\ &\le \int_{t-\tau_1}^{t} \dot{e}(s)^T Q_2 \dot{e}(s) ds + \tau_1 e(t)^T H_1 e(t) + 2e(t)^T (U - P(B_u K_2))(e(t) - e(t-\tau(t))) \\ &\quad - 2e(t)^T PE(\psi_2(e(t)) - \psi_2(e(t-\tau(t)))) \end{aligned} \quad (17)$$

The time derivative of $V_2(t)$ and $V_3(t)$ are respectively, as:

$$\begin{aligned} \dot{V}_2(t) &= e(t)^T Q_1 e(t) - (1-\dot{\tau}(t)) e(t-\tau(t))^T Q_1 e(t-\tau(t)) \\ &\le e(t)^T Q_1 e(t) - (1-\lambda_2) e(t-\tau(t))^T Q_1 e(t-\tau(t)) \end{aligned} \quad (18)$$

And

$$\dot{V}_3(t) = \lambda_1 \dot{e}(t)^T (Q_2 + Q_3)\dot{e}(t) - \int_{t-\lambda_1}^{t} \dot{e}(s)^T (Q_2 + Q_3)\dot{e}(s)ds \quad (19)$$

Therefore, using Equations 17 to 20, the following result can be obtained:

$$\begin{aligned}\dot{V}(t) = {} & 2e(t)^T P(Ae(t) + D\psi_1(e(t)) + E\psi_2(e(t-\tau(t)))) + \lambda_1 e(t)^T H_1 e(t) \\ & + 2e(s)^T (U - P(B_u K_2))(e(t) - e(t-\tau(t))) + e(t)^T Q_1 e(t) - \int_{t-\tau(t)}^{t} \dot{e}(s)^T (Q_3)\dot{e}(s)ds \quad (20) \\ & - (1-\lambda_2)e(t-\tau(t))^T Q_1 e(t-\tau(t)) + \lambda_1 \dot{e}(t)^T (Q_2 + Q_3)\dot{e}(t)\end{aligned}$$

According to Leibniz–Newton formula, for any column matrix T, the following equations hold:

$$2\Theta(t)^T T(e(t) - e(t-\tau(t) - \int_{t-\tau(t)}^{t} \dot{e}(s)ds) = 0 \quad (21)$$

where $T = col\{T_1, T_2, ..., T_5\}$ and $\Theta(t) = col\{e(t), e(t-\tau(t)), \psi_1(e(t)), \psi_2(e(t-\tau(t))), \dot{e}(t)\}$.

By adding the terms on the right sides of Equation 9a to b and left side of Equation 21 to Equation 20 and by the fact that for any $r \geq 0$ and any function $f(t)$,

$$\int_{t-r}^{t} f(t)ds = rf(t) \quad (22)$$

$\dot{V}(t)$ can be expressed as follows:

$$\begin{aligned}\dot{V}(t) \leq {} & \Theta(t)^T (\Sigma + \lambda_1 T Q_3^{-1} T^T)\Theta(t) \\ & - \int_{t-\tau(t)}^{t} (\Theta(t)^T T + \dot{e}(s)^T Q_3) Q_3^{-1} (\Theta(t)^T T + \dot{e}(s)^T Q_3)^T ds\end{aligned} \quad (23)$$

Thus, if the inequality,

$$\Sigma + \lambda_1 T Q_3^{-1} T^T < 0 \quad (24)$$

holds, it follows $\dot{V}(t) < 0$. Therefore, we conclude that under sufficient condition (Equation 24), the error dynamic (Equation 5) is asymptotically stable.

Illustrative examples

The sufficient condition for asymptotically anti-synchronization of a class of delayed neural networks presented in this paper is demonstrated by a couple of examples and numerical simulations.

Example 1

A two-dimensional cellular neural network (CNN) with time varying delays is given in (Gilli, 1993) and described by the following equation:

$$\dot{x}_i(t) = -c_i x_i(t) - \sum_{j=1}^{2} d_{ij} f_j(x_j(t)) - \sum_{j=1}^{n} e_{ij} f_j(x_j(t-\tau_j)) + B_u u_i(t), \quad i = 1,2 \quad (25)$$

where $c_i = 1$, $D = (d_{ij})_{2\times 2} = \begin{bmatrix} 1.0+\pi/4 & 20 \\ 0.1 & 1.0+\pi/4 \end{bmatrix}$, $E = (e_{ij})_{2\times 2} = \begin{bmatrix} -1.3\sqrt{2}\pi/4 & 0.1 \\ 0.1 & -1.3\sqrt{2}\pi/4 \end{bmatrix}$ and $f_i(x_i) = 0.5(|x_i + 1| - |x_i - 1|)$, respectively. The delays $\tau_1(t) = \tau_2(t) = (1 - e^{-t})/(1 + e^{-t})$ are time-varying and satisfy $0 \leq \tau_j(t) \leq 1 = \tau_1, 0 \leq \dot{\tau}_j(t) \leq 0.5 = \tau_2, j = 1, 2$. The chaotic behavior of the system with delay varying form 0.845 to 1 has been reported (Gilli, 1993). Figure 1 shows the $x_1 - x_2$ plot of the uncontrolled CNN ($u(t) = [0, 0]^T$) with the initial condition $[x_1(0), x_2(0)]^T = [0.1, 0.1]^T$ for delay 0.85.

With considering $B_u = diag\{1, 1\}$ and by solving the LMI (Equation 11) via Matlab LMI toolbox, possible solutions for the feedback gains of controller (Equation 5) are as:

$$K_1 = \begin{bmatrix} -3.8471 & -24.4628 \\ -24.4628 & -279.9206 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.0004 & 0.0018 \\ 0.0019 & 0.0207 \end{bmatrix} \quad (26)$$

In the following numerical simulation, we take the initial conditions as:

$$[x_1(0), x_2(0)]^T = [0.1, 0.1]^T, \quad [y_1(0), y_2(0)]^T = [0.2, -0.2]^T$$

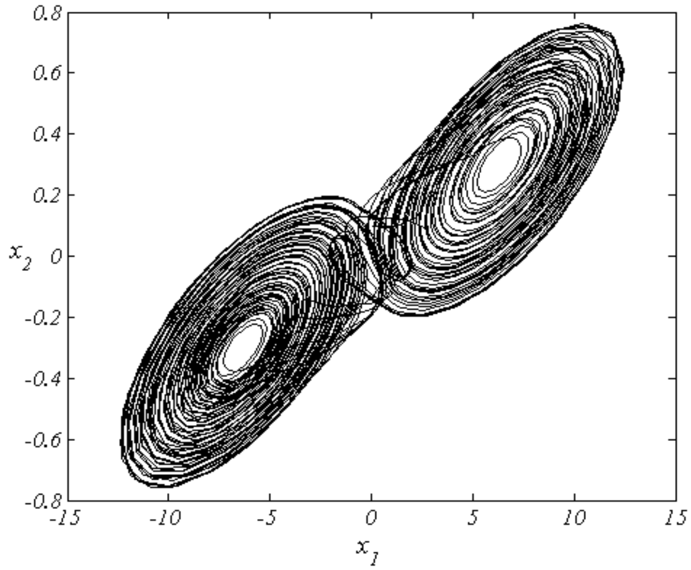

Figure 1. Chaotic behavior of drive system (Equation 25) in phase space.

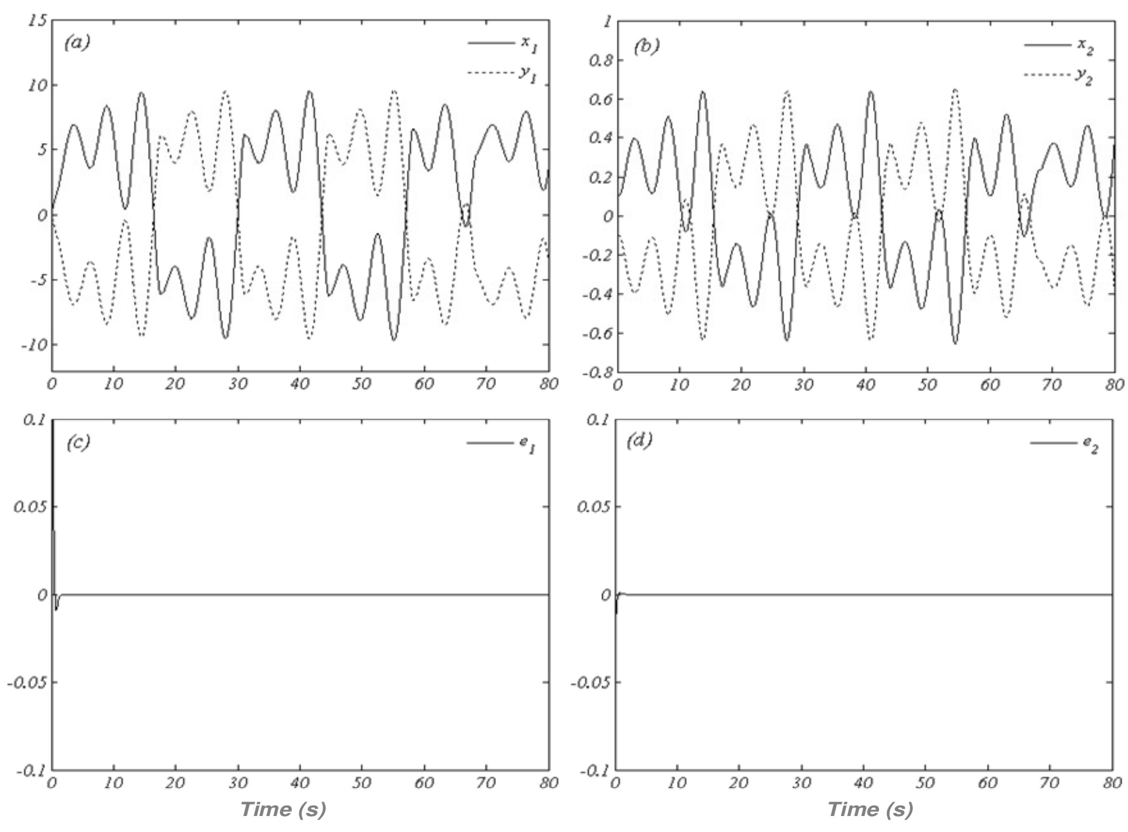

Figure 2. Simulation results of Example 1a and b state trajectories (solid line = master system, dashed line = slave system); (c) and (d) anti-synchronization errors.

Simulation results are as shown in Figure 2. The state responses of the drive and response systems are as shown in Figure 2a and b, respectively. Figure 2c and d, respectively, shows that the anti-synchronization errors

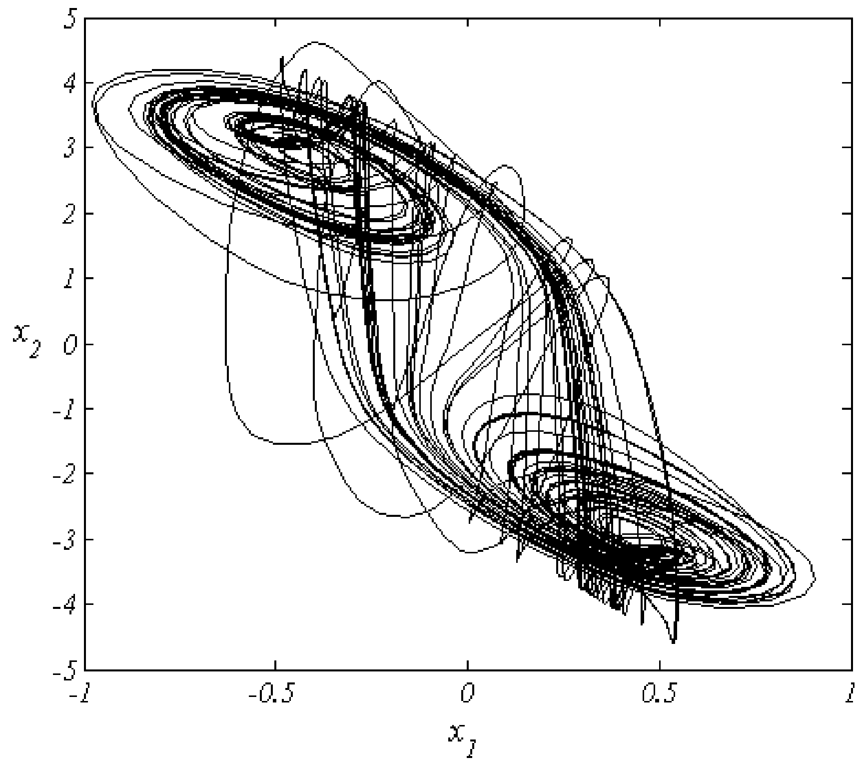

Figure 3. Chaotic behavior of drive system (Equation 27) in phase space.

$e_1(t)$ and $e_2(t)$ between drive and response systems are stabilized to zero, respectively after a short while.

Example 2

Consider the following two-order Hopfield neural network (HNN) with time-varying delay:

$$\dot{x}_i(t) = -c_i x_i(t) - \sum_{j=1}^{2} d_{ij} f_j(x_j(t)) - \sum_{j=1}^{n} e_{ij} f_j(x_j(t-\tau_j)) + B_u u_i(t), \quad i = 1,2 \qquad (27)$$

where $c_i = 1$, $D = (d_{ij})_{2\times 2} = \begin{bmatrix} 2.1 & -0.12 \\ -5.1 & 3.2 \end{bmatrix}$, $E = (e_{ij})_{2\times 2} = \begin{bmatrix} -1.6 & -0.1 \\ -0.2 & -2.4 \end{bmatrix}$ and $f_i(x_i) = \tanh(x_i)$, respectively. The delays $\tau_1(t) = \tau_2(t) = e^t/(1+e^t)$ are time-varying and satisfy $0 \le \tau_j(t) \le 1 = \tau_1, 0 \le \dot{\tau}_j(t) \le 0.5 = \tau_2, j = 1,2$. Figure 3 shows the $x_1 - x_2$ plot of the uncontrolled HNN ($u(t) = [0,0]^T$) with the initial condition $[x_1(0), x_2(0)]^T = [-0.35, 0.5]^T$ for delay 0.85.

With $B_u = diag\{1,1\}$ and by solving LMI (Equation 11) in Theorem 1, we get:

$$K_1 = \begin{bmatrix} -23.9331 & 10.0806 \\ 10.0806 & -14.9990 \end{bmatrix}, \quad K_2 = 10^{-3} \times \begin{bmatrix} 0.3027 & -0.1208 \\ -0.1224 & 0.1695 \end{bmatrix} \qquad (28)$$

The initial conditions drive and response systems are as:

$$[x_1(0), x_2(0)]^T = [0.2, 0.5]^T, \quad [y_1(0), y_2(0)]^T = [-1.3, 2.1]^T$$

The simulation results are as shown in Figure 4. From Figure 4c and d, one can see that the anti-synchronization error between the two drive and response systems state vectors asymptotically converges to zero.

To present a quantitative comparison between the proposed method and nonlinear feedback control method (Cui and Lou, 2009), the two following criteria are used:

1. Synchronization error settling time (SEST). It is the time at which $|e| < 0.005$.
2. Integral of SQUARED synchronization error (ISSE) up to SEST.

As we know, the less the SEST, the sooner the convergence. The less the ISSE the better the synchronization achieved. Table 1 presents the results. Referring to

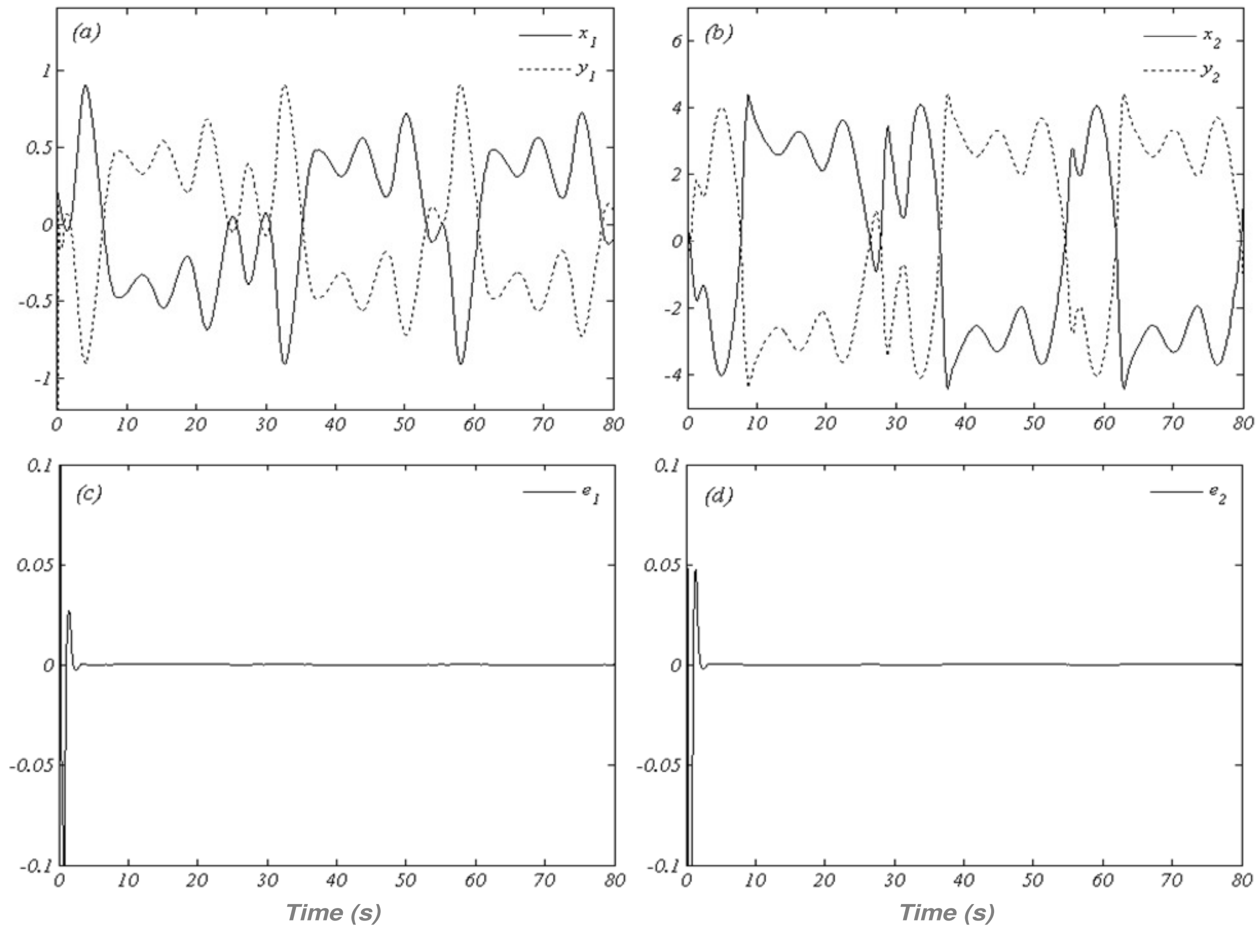

Figure 4. Simulation results of Example 2 a and b state trajectories (solid line = master system, dashed line = slave system). (c) and (d) anti-synchronization errors.

Table 1. Comparison between two different methods of anti-synchronization.

Criteria		ISSE	SEST (Sec.)
Nonlinear feedback control method (Cui and Lou, 2009)	Example 1 (CNN)	ISSE(e_1)= 139.52, ISSE(e_2)= 128.94	SEST(e_1) $\approx$ 12.78 SEST(e_2) $\approx$ 10.42
	Example 2 (HNN)	ISSE(e_1)= 1046.08, ISSE(e_2)= 1253.4	SEST(e_1) $\approx$ 14.56 SEST(e_2) $\approx$ 15.13
Proposed method	Example 1 (CNN)	ISSE(e_1)= 10.6581, ISSE(e_2)= 0.2044	SEST(e_1) $\approx$ 1.036 SEST(e_2) $\approx$ 0.3176
	Example 2 (HNN)	ISSE(e_1)=239.1924, ISSE(e_2)=136.8759	SEST(e_1) $\approx$ 1.27 SEST(e_2) $\approx$ 0.7131

the Table 1, we can conclude that the speed of synchronization with the proposed method is better than that with nonlinear feedback control methodology.

Conclusion

By designing time-delay feedback controller, this paper deals with the anti-synchronization problem of a class of chaotic neural networks with time-varying delays. An effective sufficient condition for global asymptotic anti-synchronization between the state vectors of the drive-response chaotic neural networks has been derived. These conditions, which are expressed in terms of linear matrix inequalities, are used to design suitable feedback gains in the response networks. Also, it illustrates that the speed of synchronization of the states is very fast and better than what was obtained by the nonlinear feedback control methodology. Two numerical examples with graphical illustrations are given to illuminate the presented synchronization scheme.

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