

Full Length Research Paper

Joint estimation of cochannel signals and direction finding

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The need for fast adaptive algorithms for signal separation in dense environments is essential aspect in modern communications systems. For example, enhancement of mobile systems performance to allow different traffics, high quality and minimum delay is achievable in condition that the channel impairments and cochannel interference is reduced. In this paper, the performance of two types of signal separation and interference cancellation algorithms is compared. The paper presents quantitative measures of the two methods and suggests further enhancement of their performance.

Key words: Constant modulus algorithms (CMA), iterative least square enumerator (ILSE), signal canceller and multiple signal classification (MUSIC).

INTRODUCTION

In cellular radio systems, spectral crowding and cochannel interference are becoming increasingly important issues as the number of subscribers grows. Cochannel interference results from frequency reuse, whereby multiple cells operate on the same carrier frequency (Lee, 1989). Depending on geographic considerations and environments conditions, cochannel interference can be the dominant channel impairment. It would be desirable to incorporate "smart" directional antennas into the cellular system to reducing the effects of cochannel interference and in turn allow greater frequency reuse. These antennas should be capable of simultaneously estimating the angles of arrival (AOA's) of several cochannel sources, as well as demodulating the signals themselves (referred to as signal copy).

In recent years, there has been much interest in blind cochannel signal copy algorithms for antenna arrays. For example, a class of blind adaptive algorithms was developed in Tao and Nicholas (2000) that extracts and

separates multiple signals-of-interest on the basis of their differing spectral self coherence refers to the property of a communication signal whereby it is correlated with a frequency-shifted version of itself. Another approach is the two-step procedure described in Ottersten et al. (1989) and Xu et al. (1992) that incorporates a high-resolution direction-finding algorithm followed by a maximum-likelihood scheme to estimate the sources. A signal subspace method, such as the MUSIC (multiple signal classification) algorithm (Schmidt, 1986), is employed to estimate the AOA's. More recently, a decision-feedback approach was presented in Swindlehurst et al. (1995) for the demodulation of digital signals. Symbol decisions based on preliminary signal estimates are used to regenerate the signal waveforms from which improved estimates are derived.

The CM array is an adaptive beamformer designed to blindly recover a cochannel signal (Gooch and Lundell, 1986). It has a conventional weight-and-sum beamformer

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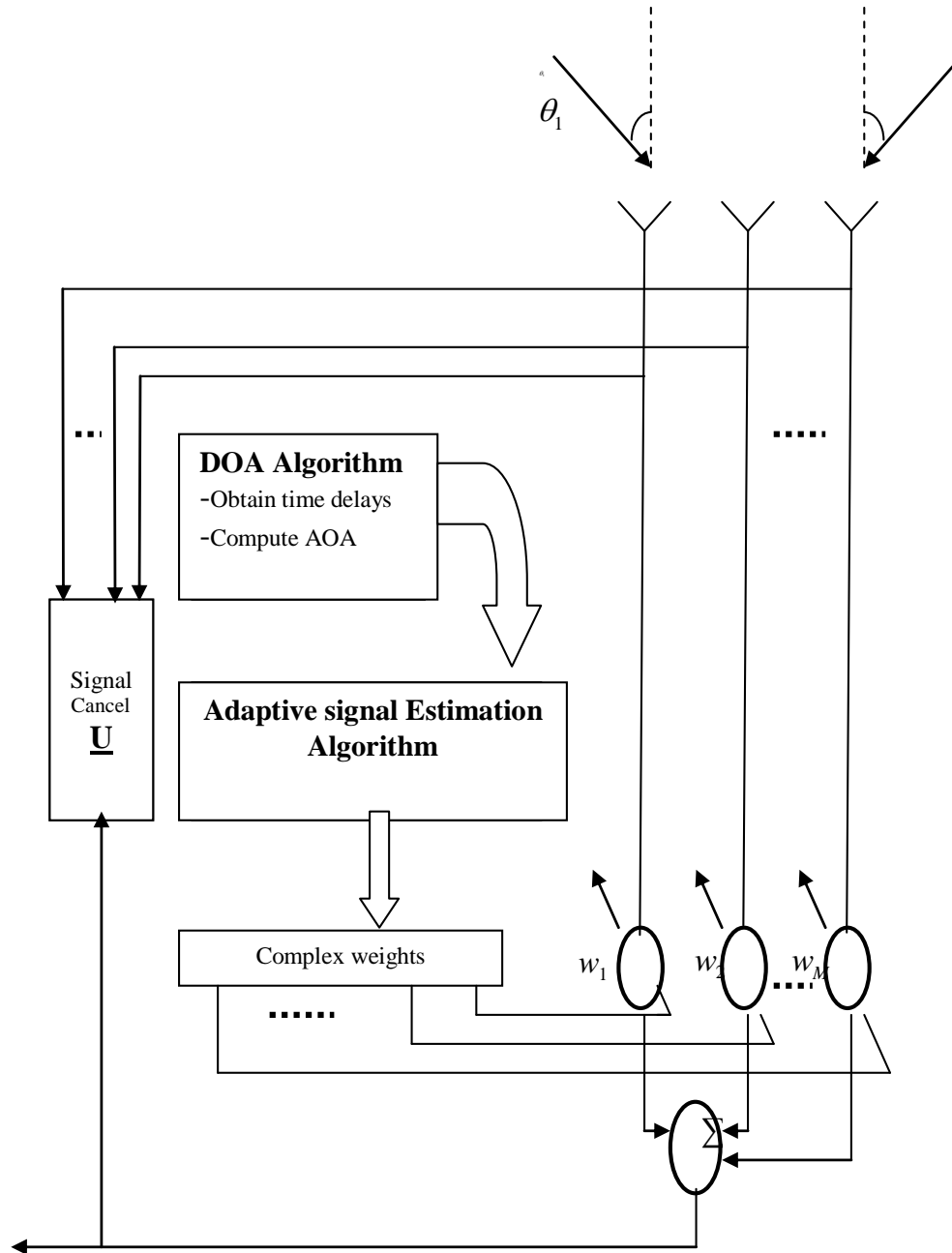


Figure 1. One stage CM array followed by signal canceller.

configuration (Widrow and Stearns, 1985) and its weights are adapted by the constant modulus algorithm (CMA) (Treichler and Agee, 1983). The CM array has fast convergence properties and low computational complexity. Moreover, its signal copy performance is insensitive to array imperfections. The multistage CM array consists of a cascade of individual CM array stages (Sansrimahachai and Constantinides, 2005; Garth, 2001; Sansrimahachai and Constantinides, 2003). An adaptive signal canceller is included in each stage to remove a captured source from the input before subsequent

processing by the follow-on stages.

In this paper, we present a comparison between the two stages algorithm, which utilizes AOA estimator followed by maximum likelihood (ML) multi dimension decision criterion and the multi stages CM array.

SYSTEM CONFIGURATION AND SIGNAL MODEL

A block diagram of the system is shown in Figure 1. Assume that the antenna elements are uniformly spaced

and omni directional so that the array input signals may be expressed as

$$x_m(t) = \sum_{l=1}^L s_l(t) e^{-j(m-1)\phi_l} + n_m(t) \quad m = 1, \dots, N \quad (1)$$

Where $\{s_l(t)\}$ are the l^{th} (baseband) sources and $\{n_m(t)\}$ are additive white Gaussian noise processes. Because the sources are narrowband, $\phi_l = 2\pi(d/\lambda)\sin(\theta_l)$ where d is the interelement spacing, λ is the wavelength of the sources, and $\{\theta_l\}$ are their angles of arrival. By collecting the signals into vectors and assuming that the $\{x_m(t)\}$ are sampled, then Equation (1) may be expressed as:

$$x(k) = A s(k) + n(k) \quad (2)$$

Where

$$x(k) \square [x_1(k), \dots, x_N(k)]^T, s(k) \square [s_1(k), \dots, s_L(k)]^T, n(k) \square [n_1(k), \dots, n_N(k)]^T$$

$$A \square \begin{pmatrix} 1 & \dots & 1 \\ e^{-j\phi_1} & \dots & e^{-j\phi_L} \\ \vdots & \vdots & \vdots \\ e^{-j(N-1)\phi_1} & \dots & e^{-j(N-1)\phi_L} \end{pmatrix} \quad (3)$$

The columns $\{a_i\}$ of the steering matrix A are known as direction vectors because they indicate the response of the array to a narrowband signal emanating from a particular direction. Note that although one is often interested in a uniform linear array as specified by Equation (3), the signal copy performance of the proposed array is independent of the array configuration (Gooch and Lundell, 1986). Our analysis applies to a more general matrix A . It is assumed that $L \leq N$ for the proposed array, unlike most direction-finding algorithms (e.g., the MUSIC algorithm) where $L < N$ must be chosen.

The correlation matrix of the array output data, is defined as

$$R_x \square E [x(k)x^H(k)].$$

It is assumed that the incident signals is independent of the additive white Gaussian noise thus R_x is given by

$$R_x = A R_s A^H + R_n \quad (4)$$

Where $R_s \square E [s(k)s^H(k)]$ and $R_n \square E [n(k)n^H(k)]$. Assume that the signals and the noise at each array element are mutually uncorrelated, thus, R_s and R_n can be represented by the diagonal matrices \sum_s and \sum_n , respectively. Furthermore, assume that the sensor noise powers are identical so that $\sum_n = \sigma_n^2 I$ and (4) becomes

$$R_x = A \sum_s A^H + \sigma_n^2 I \quad (5)$$

The i^{th} diagonal component of \sum_s is $\sigma_{si}^2 = E [|s_i(k)|^2]$, corresponding to the power of the i^{th} source. It is well known that the rank of $A \sum_s A^H$ is equal to the number of sources with different AOA's (L) so that $N - L$ eigen values of R_x are equal to σ_n^2 .

CM ARRAY AND ADAPTIVE SIGNAL CANCELLER

The CM array estimates one component, $s_i(k)$, of $s(k)$ from $x(k)$ in an on-line adaptive manner without directly estimating R_x . It also provides a correction for the estimate of the source direction vector a_i and, thus, the angle of arrival θ_i . Observe in Figure 1 that, the input vector $x(k)$ is processed by a weight-and sum beamformer, yielding the output

$$y(k) = w^H(k)x(k) \quad (6)$$

Where $w(k) \square [w_1(k), \dots, w_N(k)]^T$ are the adaptive weights adjusted by the constant modulus algorithm such that

$$w(k+1) = w(k) + 2\mu_{cma} x(k) \varepsilon_c^*(k) \quad (7)$$

with

$$\varepsilon_c(k) = y(k) / |y(k)| - y(k) \quad (8)$$

The step size $\mu_{cma} > 0$ controls the convergence rate of (7), and the superscript * denotes complex conjugate.

This update is identical to that of the complex least-mean-square (LMS) algorithm (Widrow and Stearns, 1985), except that the desired signal is replaced by $y(k)/|y(k)|$.

It has been shown for constant modulus signals that the capture behavior of the CM array depends on the initial weight vector $w(o)$ and the relative signal powers at the array output. Specifically, for $L = N = 2$ sources (and a different version of CMA), it was demonstrated that the CM array will lock onto the source with the greatest power at the output of the array while nulling the other source (Gooch and Lundell, 1986). Since the array output primarily contains the captured source, a signal canceller may be used to remove $s_i(k)$ from $x(k)$, generating a modified input vector that can be processed by a follow-on CM array stage in a multistage system (Sansrimahachai and Constantindes, 2005; Shynk et al., 1996). Figure 1 show the signal canceller processes; the array output via $u(k) = [u_1(k), \dots, u_N(k)]^T$ result is subtracted from the array input to yield an error vector

$$e(k) = x(k) - u(k)y(k) \quad (9)$$

The canceller weights may be updated by gradient-descent algorithm using

$$u(k+1) = u(k) + 2\mu_{lms} y^*(k) e(k) \quad (10)$$

This recursion implements a set of N independent signal-weight LMS algorithm updates. It is straightforward to show that for convergence in the mean, the step size is bounded by $0 < \mu_{lms} < 1/\sigma_y^2$ where $\sigma_y^2 = E[|y(k)|^2]$

is the variance of the CM array output (this variance is actually time-varying because the CM array weights are continually updated by Gooch and Lundell (1986). Thus, the convergence properties of the canceller weights depend on those of the CM array, whereas the CM array weights are independent of the adaptive canceller. All canceller weights converge with the same time constant (because of the single input $y(k)$) given approximately by

$$\tau \approx 1/(2\mu_{lms}\sigma_y^2) \quad (11)$$

DIRECTION OF ARRIVAL ESTIMATION AND SIGNALS SEPARATION ALGORITHM

A core problem in the area of blind signal separation/

equalization is the following. Consider L independent sources, transmitting binary symbols $\{+1, -1\}$ at equal rates in a wireless scenario. The signals are received by a central antenna array, consisting of N elements antenna array. Assuming synchronized sources, equal transmission delays, negligible delay spread, and sampling at the bit rate, each antenna receives a linear combination of the transmitted symbol sequences and a weighted combination of the antennas output is obtained. The blind CM array depends on restoring the constant amplitude property for capturing one of the L sources without determination of the order of this source within the combined received signal. Usually in mobile system the direction of arrival and the power of the received signal are variable parameters. This variation of individual sources limits the capturing capability of the CM array. A promising method to overcome this problem is the utilization of two stage system based on AOA estimator allowed by iterative least square enumerator (ILSE) for simultaneous detection of all the L incident signals. The subspace methods such as MUSIC algorithm provides an accurate AOA estimator which are utilized as an initial estimate of the array weights. The simultaneous signals detection is then carried out by examining the most likelihood vertex of 2^L vertices of the hybrid cube represents the all possible signals.

MUSIC Algorithm

The multiple signal classification (MUSIC) method is a profit from the eigen structure properties of the array correlation matrix to obtain very-high-resolution estimates with lower computational complexity when compared to ML estimation schemes. The basic idea of the MUSIC method is to separate signal from noise by the orthogonal property of their spaces through eigen-decomposition of the correlation matrix of the received signal.

Let us analyze the properties of the spatial correlation matrix R_x described in Equation (4). It is clear that if the number of array sensors is large than the number of signal sources (that is, $N > L$), when R_s is positive definite (that is, the signals $s_i(t)$ are not fully correlated), the matrix $R_x - \sigma^2 I$ will have rank L and a null space of dimension $N-L$. Then matrix R_x will have L eigen values greater than σ^2 and $N-L$ eigen values equal to σ^2 ; these eigen values may be sorted from largest to smallest such that

$$\lambda_1 > \lambda_2 > \dots > \lambda_L > \lambda_{L+1} = \lambda_{L+2} = \dots = \lambda_N = \sigma^2$$

The eigenvectors $\{e_1, e_2, \dots, e_L\}$ are corresponding to the largest eigen values span the L -dimensional signal

subspace. These eigen vectors can be grouped in the columns of matrix E_s .

The eigen vectors $\{e_{L+1}, e_{L+2}, \dots, e_L\}$ corresponding to the smallest eigen values span the $(N-L)$ -dimensional noise subspace. These signal eigen vectors can be grouped in the columns of matrix E_N .

Then it is clear that, as the columns of matrix A are orthogonal to the eigen vectors that span the noise subspace.

For an exactly known R_x , the desired angles $\theta_i, i = 1, 2, \dots, L$ can be found by evaluating the MUSIC spatial spectrum defined as

$$P_{MUSIC}(\theta) = \frac{1}{a(\theta)^H E_N E_N^H a(\theta)} \quad (12)$$

Ideally, $P_{MUSIC}(\theta)$ will peak to infinity each time a true $\theta_i, i = 1, 2, \dots, L$ angle is tested. The MUSIC method works only when the rank of matrix $R_x - \sigma^2 I$ is equal to L , that is, when the signals are uncorrelated.

Iterative least square with enumeration algorithm

In the blind signal separation scenario, both A and S are unknown, and the objective is, given X, to find the factorization $X = AS$ such that S belongs to the binary alphabet. Alternatively, we try to find a weight matrix W of full row rank L such that $S = W^T X$. Uniqueness of this factorization is important, and was established in Lee (1989) if A is full rank and the columns of S exhaust all 2^L distinct (up to a sign) possibilities, then this is sufficient for the factorization to be unique up to trivial permutations and scaling by ± 1 of the rows of S and columns of A. Hence, once any such factorization of X is found, S contains the binary signals that were originally transmitted, or their negative, but not some ghost signal. This scenario by itself is perhaps naive, but it is the core problem in more realistic blind (FIR-MIMO) scenarios (Tao and Nicholas, 2000), where long delay multi path is allowed, and sources are not synchronized and are modulated by arbitrary pulse shape functions.

One of the first papers to consider this problem appeared in full in Lee (1989). In that paper, arbitrary finite alphabets are considered although only BPSK was tested extensively. A fixed-point iteration algorithm is proposed, it is called ILSE which is based on clever enumeration of candidate matrices S. Clearly, ILSE is a conditional maximum likelihood estimator. The ILSE algorithm utilizes an accurate initialization of the array weights by the aid of the MUSIC Algorithm. The iterative

solution of this LS optimization problem is given by the following steps

$$\min_S \|X - AS\|_F^2 = \min_{S(1)} \|X(1) - AS(1)\|_F^2 + \dots + \min_{S(N)} \|X(N) - AS(N)\|_F^2$$

ILSE,

1. Given $A_0, k = 0$
2. $k = k + 1$
 - Minimize (1) for S_k (by enumeration)
 - $A_k = X S_k^+ (S_k S_k^+)^{-1}$
3. Continue until $(A_k - A_{k-1}) = 0$

The basic idea behind ILS solutions of is simple, that each time, compute an LS update for one of the unknown matrices conditioned on a previously obtained estimate for the other matrix, proceeds to update the other matrix, and repeat until convergence of the LS cost function is reached.

COMPUTER SIMULATIONS

Computer simulations are performed using MATLAB to verify the performance of the MUSIC, the ILSE and the CMA Algorithms. Assume that three signals arrive from faraway signal sources from directions $\theta = 10^\circ, 30^\circ, 50^\circ$. The three signals have equal power and they have the same signal to noise ratio. The array is a linear array consists of 4 elements separated by half the wavelength. Only 256 snapshots are taken into consideration to estimate the correlation matrix of the received signal. The performance of the MUSIC method is evaluated where, the incident signals are assumed to be uncorrelated. The output of the MUSIC method under these conditions is presented in Figure 2. It is clear that the MUSIC method has successfully determined the correct DOA of the incident signals. These results are applied as initial estimate of the array weights vector in iterative manner for both the CM array with interference canceller as well as the ILSE where both of them were able to estimate the signals and their corresponding probability of error are plotted in Figures 3 and 4, respectively. One can see although the ILSE is less complex compared with the CM array, followed by signal canceller, it provides a similar behavior for the considered environments. Moreover the ILSE is capable to tolerate the DOA estimation error; this is indicated in Figure 5, where the ILSE was provided with 5% error in the DOA estimate.

Conclusions

The Multi stage CM array with signal canceller provides an acceptable performance for successive signal separation and interference cancellation in stationary environments. The CMA is completely blind algorithm and it depends on restoring the constant envelope

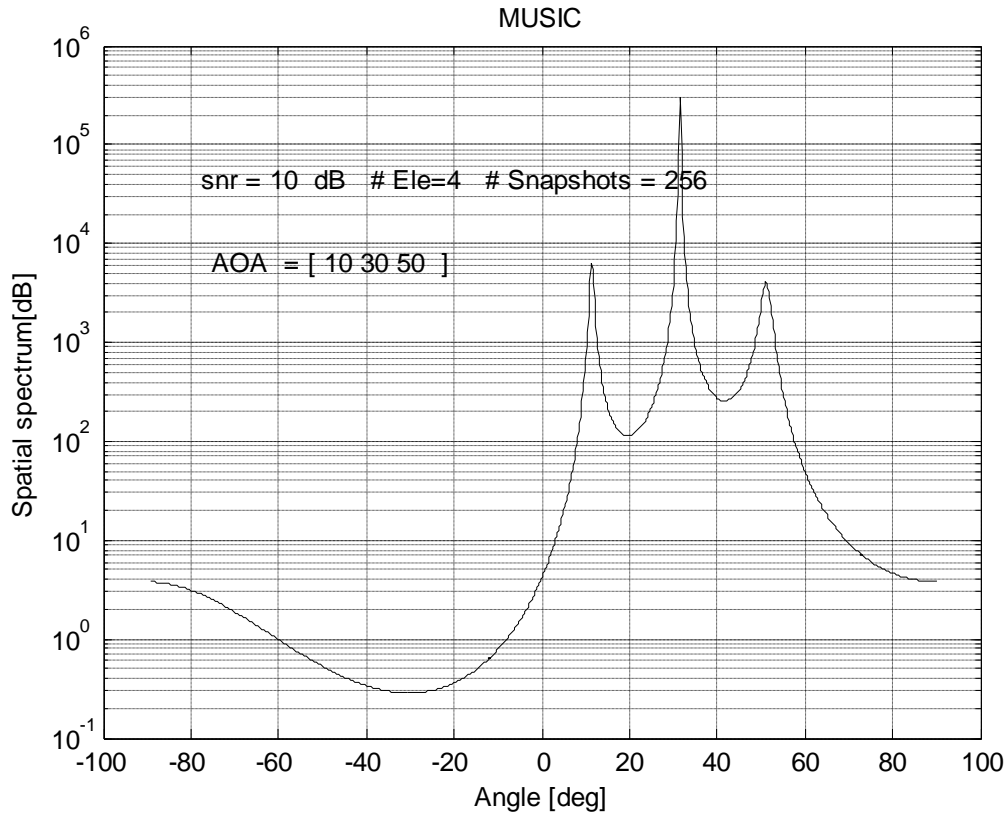


Figure 2. AOA estimation based on MUSIC spatial spectrum estimation.

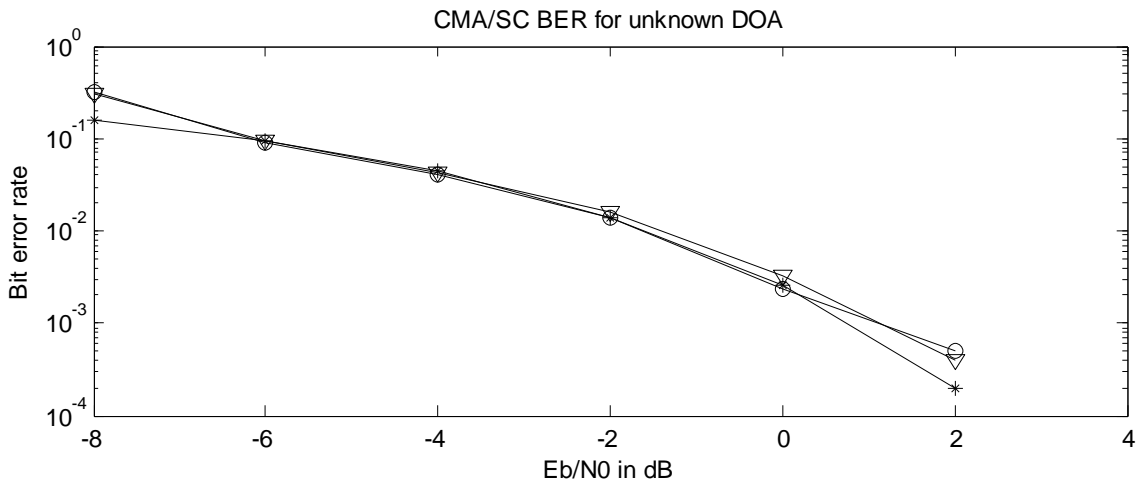


Figure 3. Bit error rate performance of 3 stages CM array with signal Canceller

property of capturing the incident signals in successive manner with the aid of LS signal canceller. The CM array performance decays in rapid changing environments such as cellular channels where the direction and power relations between signals are changed rapidly. The ILSE is a promising algorithm for joint DOA estimation and

signal copy. Convergence of the ILSE cost is guaranteed because each (conditional LS) update may either improve or maintain, but cannot worsen the fit. The final output is generally dependent on the initialization. For that reason, an initial weight vector based on MUSIC algorithm enhances the performance of the algorithm, An

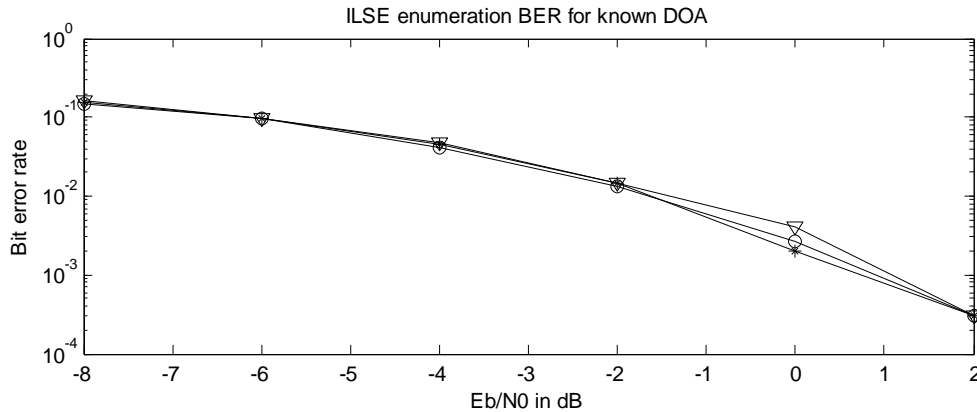


Figure 4. Bit error rate for 3 detected users by ILSE algorithm with perfect AOA estimate.

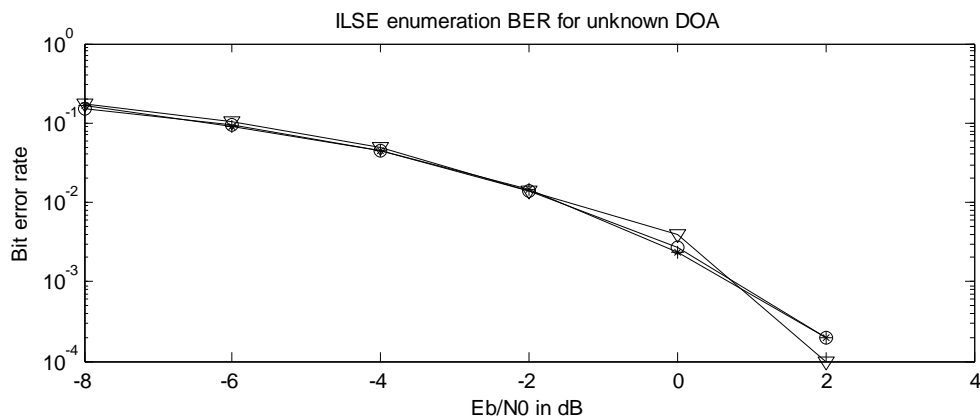


Figure 5 Bit error rate for 3 detected users by ILSE algorithm with 5% error in AOA estimate.

interesting point for further research is to find out a fast AOA estimator, which needs a few array snapshot to provide an accurate AOA estimate to be utilized as an initial weight vector for the signal separation algorithm. Of course such fast initialization will enhance the possibility of real time processing in the fast changing and dense environment encountered in cellular communications.

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