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Robust location algorithm based on weighted leastsquares support vector machine (WLS-SVM) for nonline-of-sight environments

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One of the main problems facing accurate location in wireless communication systems is non-line-ofsight (NLOS) propagation. Though learning location methods perform well in NLOS environments, learning location methods may be improved further since these methods do not consider outliers in the training data set. In this paper, we extend weighted least squares support vector machine (WLS-SVM) algorithm to mobile location problem. The proposed method can effectively suppress outliers with different weights. In simulation, we analyze the effects of the number of training points, the percentage of outliers in training data set, the standard deviation and mean of outliers, and the standard deviation of measurement error. Simulation results show that the proposed algorithm clearly outperforms three other algorithms (LS method, kernel method and LS-SVM based method).

Key words: Weighted least-squares support vector machine (WLS-SVM), non-line-of-sight (NLOS).

INTRODUCTION

Geolocation, or location estimation in terms of geographic coordinates of a mobile station (MS) with respect to base stations (BSs) in wireless communication systems, has gained considerable attention over the past decade, especially since the Federal Communication Commission (FCC) passed a mandate requiring cellular providers to generate accurate location estimates for Enhanced-911 (E-911) services [FCC, 1999]. This has boosted the research in the field of wireless location as an important public safety feature, which can also add many other potential applications [Sayed et al., 2005]: Locationsensitive billing, fraud protection, person/asset tracking, fleet management, intelligent transportation systems (ITS), mobile yellow pages, wireless system design, and radio resource management, etc.

Currently, the most often used techniques for wireless location are time-of-arrival (TOA), time-difference-ofarrival (TDOA), signal strength (SS), angle-of-arrival (AOA) based methods, or combination of these. Unfortunately, the main shortcoming of these techniques is that they require line-of-sight (LOS) propagation for accurate estimates [Gui et al., 2011; Zhangxin et al. (2011)]. However, errors due to NLOS propagation are dominant errors in location estimation in badurban or urban areas where people are more interested in the MS's location [Caffery and Stuber, 1998]. For NLOS situation, the propagating signal between the MS and the BS goes through reflections and refractions off many objects in its path. This causes the signal to arrive the receiver from a different angle than the direct path between the MS and the BS, and for ranging measurements (or equivalently, TOA), it will add a large positive error in addition to standard measurement error [Caffery and Stuber, 1998].

To suppress the NLOS error, different methods have been addressed in the literature [Wylie and Holtzman, 1996; Liao and Chen, 2006; Wuk et al., 2006; Venkatraman et al., 2004; Wang et al., 2001; Cong and Zhang, 2001; Chen, 1999]. The methods for NLOS mitigation considered in Wylie and Holtzman (1996), Liao and Chen (2006), and Wuk et al. (2006) which require a time series of range measurements from a moving user,

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work well when MS is moving. The method in Venkatraman et al. (2004) uses a constrained condition to suppress NLOS errors. The algorithm presented in Wang et al. (2001) replaces the nonlinear terms in the measurement equation with a single variable, and adds a loose variable to the equation, in order to simplify the problem and make it a mathematical programming problem. The method in Cong and Zhang (2001) generally assumes that there is a large number of BSs available for location, with only a small subset of those being NLOS. The method in Chen (1999) attempts to selectively remove or weight NLOS corrupted measurements based on their deviation from the majority-LOS BSs' estimate using residual weighting algorithms.

Unfortunately, none of these methods can solve the NLOS problem adequately, since too many elements affect the signal propagation, and the propagation environment varies from place to place. Especially in badurban or urban areas, NLOS propagation is common for radio frequency signal. Often, only one or two, even none of the signals from BSs (or MS) can propagate in LOS scenario. However, another set of location algorithms presented in Binghao et al. (2005), Michael et al. (2003), and Sun and Guo (2005), based on the learning theory and the information of training points, seem immune to such obstacles. The method in Binghao et al. (2005) first generates the NLOS correction map based on Kriging method, and then use the correction map to rectify the distorted MS location. The method presented in Michael et al. (2003) introduces the use of nonparametric kernel-based estimators for location of MS using measurements of propagation delays. Further, a least square support vector machine (LS-SVM) based location method in Sun and Guo (2005) is proposed to learn the relationship between the raw signal TOA measurements and the MS's location. However, all of these methods mentioned in Binghao et al. (2005), Michael et al. (2003) and Sun and Guo (2005) do not consider outliers in the training data set. In practice, a global positioning system (GPS) receiver may be used to provide the location estimates of training points. Because of multipath and NLOS situations, the performance of GPS is severely degraded, and the location estimates of training points will appear outliers, especially in badurban or urban areas. As a result, the position accuracy of the MS will be corrupted by outliers, and we need to consider another better learning algorithm, which can make the location algorithm robust for outliers.

WLS-SVM algorithm presented in Suykens et al. (2002) is a powerful tool to obtain a robust estimation for function estimation and density estimation. It can effectively suppress outliers by weighting training points based on the error variables estimated by LS-SVM. In this paper, we extend WLS-SVM algorithm to mobile location problem. The proposed method has more tole-rance for outliers and simulation results show the good

performance of this new method.

LS-SVM for mobile location

Here, we present the LS-SVM based location algorithm [Sun and Guo, 2005]. In general, learning location algorithms consist of two phases: Training and positioning. During the training phase, the parameters of learning algorithms are estimated using measurements at some known points (training points). During the positioning phase, the measurement of MS at an unknown location is performed, and then the position of MS at an unknown location can be obtained using the parameters estimated in the training phase. For simplify-cation, we consider TOA based method. Assuming that (\hat{x}_k, \hat{y}_k) is the position of the k th training point, which

can be estimated by GPS receiver, r_{ki} is the range measurement of the i th BS at the k th training point. Given a training data set of N points:

$$D = \{ (I_k, O_k) \mid k = 1, 2, \cdots, N \}$$
(1)

With input data $I_k \in \mathbb{R}^n$ and output data $O_k \in \mathbb{R}$; where $I_k = [r_{k1} \cdots r_{kn}]$ is the vector of the range measurements at the k th training point, n is the number of BSs, and $O_k = \hat{x}_k$ or $O_k = \hat{y}_k$, depending on different outputs. Here, LS-SVM model has N inputs and two outputs. The algorithm can be directly scaled up to other location techniques such as TDOA, SS, and AOA. For other location techniques, only difference is that the input data vector I_k is the vector of the other measurements according to different location techniques. One considers the following optimization problem in primal weight space:

$$\min_{w,b,e} J(w,e) = \frac{1}{2} w^T w + \frac{1}{2} \gamma \sum_{k=1}^{N} e_k^2$$
(2)

Subject to

$$O_k = w^T \varphi(I_k) + b + e_k, k = 1, \cdots, N$$
(3)

where $\varphi(): \mathbb{R}^n \to \mathbb{R}^{n_h}$ is a nonlinear mapping in kernel space, $w \in \mathbb{R}^{n_h}$, error variable $e_k \in \mathbb{R}$, and b is a bias. J is a loss function and γ is an adjustable constant. The aim of the mapping function in kernel space is picking

out features from primal space and mapping training data into a vector of a high dimensional feature space in order to solve the nonlinear regression problem.

According to optimal function (2), we define the Lagrangian function as:

$$L(w,b,e,a) = J(w,e) - \sum_{k=1}^{N} a_k \{ w^T \varphi(I_k) + b + e_k - O_k \}$$
(4)

where a_k are Lagrangian multiplier, also known as support vectors $(a_k \in R)$. The optimality of upper function is as following sets of linear equation instead of quadratic program in the traditional SVM.

$$\frac{\partial L}{\partial w} = 0 \rightarrow w = \sum_{k=1}^{N} a_k \varphi(I_k)$$
$$\frac{\partial L}{\partial b} = 0 \rightarrow w = \sum_{k=1}^{N} a_k = 0$$
$$\frac{\partial L}{\partial e_k} = 0 \rightarrow a_k = \gamma e_k$$
$$\frac{\partial L}{\partial a_k} = 0 \rightarrow w^T \varphi(I_k) + b + e_k - O_k = 0$$
(5)

Here, $k = 1, \dots, N$. After eliminating variables (w, e), we get the following matrix equations:

$$\begin{bmatrix} 0 & 1_{v}^{T} \\ 1_{v} & \Omega + \frac{1}{\gamma}I \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ O \end{bmatrix}$$
(6)

where $O = \begin{bmatrix} O_1 & \cdots & O_N \end{bmatrix}^T$, $1_v = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}_N$, $a = \begin{bmatrix} a_1 & \cdots & a_N \end{bmatrix}^T$, and $\Omega_{kl} = \varphi(I_k)^T \varphi(I_l)$, $k, l = 1, \cdots, N$

According to Mercer's condition, there is mapping φ and kernel function:

$$K(I_k, I_l) = \varphi(I_k)^T \varphi(I_l)$$
⁽⁷⁾

After training phase, the parameters a, b can be obtained by solving (6). Assuming that (\hat{x}, \hat{y}) is the position estimates of the MS at an unknown location, r_i is the range measurement of the i th BS when the MS locates at the unknown location, and $I = [r_1 \cdots r_n]$ is the corresponding vector of range measurements. During the positioning phase, the location of MS at an unknown location can be obtained:

Many kernel functions such as poly-nominal, MLP, splines, RBF have similar performance. RBF was chosen due to its similarity with the Euclidean distance and also since it gives better smoothing and continuous properties even with a small number of samples [Elgammal et al., 2002]. Another reason for using RBF kernel is that the RBF kernel function is easy to integrate and differentiate and can lead to mathematically tractable solution.

Generally, a grid search using leave-10-out crossvalidation [Caffery, 2000] is employed to tune these parameters during the training phase. Grid search algorithm:

1. For each set of values of the parameters, leave-10-out cross-validation on the training set is performed to predict the prediction error.

2. Select the set of values of the parameters that produced the model that gave the smallest prediction error (optimal parameter settings).

3. Train the model with the optimal parameter settings with the model training set and test it with a test set (test is not used for training).

WLS-SVM FOR MOBILE LOCATION

Here, we extend WLS-SVM algorithm to mobile location problem to mitigate the effect of outliers in the training data set. "LS-SVM for mobile location shows that the parameters a, b may be corrupted by outliers since every training points is weighted by the same value in the LS-SVM. Here, in order to obtain a robust estimate based on the previous LS-SVM solution, in a subsequent step,

we can weight the error variables $e_k = a_k / \gamma$ by

$$O(I) = \sum_{k=1}^{N} a_{k} K(I, I_{k}) + b$$
(8)

where $O = \hat{x}$ or $O = \hat{y}$, depending on different outputs.

There are only two parameters to be tuned: The kernel setting and γ . Kernel function has different types, such as poly-nominal, multi-layered perceptron (MLP), splines, radial basis functions (RBF) and soon. In Sun and Guo (2005), the authors focus on RBF kernels which corresponds to

$$K(I_k, I_l) = \exp(-\frac{\|I_k - I_l\|_2^2}{2\sigma^2})$$
(9)

weighting factors V_k . This leads to the optimization problem.

$$\min_{w',b',e'} J(w',e') = \frac{1}{2} w'^T w' + \frac{1}{2} \gamma \sum_{k=1}^{N} v_k e'_k^2$$
(10)

Subject to

$$O_k = w^T \varphi(I_k) + b' + e'_k, k = 1, \cdots, N$$
 (11)

The Lagrangian function becomes:

$$L(w', b', e', a') = J(w', e') - \sum_{k=1}^{N} a'_{k} \{ w^{T} \varphi(I_{k}) + b' + e'_{k} - O_{k} \}$$
(12)

The unknown variables for this WLS-SVM problem are denoted by the 'symbol. From the conditions for optimality and elimination of W, e we get the following matrix equations:

$$\begin{bmatrix} 0 & \mathbf{1}_{\nu}^{T} \\ \mathbf{1}_{\nu} & \boldsymbol{\Omega} + V_{\gamma} \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ O \end{bmatrix}$$
(13)

where the diagonal matrix V_{γ} is given by:

$$V_{\gamma} = diag\{\frac{1}{\mathcal{W}_1} \quad \cdots \quad \frac{1}{\mathcal{W}_N}\}$$
(14)

The choice of the weights v_k is determined based on the error variables $e_k = a_k / \gamma$ from the LS-SVM equation (6). Robust estimates are obtained then (Suykens et al., 2002) by taking:

$$v_{k} = \begin{cases} 1 & |e_{k}/\hat{s}| \leq c_{1} \\ \frac{c_{2} - |e_{k}/\hat{s}|}{c_{2} - c_{1}} & c_{1} \leq |e_{k}/\hat{s}| \leq c_{2} \\ 10^{-4} & otherwise \end{cases}$$
(15)

where \hat{s} is a robust estimate of the standard deviation of the LS-SVM error e_k . \hat{s} can be estimated by [Andrzej and Shunichi, 2005]:

$$\widehat{s} = 1.483 Med\{\left|e_{k} - Med\left(\left|e_{k}\right|\right)\right\}$$
(16)

It can be seen from (6) and (13) that the main difference

between LS-SVM and the proposed method is the weights v_k . LS-SVM gives the same v_k for each training points while different v_k are arranged for different training points in the proposed method. Since the outliers appear in the positions of the training data,

the weights v_k in the proposed method can help to suppress the outliers such that the positioning accuracy can be improved.

The cost function of (2) in the unweighted LS-SVM formulation is optimal under the assumption of a normal

Gaussian distribution for e_k . The procedure (15) corrects for this assumption in order to obtain a robust estimate when this distribution is not normal. Eventually, the procedure (10) (15) can be repeated iteratively, but in practice, one single additional WLS-SVM step will often

be sufficient. The constants C_1 , C_2 can be determined by the percentage of outliers in the training data set [Chen and Jain, 1994]. However, the percentage of outliers in the training data set is generally unknown in practice

situation. Here, $c_{\rm 1},~c_{\rm 2}$ are typically chosen as $~c_{\rm 1}\,{=}\,2.5$

and $c_2 = 3$ [Suykens et al., 2002]. This is a reasonable choice taking into account the fact that for a Gaussian distribution, there will be very few residuals larger than $2.5\hat{s}$. Using these weightings, we can correct for y-

outliers (outliers in the location estimates of training points) or for a non-Gaussian instead of Gaussian error distributions.

This leads us to the following algorithm:

1. Given training data $\{I_k, O_k\}_{k=1}^N$, find an optimal (γ, σ) combination by solving linear systems (6). For the optimal (γ, σ) combination one computes $e_k = a_k / \gamma$ from (5).

- 2. Compute \widehat{s} from (16).
- 3. Determine the weights V_k based on e_k , \hat{s} .

4. Compute a' and b' from (13), estimate the position of MS at an unknown location using $O(I) = a'^T K(I, I_k) + b'$

SIMULATIONS

The estimators are evaluated for location accuracy when the MS is located outdoors in urban microcells, since this is the region of greatest interest to cellular network providers. Assuming in a Manhattan-like urban environment, the geometry of the base-station configuration is shown in Figure 1. The square regions represent buildings, and other regions represent streets. This



Figure 1. Manhattan-like urban environment

configuration was used since similar configurations have been used to evaluate other MS location schemes [Wuk et al., 2006; Binghao et al., 2005; Michael et al., 2003]. The MS location is sampled from a uniform distribution over the street. This is the worst case because there is no prior information about MS location other than cell

residence. We model the range measurement r_i of the i th BS as:

$$r_i = r_i^0 + n_i + NLOS_i$$

where r_i^{0} is the true distance between the MS and BS i, n_i represents the standard measurement error subjected to zero mean Gaussian distribution with standard deviations σ , $NLOS_i$ is a random variable representing the error due to NLOS propagation. There are, broadly, three types of methods to generate the NLOS error [Binghao et al., 2005]. The first method is based on

deterministic, Gaussian or other distribution model. Though this method is convenient, nevertheless, it can hardly describe the real NLOS error since it cannot describe the spatial correlation of the real environments. On the contrast, 3D ray tracing plus Poisson or Rician model can accurately generate the NLOS error in a special environment, but it is a very complex method. It is time consuming and also costly. The chosen method in this paper is the medium accuracy model by Dijkstra algorithm (2D only), which was also used in Wuk et al. (2006), Binghao et al. (2005) and Michael et al. (2003).

The location estimates of the k th training point can be modeled as:

$$\hat{\theta}_k = \theta_k^0 + e_k + outliers_k$$

where $\hat{\theta}_k = [\hat{x}_k, \hat{y}_k]$ is the estimated position coordinates of the training point k, which is provided by



Figure 2. Performance comparison with different number of training points.

GPS. $\theta_k^0 = [\hat{x}_k^0, \hat{y}_k^0]$ is the true position coordinates of

the training point k. e_k represents the standard measurement error, which is brought by the GPS receiver equipments and could be taken as a Gaussian random variable with zero mean and 10 m standard deviation.

*outliers*_k represents the outlier, brought by multipath and NLOS situations. We model *outliers*_k as a Gaussian random variable with 100 m mean and 100 m standard deviation. The percentage of outliers in the training points set is denoted as P.

The WLS-SVM based location method proposed in this paper is compared with conventional LS method [Caffery, 2000] and two learning location methods (kernel method [Michael et al., 2003] and LS-SVM based location method [Sun and Guo, 2005]). Here, we evaluate the performance of the proposed method through comparing its mean location errors (MLE) with those algorithms.

 $MLE = E[\sqrt{(\hat{x} - x_{MS})^2 + (\hat{y} - y_{MS})^2}]$ is obtained from the average of 5000 independent runs. Where (x_{MS}, y_{MS}) is the real position coordinates of MS, and (\hat{x}, \hat{y}) is the estimated position coordinates of MS.

Effects of the number of training points

The number of training points N is a critical factor on

the performance of the system. The purpose of this experiment is to compare the performance of three learning location methods with different sizes of training set. The 20% training points have outliers, and the standard deviation of measurement error is 30 m. The number of training points is varied from 10 to 50. Figure 2 shows the mean location error, for comparison, the results of other two learning location method (kernel method and LS-SVM method) are also given. It is observed from Figure 2 that the mean location error decreases with the number of training points and the proposed method clearly outperforms the other two. Figure 2 also shows that the WLS-SVM based location algorithm is robust to outliers. Once the training set reaches a certain size, new training points are adding mostly redundant information. This point appears to be around N = 30 for this environment. The decision of how many training points are needed is a trade off between the cost of taking the measurements of training points and how much accuracy is desired.

Effects of the percentage of outliers in the training data set

Simulations are performed to study how the mean location error is affected by the percentage of outliers in the training data set. This performance is also compared with kernel method and LS-SVM method, as shown in Figure 3. The number of training points is 30, and the standard deviation of measurement error is 30 m. The percentage of outliers in the training data set is varied



Figure 3. Performance comparison with difference percentage of outliers in the training data set.



Figure 4. Mean location error versus he standard deviation and mean of outliers in kernel method.

from 0 to 50%. It can be seen from Figure 3 that the mean location error increases with the percentage of outliers in the training data set and the WLS-SVM method performs better than kernel method and LS-SVM method when $P \neq 0$. Figure 3 also shows that the WLS-SVM method has the similar performance as the LS-SVM method when P = 0. In other words, the WLS-SVM method can also work well in the situation where the training data set has not outliers.

Effects of the standard deviation and mean of outliers

The purpose of this experiment is to study how the mean location error is affected by the standard deviation and mean of outliers. Outliers are modeled as Gaussian random variables with different means and standard deviations in this experiment. The number of training points is 30, and the standard deviation of measurement error is 30 m. The percentage of outliers in the training data set is 20%. Figures 4 to 6 shows the mean location



Figure 5. Mean location error versus the standard deviation and mean of outliers in LS-SVM method.



Figure 6. Mean location error versus the standard deviation and mean of outliers in WLS-SVM method.

error of kernel method, LS-SVM method, and WLS-SVM method, versus the standard deviation and mean of outliers. It can be concluded from Figures 4 to 6 that the proposed method deals with large outliers more

effectively than the other algorithms. As the standard deviation of outliers become large, the improvement in performance of the proposed method becomes more apparent.



Figure 7. Performance comparison with difference standard deviation of standard measurement error.

Effects of the standard deviation of range measurement error

Here, we consider how the mean location error is affected by the standard deviation of the range measurement. The 20% training points have outliers, and the number of training points is 30. The standard deviation of standard measurement error is varied from 10 to 60 m. Simulation results in Figure 7 show the relationship between the mean location error and the standard deviation of standard measurement error. Results in Figure 7 also declare that the WLS-SVM method has the best performance among four methods while LS method, which does not use the information of training points, provides extremely poor performance.

Conclusions

The localization of an MS can have significant errors when NLOS measurements are present. Though learning location methods perform well in NLOS environments, learning location methods may be improved further since these methods do not consider outliers in the training data set. This paper has proposed a method, based on WLS-SVM, to mitigate NLOS errors. The proposed method can effectively suppress outliers with different weights. A comparison is performed between proposed method and three other methods (LS method, kernel method and LS-SVM method). Simulations are performed in different cases and it can be seen from simulations and theoretic analysis that the proposed algorithm clearly outperforms the other three.

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