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Full Length Research Paper

Optimal power control game for primary-secondary user in cognitive radio network

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In cognitive radio network (CRN), the utilities results in Nash equilibrium of power control game without using pricing are inefficient. In this paper, a distributed power control algorithm is proposed to improve the utilities of both primary user (PU) and secondary users (SUs) in the CRN based on game theoretic framework. A distributed power control is a non-cooperative power control game, and the quality of service (QoS) received by PU and SUs terminals are referred to as the utility function. PU and SUs act as decision makers in the game and they maximize their utilities in a distributed fashion. We introduce a new pricing function for SUs as a function of transmit power and square amount of interference in order to guide SUs to an efficient Nash equilibrium point. Analysis of the existence and uniqueness of Nash equilibrium for the proposed power control game with pricing is presented. Simulation results show that the proposed power control algorithm via a new pricing function maximizes the number of SUs access the unused spectrum, and improves the utilities of PU and SUs.

Key words: Cognitive radio, power control, game theory, pricing.

INTRODUCTION

The increase in demand for wireless services coupled with the limited network resources available in wireless network makes it necessary to regulate the network resources efficiently. To address this issue, cognitive radio (CR) has been introduced as a promising platform to improve the spectrum utilization efficiency by allowing secondary users (SUs) to sense and access the unused part of the licensed spectrum. It has the ability to perceive its radio frequency environment, learns, adapt, and then reconfigure the system operation to capitalize the radio spectrum and guarantee a highly reliable communication (Haykin, 2005). To attain efficient spectrum utilization in CRN, dynamic spectrum access (DSA) has been used. In DSA, the licensed spectrum is assigned to the primary users (PUs); however there will always be instances whereby the licensed spectrum is unutilized. When SUs utilize the unused part of the spectrum, they become a source of interference to others PUs and SUs. Therefore, an efficient power control is necessary to reduce the amount of interference by reducing the power consumed at PU and SUs terminals.

Recently, game theory is emerging as a successful tool used to study the problem of power control in wireless data networks. Goodman and Mandayam (2000) modeled the power control in wireless data network as a non-cooperative power control game and defined the user's utility function as the ratio of throughput to transmit power. MacKenzie and Wicker (2001) provided some motivation for using game theory in communication system, especially in power control problem. Saraydar et al. (2001) and Saraydar et al. (2002) used pricing function to obtain a more efficient solution for the power control game. Xiao et al. (2001) defined the utility function as S-shaped (sigmoidal) function of the user's signal to interference ratio (SIR). Alpcan and his co-authors (2002)

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Abbreviations: CRN, Cognitive radio network; **PU,** primary user; **SU,** secondary user; **QoS,** quality of service; **DSA,** dynamic spectrum access; **IC,** interference cap; **SIR,** signal to interference ratio.

defined a utility function as a logarithmic, concave function of the user's SIR to maximize the spectral efficiency. In addition, many authors studied power control game in several works in CRN, which considered the SUs as decision makers. Huang et al. (2006) studied the coexistence between PUs and SUs under interference temperature constraints. Jia and Zhang (2007) studied the spectrum sharing optimal power control for SUs based on the system that was studied by Huang et al. (2006). Wang et al. (2007a) investigated the power control with and without interference temperature constraints. Wang et al. (2007b) studied the power control with exponential pricing function among the SUs. In all the works (Huang et al., 2006; Wang et al., 2007a; Wang et al, 2007b; Jia and Zhang, 2007), QoS guarantee of SUs was the main goal, leaving behind the QoS of PU unattended. The excellent work of spectrum sharing power control game studied by Li and Jayaweera (2008a), in which both the PU and SUs acted as decision makers. The system reward PU for allowing SUs to share their licensed spectrum, and penalize it when the amounts of interference becames greater than the interference cap (IC). Interference cap (IC) is defined as the maximum value of interference that the PU could tolerate (Li and Jayaweera, 2008a). Li and Jayaweera, (2008a) considered a matched filter (MF) detector to be at the PU and SUs receivers. On the other hand, Li and Jayaweera (2008b) considered a linear minimum mean squared error (LMMSE) detector for PU and SUs receivers. Next, Jayaweera and Li (2009) proposed a dynamic spectrum leasing (DSL) concept between PU and SUs, in which the PU, who owns the spectrum property right, has an incentive to allow SUs to operate in its spectrum band. All SUs adapt their transmission powers to achieve a certain transmission quality, and ensure low interference to the PU and other SUs (Li and Jayaweera, 2008a; Jayaweera and Li, 2009). However, the results of Nash equilibrium point are not efficient.

The main contribution of this paper is the new pricing function, which is used to guide SUs terminals to achieve their transmission using less power, and consequently reduce the amount of interference in the system. Then, we prove that there is a unique Nash equilibrium in our game via pricing, and we propose an algorithm to achieve an optimal solution. Numerical analysis shows that the power control game via our new pricing function improves the performance of PU and SUs. In the next section, we introduce system and game models. Then the PU utility function and the SUs net utility function (utility - price) are defined. Next section proves that the Nash equilibrium exists and is unique in the proposed power control game with pricing. In the last section, numerically we analyze the performance of our proposed power control game with pricing and compare the results with Li and Jayaweera (2008a).

MATERIALS AND METHODS

System model

We consider a cognitive radio system within the range of 3G cellular network, which consists of one PU base station and one SUs base station Figure (1). We consider one PU who owns the licensed spectrum, and K unlicensed SUs that communicate with their base station using DS-CDMA scheme. The code correlation coefficient between the signaling waveforms of PU and k -th SU is c_{0k}, between the k -th SU and PU is c_{k0}, and between j -th SU and k -th SU is c_{jk}, where $\forall\, j,k \in K$. The code correlation coefficients between two users j,k is computed as $c_{jk} = \mathbf{s}_k^T \mathbf{s}_j$, where $\mathbf{s}_j, \mathbf{s}_j$ are the code signatures of user j and user k. We denote h_{sk} as the path gain between k -th SU and SUs base station and h_{pk} as the path gain between k -th SU and PU base station. Similarly, the path gains between PU and PU base station and between PU and SU base station are denoted as h_{p0}, h_{s0} respectively.

For simplicity, we neglect the interference from other adjacent cells and the interference from others PUs. Thus, the SINR of PU can be writen as:

$$\gamma_0 = \rho_0 \frac{h_{p,0}^2\, p_0}{\sum_{j=1}^{K} h_{pj}^2\, c_{sp}^2\, p_j + \sigma^2} \tag{1}$$

where p_0, σ^2 are the PU transmission power in watts and Additive White Gaussian Noise power (watts) respectively. ρ_0 is a parameter for power control in $[0, P_0^{max}]$, where p_k^{max} is the PU maximum power . Similarly, the SINR of the k -th SU at the SU receiver is

$$\gamma_k = \rho_k \frac{h_{sk}^2 p_k}{\sum_{j\neq k}^{K} h_{sj}^2\, c_{j,k}^2 p_j + h_{s0}^2 c_{ps}^2 p_0 + \sigma^2}\ \forall\, k \in K \tag{2}$$

where p_k is the k -th SU transmission power in watts and ρ_k is a parameter that we used in this paper for power control between $[0, \quad P_k^{max}]$, where $P_k^{\max}$ is the k -th SU maximum power.

Game model

Let $G = [K, \{P_{0k}\}, \{u_0(.), u'_k(.)\}]$ denoted the non-cooperative power control game for cognitive radio networks, where $k = \{0, 1, 2, \ldots, k\}$ is the index set for the PU and SUs, which 0-th indicates to PU and $k = 1,2,\ldots,K$ represents the k -th SU. $P_{0k} = Q_0 \times P_1 \times P_2 \times ..\times P_k$ is the action space of PU and the $k -$ th SUs, where Q_0 represents the PU's action set and P_k for $k = 1,2,\ldots,k$ represents the k -th SU's action set.

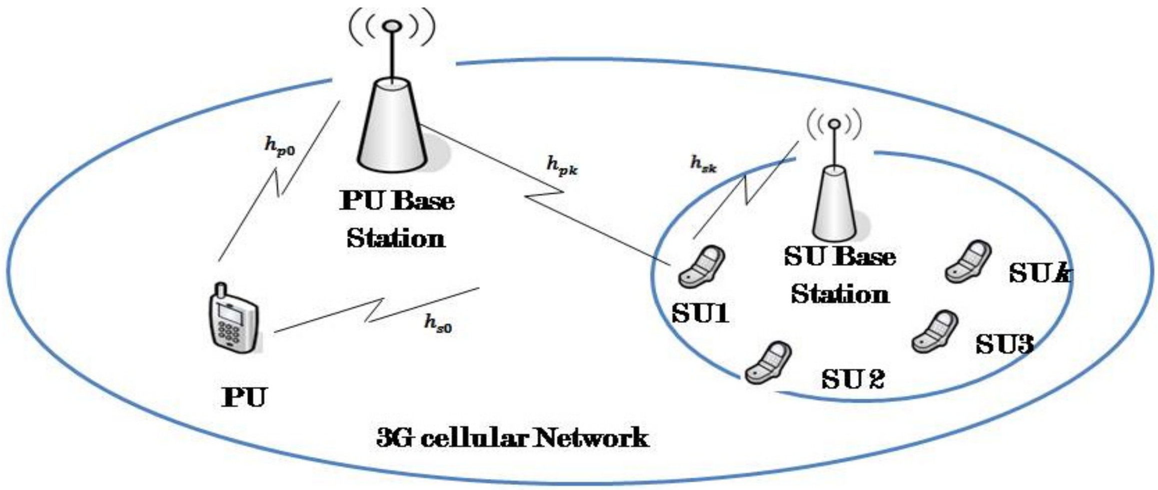

Figure 1. System model of cognitive radio network.

$u_0(.)$ is the utility function of PU, and $u'_k(.)$ is the net utility of the k -th SU. PU selected interference level Q_0 such as $Q_0 \in [0,\ Q_0^{max}]$, where Q_0^{max} is the maximum interference level that the PU can tolerate. Each SU selected a power level $p_k \in [0, P_k^{max}]$, where P_k^{max} is the maximum power of SUs.

Primary user utility function

PU utility function is defined as a function of interference and interference cap as in Li and Jayaweera (2008a) work:

$$u_0(Q_0,p_0) = Q_0 - \mu_1[(Q_0 - I_0)^2 u(Q_0 - I_0)] - \mu_2[(e^{(I_0-Q_0)} - 1)\, u(I_0 - Q_0)] \tag{3}$$

where $I_0 = \sum_{j=1}^{k} h_{pj}^2 c_{s0}^2 p_j$ is the amount of interference from secondary users, μ_1, μ_2 are pricing coefficients for primary user, and $\mathbf{u}(.)$ is step function that is define as follows:

$$u(x) = \begin{cases} 1 \text{ if } x \geq 0 \\ 0 \text{ otherwise} \end{cases} \tag{4}$$

PU utility function that is used in equation (3) should satisfy the transmission quality of PU. PU will be significantly penalized when it cannot achieve its QoS requirement on occasion of $Q_0 < I_0$. PU utility will also be relatively penalized when SINR is greater than the target SINR that is $Q_0 > I_0$ because it does not need to transmit at too high power causing more interference to other users.

Secondary user utility function

When SUs access the licensed spectrum, they become a source of interference to others PU and SUs. Li and Jayaweera (2008a) considered a utility function for SUs, measured the amount of information bits transmitted successfully per joule of energy consumed, that used by Saraydar (2002). Saraydar (2002) proved that the Nash equilibrium solution using this utility function is inefficient because each user in the system maximized its utility selfishly, thus harming other users in the system. The Nash equilibrium in this case is not optimal and less efficient than the power allocation obtained by a centralized optimization system, in which users can cooperate between themselves. We consider a suitable energy efficient utility function, which is decoupled from layer decision such as modulation and coding. SUs utility function is given as Musku et al. (2006):

$$u_k(p_k, \mathbf{p}_{-k}) - \frac{R_k f(\gamma_k)}{p_k} \tag{5}$$

where R_k, p_k are the transmission rate and transmission power of the k -th SU respectively, γ_k is the secondary user's SINR. $f(\gamma_k) = \ln(\lambda \gamma_k)$ is the efficiency function and λ is a constant to determine the quality of service requirement.

In this paper, we introduce a new pricing function for SUs to reduce the amount of interference among SUs, and guide the SUs to an efficient Nash equilibrium point. The Pricing function is as follows:

$$v(p_k, \mathbf{p}_{-k}) = v p_k \left(\sum_{\substack{j=1 \\ j \neq k}}^{K} h_{sj}^2 c_{j,k}^2 p_j + h_{s0}^2 c_{ps}^2 p_0 + \sigma^2 \right)^2 \tag{6}$$

where v is the pricing factor, which should be tuned such that each user's self-interest leads to overall improvement of the system. We refer the interference of k -th user as

$$I_{-k}(p) = \sum_{j \neq k}^{K} h_{sj}^2 c_{j,k}^2 p_j + h_{s0}^2 c_{ps}^2 p_0 + \sigma^2 \tag{7}$$

Thus, the equation (6) can be written as:

$$v(p_k, \mathbf{p}_{-k}) = v\, p_k (I_{-k})^2 \tag{8}$$

Therefore, the general expression of the utility function with pricing for the k -th SU is

$$u_k'(p_k, \mathbf{p}_{-k}) = \frac{R_k\, f(\gamma_k)}{p_k} - v\, p_k (I_{-k})^2 \tag{9}$$

Existence of Nash equilibrium

The game defined above has at least one Nash equilibrium if the action set Q_0 is nonempty, compact, convex subset of a Euclidean space, and the utility function u_0 is continuous in $\mathbf{p}$ and concave in Q_0. The PU power action set is nonempty, compact, closed subsets of R by definition and the utility function of the PU is continuous in $\mathbf{P}$. Furthermore, PU utility function u_0 reduces to $u_0(Q_0, p_0) = Q_0 - \mu_2(e^{(I_0 - Q_0)} - 1$, when $0 \leq Q_0 \leq I_0$. The second order derivative of u_0 with respect to Q_0 is Q_0 is $\partial^2 u_0 / \partial Q^2 = -\mu_2 e^{(I_0 - Q_0)} < 0.$. Thus, it is concave in Q_0. On the other hand, PU utility function is reduced to $u_0(Q_0, p_0) = Q_0 - \mu_1[(Q_0 - I_0)^2\, u(Q_0 - I_0)]$, when $I_0 < Q_0$. The second order derivative of u_0 with respect to Q_0 is $\partial^2 u_0 / \partial Q^2 = -2\, \mu_1 < 0$. Thus, PU utility function is concave in Q_0. Therefore, the utility function of PU satisfied all conditions, so the Nash equilibrium exists in this game (Li and Jayaweera, 2008a).

To derive an algorithm for our power control game we adopt a power control algorithm in which each SU maximizes its net utility $u_k'(p_k, \mathbf{p}_{-k})$. This can be achieved at a point for which the partial derivative of $u_k'(p_k, \mathbf{p}_{-k})$ with respect to p_k is equal to zero.

$$\frac{\partial u_k'}{\partial p_k} = \frac{-1 + \ln(\lambda\, \gamma_k)}{p_k^2} - v.\, I_{-k}^2 = 0 \tag{10}$$

By rearranging (10), the condition for maximizing utility function with new pricing function becomes

$$\ln(\lambda\, \gamma_k) - 1 + v.\, I_{-k}^2.\, p_k^2 = 0 \tag{11}$$

Definition 1: A power vector $\mathbf{p} = (p_1, p_2, \ldots, p_k)$ is a Nash equilibrium of the power control game with pricing $G = [K, \{P_{0k}\}, \{u_0(.), u_k'(.)\}]$ if, for every $k \in K$, $u_k(p_k, p_{-k}) \geq u_k(p_k', p_{-k})$ for all $p_k' \in P_k$.

To prove the existence of Nash equilibrium, it is necessary to prove that the proposed game satisfies the requirements of the theorem 1.2, which is given by Fudenberg and Tirole (1991).

Theorem 1 (Existence): A Nash equilibrium in the transmit powers exists in the game $G = [K, \{P_{0k}\}, \{u_0(.), u_k'(.)\}]$ if, for all $k = 1, 2, \ldots, K$:

1) The action sets P_k is nonempty, convex, and compact subset of sum Euclidean space R^N.
2) $u_k'(p_k, \mathbf{p}_{-k})$ is continuous in $\mathbf{p}$ and quasi-concave in p_k.

Proof: We assumed that the transmit power strategy for each SU in our game is defined by minimum and maximum powers, and all values of SU transmit power in between. Therefore, the first condition of the strategy space P_k is satisfied.

To show that the SU utility function is quasi-concave in p_k, the second derivative of $u_k'(p_k, p_{-k})$ is computed with respect to p_k.

$$\frac{\partial^2 u_k'}{\partial p_k^2} = \frac{2\ln(\lambda\, \gamma_k) - 3}{p_k^3} \tag{12}$$

In a real system, the probability of reception is always less than or equal to 1 ($P_c \leq 1 \Rightarrow \ln(\lambda\, \gamma_k) \leq 1$). If we use this condition in equation (12), we conclude that, $\partial^2 u_k' / \partial p_k^2 \leq 0$ which implies that $u_k'(p_k, \mathbf{P}_{-k})$ is a quasi-concave function in P_k. This proves condition 2, and guarantees an existence of a Nash Equilibrium.

Uniqueness of Nash equilibrium

The game above has always at least one Nash equilibrium. According to Yates (1996), if the best response correspondences of PU and SUs are standard functions, then the Nash equilibrium in this game will indeed be unique. Li and Jayaweera (2008a) proved that the best response function for PU is a standard function, and PU can maximize its utility function as

$$\tilde{Q}_0 = \operatorname{argmax} u_0 \tag{13}$$

In the case of SUs, we need to prove that the SUs best response function is a standard function.

Proposition 1: For a game $G = [K, \{P_{0k}\}, \{u_0(.), u_k'(.)\}]$, the best response of the k -th SU, given the transmission power vector of other SU p_{-k}, is given by $r_k(p_{-k}) = \min(\tilde{p}_k, p_k^{max})\ \forall\, k \in K$, where p_k^{max} is the maximum transmission power of $k's$ strategy space P_k.

Proof: Let $r_k(p_{-k})$ be the best response function of the k -th SU as a best action that the user k can take to attain the maximum utility given the other users' strategy p_{-k}. Formally, user k 's best response $r_k\colon P_{-k} \rightarrow P_k$ is the mapping that assigns to each $p_{-k} \in P_{-k}$ the set

$$r_k(p_{-k}) = \{p_k \in P_k : u_k(p_k, p_{-k}) \geq u_k(p_k', p_{-k}), \forall p_k' \in P_k\} \quad (14)$$

where this is a set containing only one point. Therefore, $\tilde{p}_k$ is the unconstrained maximum of the SU utility function u_k.

$$\tilde{p}_k = \arg\max u_k' \quad (15)$$

Moreover, as seen from equation (12), $\partial^2 u_k'/\partial p^2 \leq 0, \forall p_k \in \mathrm{R}^+$, which implies that the maximum is unique.

Theorem 2 (Uniqueness)*:* The Nash equilibrium of the game $[K, \{P_{0k}\}, \{u_0(.), u_k'(.)\}]$ is unique.

Proof: the key aspect of Nash equilibrium uniqueness is to show that the best response function $r(\mathbf{p}_{-k})$ is a standard function. If we let, $\mathbf{r(p)} - (r_1(\mathbf{p}_{-1}), r_2(\mathbf{p}_{-2}), \ldots, r_k(\mathbf{p}_{-k}))$ is the best response vector for all SUs. To prove that the Nash equilibrium is unique, the best response function should be a standard function and has to satisfy the following properties:

- Positivity: $\mathbf{r(p)} > 0$
- Monotonicity: if $\mathrm{p}_1 \leq \mathrm{p}_2$, then $\mathbf{r(p_1)} \leq \mathbf{r(p_2)}$
- Scalability: for all $\alpha > 1$, then $\alpha\, \mathbf{r(p)} > \mathbf{r}(\alpha\mathbf{p})$.

These properties can be easily verified for $\mathrm{r(p)}$. Every SU update it's transmit power using equation (15) depending on the knowledge of all other SUs and PU in the system, in reality the term $I_{-k}(p) = \sum_{j \neq k}^{K} h_{sj}^2 c_{j,k}^2 p_j + h_{s0}^2 c_{ps}^2 p_0 + \sigma^2$. This term represents the interference plus noise experienced by user k 's signal at the SU base station. It is shown in Yates R.D. (1996) that the fixed point $I_{-k}(\mathbf{p})$ is a standard function.

From Yates (1996), we can see that the positivity property is implied by a nonzero background receiver noise.

The Monotonicity property: if $\mathrm{p}_1 \leq \mathrm{p}_2$

$$r(\mathrm{p}_1) - r(\mathrm{p}_2) = I_{-k}(\mathrm{p}_1) - I_{-k}(\mathrm{p}_2)$$

$$= I_{-k}(\mathrm{p}_1 - \mathrm{p}_2) \leq 0 \quad (16)$$

The scalability property: if $\alpha > 1$

$$\alpha r(p) - r(\alpha p) = \alpha I_{-k}(\mathrm{p}) - I_{-k}(\alpha\mathrm{p})$$

$$= (\alpha - 1)\, I_{-k}(\mathrm{p}) > 0 \quad (17)$$

Thus, the best response function is a standard function and therefore the Nash equilibrium is unique.

Proposed power control algorithm

We present a synchronous power control algorithm which converges to the unique Nash equilibrium. We assume that the k -th SU update their transmit powers at time instances in the set $\{T_k = t_{k1}, t_{k2} \ldots\}$, with $t_{ki} < t_{ki+1}$, $t_{k0} = 0$. Let ϵ be a small number (e.g. 10^{-7}). Generate a sequence of powers as follows:

1) Start number of secondary users $k = 1$.

2) Initialize power vector for PU and SUs randomly at time t_0.

3) For all k at time instant t_i.

a) Update secondary user k 's power by compute $\tilde{p}_k = \arg\max u_k'$ and set the transmission power $r_k(t_i) = \min(\tilde{p}_k, p_k^{max})$.

b) Update primary user IC by compute $\tilde{Q}_0 = \arg\max u_0$ and set the interference cap $r_0(t_i) = \min(\tilde{Q}_0, Q_0^{max})$

c) Compute SU SIR

4) If $|| p(t_i), p(t_{k-1}) || \leq \epsilon$, then STOP and declare the Nash equilibrium. Else, make $i = i + 1$ and go step 3.

5) If $\gamma_k < \gamma_k^{tar}$ or $I_k > Q^{max}$, then STOP. Else, Make $k = k + 1$ and go to step 2.

RESULTS AND DISCUSSION

The performance of our proposed power control algorithm with a new pricing function is compared with the primary-secondary user power control game studied by Li and Jayaweera (2008a). The same system parameters given by Li and Jayaweera (2008a) were used in our simulation except the parameter $\rho_k = 4.2$ that is used to allow all SUs to achieve their optimal SIR under the interference cap constrained. The path gains for both PU and SUs are $h_{pk} = h_{sk} = h_{s0} = h_{p0} = 1\, \forall k \in K$. $P_k^{max} = 20$, $\mu_1 = 10, \mu_2 = 100, Q_0^{max} = 5$, $\sigma^2 = 1$, and the length of packet is $M = 80$.

The cross correlation codes between PU and SUs are $c_{0k} = c_{k0} = c_{jk} = 0.1$ for all $k \in \{1, \ldots, k\}$. Optimal SIR is considered equal to 12.42, and the value of λ is calculated depended on the value of SIR, in which $\lambda = 0.21886$. The coefficient of pricing is considered to be ($v = 0.00013$). SUs base station in our system broadcasts the sum of interference value (I_{-k}) and the pricing factor value to all SUs and then each SU trying to optimize its own net utility depending on that local information and controls the outcome of the game.

Each SU in this game is self interested and it does not need to know the channel conditions information of the other users, and hence no need to perform signaling to collect all information of other users in a centralized way.

Pricing in this case reduces the overhead, thus increases spectrum utilization and improves the system performance.

The proposed power control algorithm is simulated by using MATLAB. In our algorithm, we let $\gamma_k^{tar} = 2\ln M = 8.764$. The results obtained in Figure 2 show a significant difference between our pro-

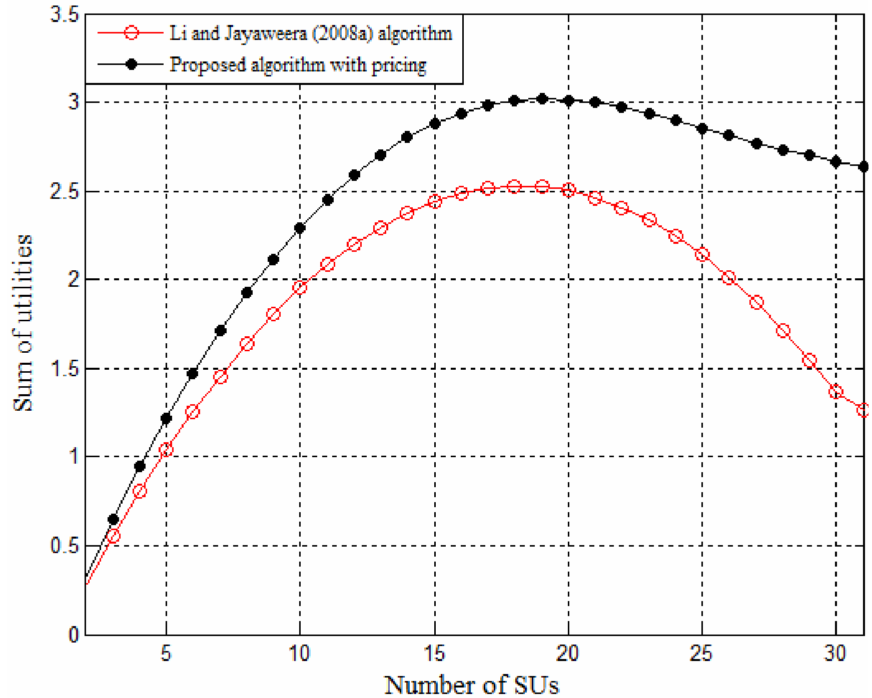

Figure 2. The maximum value of sum of utilities occurs when the number of SUs is 18 (SUs sum of utilities).

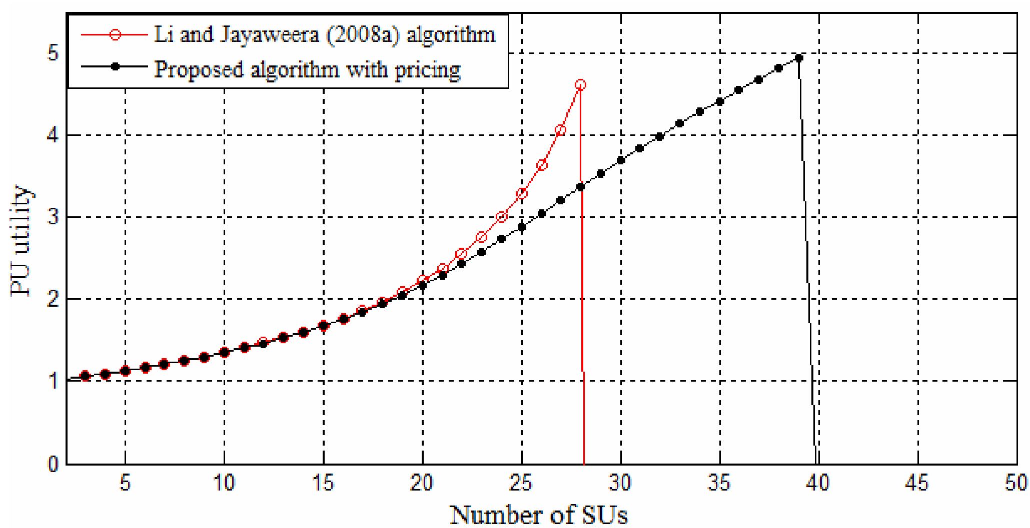

Figure 3. Proposed algorithm with pricing maximize the system capacity to 40 SUs compared to 28 SUs resulted from other algorithms (PU utility function).

posed algorithm using pricing function and Li and Jayaweera (2008a) algorithm. In Figure 3, PU utility function of Li and Jayaweera (2008a) is fall down when the number of SUs reached to 28 SUs, which means that the amount of interference generated by 28 SUs in that algorithm reaches the maximum interference cap. When any other SU entered the system $(I_0 > Q_0^{max})$, PU utility function is decreased. We show that the new pricing function allowed PU to tolerate 40 SUs by using the same maximum interference cap ($Q_0^{max} = 5$). In other words, PU can sells or leases its unused spectrum to more numbers of SUs and this proves that our proposed algorithm able to manage network resources more efficiently.

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