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Mean-Gini portfolio selection: Forecasting VaR using GARCH models in Moroccan financial market

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This paper focuses on Mean-Gini (MG) method for optimum portfolio selection. The MG framework, introduced by Shalit and Yitzhaki, is an attractive alternative as it is consistent with stochastic dominance rules regardless of the probability distributions of asset returns. Therefore, a MG framework is similar to a corresponding Mean-Variance (MV) framework in that it also uses two summary statistics- the mean and a measure of dispersion to characterize the distribution of a risky prospect. The goal of this paper is to test MG strategy, based on Moroccan financial market data from turbulent market period of the years 2011, 2012, 2013 and 2014. In addition, those outcomes are explicitly tested in terms of Value-at-risque (VaR). The results show that MG strategy is profitable for investors. Moreover, we consider MG strategy to be safer in turbulent times.

Key words: Diversification, MG, mean-variance, Morrocan financial market, portfolio selection, value-at-risk.

INTRODUCTION

The early '50s marked the starting point for the development of modern finance theory, with the work of Markowitz (1952a, 1952b, 1959). The model is supposed to be a reference in efficient portfolio construction.

The investors have the ultimate goal of combining a set of assets with maximum return and a given level of risk amounts to the same thing, or what amounts the same minimal risk for a given level of return. However the application of MV optimization is questionable. In fact a MV optimization does not consider the direction of the

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price movement. Thus, optimizing the variance can prevent investor from losses in same manner as from gains. Moreover, Roll (1977, 1978, and 1979) firstly pointed out other weaknesses of the theory. This evidence forced several theorists to search for other more appropriate models to modelize the best possible return / risk relationship. For instance, Markowitz (1991), Fishburn (1977) and Bawa (1977) proposed mean-lower partial moment approach; Yitzhaki (1982) and Yitzhaki (1984) proposed MG portfolio selection model; Konno and Yamazaki (1991) proposed Mean-Absolute Deviation (MAD) approach; Uryasev (2000) proposed Mean-VaR (Mean-CVaR) type models.

The restrictive character of the variance as a risk parameter led us to choose MG method. The MG strategy uses Gini as a parameter of risk instead of variance. Concept of MG was proposed by Shalit and Yitzhaki (1984; 2005) in finance as an alternative method to the MV approach of Markowitz (1952) because it can outstrip normal assumptions of return distribution and utility function quadratic. Yitzhaki (1982) has shown that the Gini coefficient satisfies the second degree stochastic dominance, which makes the MG model compatible with the theory of expected utility.

In the last twenty years, alternative measures of market risk have been proposed in the literature. The VaR has attracted particular attention. VaR refers to a portfolio's worst outcome that is expected to occur over a predetermined period and at a given confidence level. VaR assumes that returns follow a normal distribution. Particularly, in the case of skewed and fat-tailed returns, the assumption of normality leads to substantial bias in the VaR estimation and results in an underestimation of volatility. Hence, other distributions, such as the student-t and the Generalized Error Distribution (GED), are applied instead.

The above models do not, however, incorporate the observed volatility clustering of returns, first noted by Mandelbrot (1963). The most popular model taking account of this phenomenon is the Autoregressive Conditional Heteroscedasticity (ARCH) process, introduced by Engle (1982) and extended by Bollerslev (1986).

The purpose of our paper is to implement GARCH (1.1) model to forecast quasi-analytic VaR under two different distributional assumptions of returns in order to estimate the 1, 5 and 10% one-day VaR for completely diversified portfolio composed only of assets from the MADEX index over a period of financial crisis on Moroccan financial market. Combined with higher moments using Cornish-Fisher expansion and the Johnson SU distribution, our basic idealization is that financial return series follow a stationary time series model with stochastic volatility structure.

Our study shows that GARCH combined with higher moments using Cornish-Fisher expansion and the Johnson SU distribution provide better estimators of VaR.

The rest of the paper is organized as follows: First, we present the framework of the two models: MV versus MG and value-at-risk estimation by GARCH (1.1) model. Hence we provide a comprehensive data and methodology. Finally, we apply MG model and calculate VaR estimation by GARCH (1.1) model with Cornish-Fisher expansion and the Johnson SU approximation; and we recall some definitions.

METHODOLOGY

Mean-variance

A portfolio is defined to be a list of weights x_i for assets $S_i;\ i=1;\ldots n$, which represent the amount of capital to be invested in each asset. We assume that one unit of capital is available and requires that capital to be fully invested. Thus, we must respect the constraint that $\sum_{i=1}^{n} x_i$ = 1. The return of portfolio (Rp), obtained by Rp = $\sum_{i=1}^{n} x_i r_i$ (xi is the amount invested in asset i, ri is the expected return of asset i per period).

In the traditional Markowitz portfolio optimization, the objective is to find a portfolio which has minimal variance for a given expected return. More precisely, one seeks such that:

$$\text{Min}\ \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j \rho_{ij} \sigma_i \sigma_j$$

Subject to:

$$\begin{cases} R_p \geq \mu \\ \sum_{i=1}^{n} x_i \\ x_i \geq 0,\ 1 \leq i \leq n \end{cases} \qquad (1)$$

Where σ_{ij} is the covariance between the returns of Si, Rp is the return of portfolio and Sj and μ are the minimal rate of return required by an investor.

Mean-Gini

The MG approach, consistent with stochastic dominance for decisions under risk, is ideal for portfolio analysis for a great variety of financial assets. The MG analysis introduced by Shalit and Yitzhaki (1984) in finance defines the Gini coefficient as an index of variability of a variable random.

The idea used by these authors assumes that the cumulative distribution corresponding to the observation with rank t is t/T.

Specifically, Dorfman (1979) and Shalit and Yitzhaki (1984) retain as a measure of the Gini coefficient :

$$\Gamma_p = 2\operatorname{cov}\left(R_p, F\left(R_p\right)\right)$$

Where Rp is the return of portfolio and F is the cumulative distribution function.

$$\Gamma_p = 2\operatorname{cov}\left(R_p, F\left(R_p\right)\right) = 2\sum_{i=1}^{n} x_j \operatorname{cov}\left(r_i, F\left(R_p\right)\right)$$

The MG mathematical model is presented as follows:

Minimize: Γ_p

Subject to:

$$\begin{cases} R_p \geq \mu \\ \sum_{i=1}^{n} x_i \\ x_i \geq 0, 1 \leq \mathrm{i} \leq \mathrm{n} \end{cases} \tag{2}$$

Where Γ_p is the portfolio Gini, xi is the amount invested in asset Si, ri is the expected return of asset Si per period, μ is the minimal rate of return required by an investor (Cheung et al., 2007; Jaaman and Lam 2012).

Value-at-risk (VaR)

Value-at-risk is a measure of risk. It represents the maximum loss of the portfolio with a certain confidence probability $1-\alpha$, over a certain time horizon. Approaches to the estimation of VaR fall into one of four categories: the variance-covariance (or parametric or analytic) approach, historical simulation (or the non-parametric approach), Monte Carlo simulation, and the extreme value. The most widely used of these is variance-covariance approach, popularized by Morgan (1996). Formally, if the portfolio's price P(t) at time t is a random variable where S(t) represents a vector of risk factors at time t, then the value-at-risk (VaR_α) is implicitly given by the formula :

$$\Pr ob\{-P(t) + P(0) > VaR_\alpha\} = \alpha$$

In the case of normal distribution, the parametric VaR is calculated by:

$$VaR_\alpha = \bar{R} - \sigma \times z_\alpha$$

Where z_α is the quantile from a normal distribution.

Zangari (1996) and Favre and Galeano (2002) provide a modified VaR calculation that takes the higher moments of non normal distributions (skewness, kurtosis) into account through the use of a Cornish-Fisher expansion.

$$z_{\alpha f} = z_\alpha + \frac{\left(z_\alpha^2 - 1\right)S}{6} + \frac{\left(z_\alpha^3 - 3z_\alpha\right)K}{24} - \frac{\left(2z_\alpha^3 - 5z_\alpha\right)S^2}{36}$$

$$VaR_{CF} = \bar{R} - \sigma \times z_{\alpha f}$$

Where S is the skewness of R and K is the excess kurtosis of R.

The Johnson SU distribution we used here differs from the Cornish-Fisher approach. It transforms a random variable z in a standard normal variable x, and writing in general:

$$x = \xi + \lambda \sinh\left(\frac{z - \gamma}{\delta}\right)$$

Where z is a standard normal variable; ξ and λ shape parameters; γ the location parameter and δ scale parameter.

The Johnson SU value-at-risk is obtained by:

$$VaR_{JSU} = -\lambda \sinh\left(\frac{z_\alpha - \gamma}{\delta}\right) - \xi$$

GARCH (1.1) model

For the present study, volatility was estimated by fitting a GARCH (1.1) model to each portfolio. This is a familiar model in econometrics (Shephard, 1996; Hartzet, 2006: Alexander et al., 2013). If y_t denotes the observed series (in this case, the observed daily return) on day t, assumed standardized to mean 0, then the model represents y_t in the form:

$$y_t = \sigma_t \varepsilon_t,$$

Where ε_t are i.i.d. N(0; 1) random variables, and the volatility σ_t is assumed to satisfy an equation of the form :

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

Data and description statistics

This paper focuses on Moroccan financial market. We consider a portfolio constituted solely by assets from the MADEX index, over the period from January 1, 2011 to November 14, 2014 (Figure 1).

We use the first 714 daily returns, corresponding to the period from March 1, 2011 to November 12, 2013, to estimate the volatility

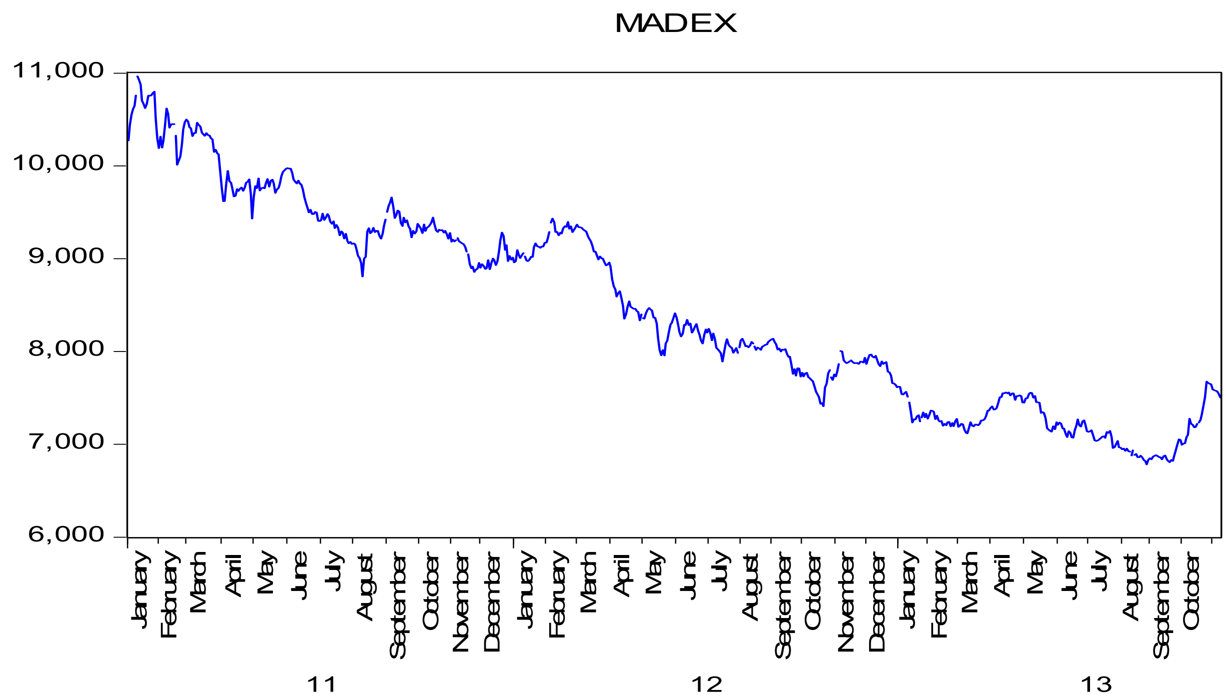

Figure 1. Evolution of MADEX.

using a GARCH model.

For the second period, from November 12, 2013 to November 14, 2014 (250 values), we estimated VaR at horizon h = 1 day and level α = 1, 5 and 10% using a GARCH (1, 1) on the first period, and kept this GARCH model for all VaR estimations of the second period. The estimated VaR and the effective losses were compared for the 250 data of the second period.

The six risky assets selected from are those most sensitive during this period: Addoha, Atlanta, BCP, Delta Holding, Mangem and Maroc Telecom. This study begins with an analysis of the characteristics of six selected assets that allows the construction of a portfolio using the MG strategy to determine the weights of the six assets. Figure 2 is plotted to illustrate stock returns and descriptive statistics are presented in Table 1.

The results of normality tests (Jarque-Bera) strongly for every stock led us to reject the null hypothesis of the normality test at 99% confidence level. These results are evidenced well-known property of financial data series, i.e. returns are usually not normally distributed. In addition, skewness and kurtosis, other properties of risky assets have been discovered that are true for our data. Since both problems are true to our data, we assume that, using the Mean- Gini strategy should end with the best portfolio, due to the fact that the Gini exceeds normal return distribution assumptions. Based on these results, we assume that in the context of our data, MG strategy must be better than the MV strategy.

After the resolution of optimization programs of the MG strategy, we obtained the optimum portfolio.

Table 2 presents percentage of stocks in optimal MG portfolio. Results show that the composition MG portfolio is diversified by combining all shares. BCP is the dominant share with 48. 90% of the funds invested in the MG portfolio that is due to the fact that its return is close to that required and risk is the smallest. While, Mangem is the smallest component of the MG portfolio (1. 19%).

Table 3 presents summary statistics of optimal portfolios obtained by the resolution of optimization programs (2). The return target for MG portfolio is -0.0002, higher than the MADEX index average return during this study period.

Table 4 presents the different tests of stationary, so we accept the alternative hypothesis that the series of returns of the two portfolios are stationary. The result of the test ARCH impels us to reject the null hypothesis. Therefore, it is assumed that the residual variance is not homoscedastic. So the prediction of VaR for MG portfolio will be made on a GARCH (1.1) model (Figure 3).

NUMERICAL RESULTS

The results of the goodness-of-fit tests for different models ARMA/GARCH show clearly that a combination

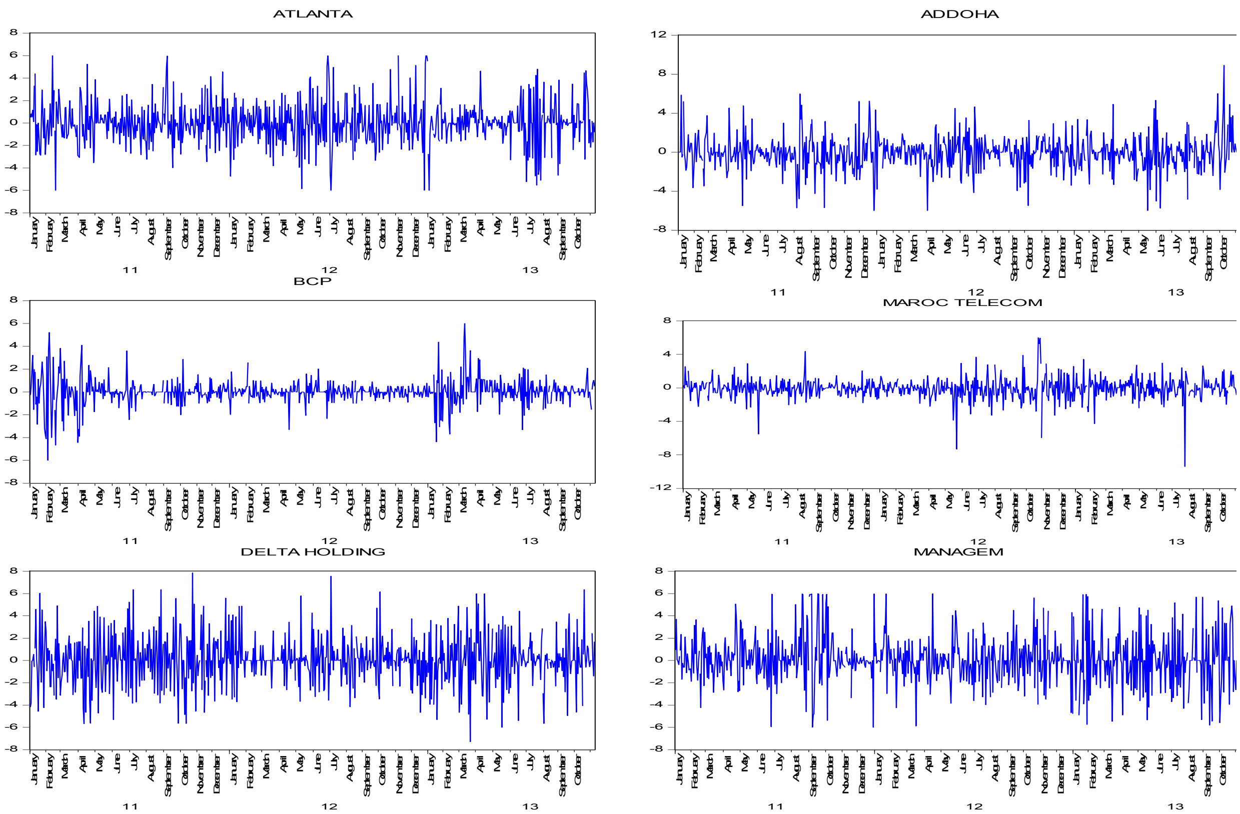

Figure 2. Evolution of returns of different assets.

Table 1**.** Descriptive statistics.

	Addoha	Atlanta	BCP	Delta Holding	Mangem	Maroc Telecom
Mean	-0.8228	0.0029	-0.0127	0.0607	0.1632	-0.0574
Std. Dev %	1.7076	1.9633	1.0677	2.1724	2.0231	1.2282
Gini %	0.0091	0.0106	0.0050	0.0118	0.0108	0.0061
Skewness	0.0087	0.3413	-0.4397	0.2658	0.4038	-0.0065
Kurtosis	4.9631	4.1469	9.8029	3.8739	4.5224	9.8548
Jarque-Bera	81.0994	37.4845	990.0570	22.0168	62.4877	988.7219

of AR (1)-GARCH (1.1) with Gaussian residuals and student-t residuals are the appropriate models from a statistical point of view for the portfolio.

The process used to estimate the parameters of the GARCH (1.1) model from historical data is known as the maximum likelihood method. This method involves choosing values for the parameters that maximize the likelihood of the data occurring. The problem is to estimate a variance of the returns from m observations of the variable when the underlying distribution is normal

Table 2. Percentage of stocks in optimal portfolios.

Addoha	5.98
Atlanta	6.32
BCP	48.90
Delta Holding	8.96
Mangem	1.19
Maroc Telecom	28.65

Table 3. Summary statistics of optimal portfolios.

Mean	-0.020
Median	-0.051910
Maximum	2.928983
Minimum	-3.178777
Std.Dev	0.779238
Gini	0.4112586
Skewness	-0.027186
Kurtosis	5.398.624
Jarque-Bera	121.1233

Table 4. Unit root tests of the series of returns.

Tests	MG	Test critical value : 1% level
ADF	-21.263317	-2.568790
KPSS	0.039752	0.739000
ERS	0.128421	1.99
ARCH	68.57045	

Table 5. Estimating parameters of GARCH (1.1) model with normal distribution.

Parameters	Coefficient	Probability
C	-0.084234	0.0929
α_0	0.046909	0.0000
α_1	0.136320	0.0001
β_1	0.769239	0.0000

Table 6. Estimating parameters of GARCH (1.1) model with student-t distribution.

Parameters	Coefficient	Probability
C	-0.099031	0.0362
α_0	0.040060	0.0102
α_1	0.156102	0.0000
β_1	0.770181	0.0000

Table 7. Unconditional coverage and conditional coverage of VaR.

	Cornish-fisher vaR		
Significant level	Coverage test	Normal	Student-t
1%	N	0	0
	Rate	0	0
	LRuc	NA	NA
	LRind	NA	NA
	LRcc	NA	NA
5%	N	8	9
	Rate	0.032	0.036
	LRuc	1.90673	1 .10914
	LRind	NA	NA
	LRcc	NA	NA
10%	N	20	34
	Rate	0.08	0.136
	LRuc	1.14094	3.35613
	LRind	1.37955	3.97297
	LRcc	2.52049	7.32911

with zero mean (Tables 5 and 6).

Back-testing VaR estimates

We evaluate the accuracy of the proposed VaR estimates over 250 day using the now standard coverage tests of Christoffersen (1998). We combine the GARCH (1.1) model with two approximation methods, the Johnson SU distribution and the Cornish-Fisher expansion, and derive the VaR estimates for the portfolio for α = 10, 5 and 1%.

In the finance literature there are basically two test procedures to compare the performances of VaR: Unconditional and Conditional.

We make use of Kupiec's (1995) test to evaluate GARCH specifications for unconditional coverage and Christoffersen test to embrace both unconditional coverage and the independence of violations. Kupiec Test and Christoffersen test results for the portfolio are reported in Tables 7 and 8.

Tables 7 and 8 present the different tests of VaR. The results show the great accuracy for all significance levels that we considered for GARCH VaR forecasting. The results are better for this sample. None of the normal models and the student-t models are rejected in the

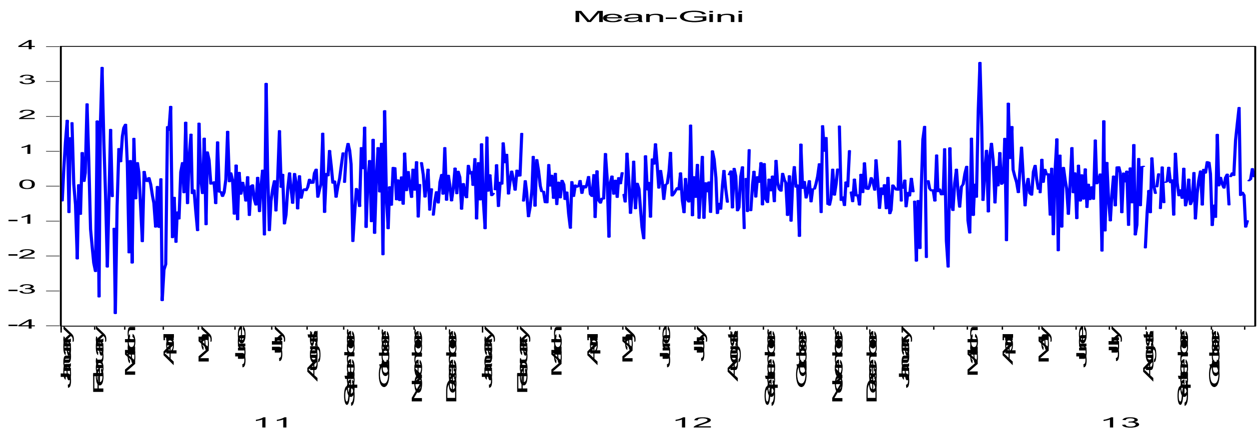

Figure 3. Evolution of MG portfolio.

Table 8. Unconditional coverage and conditional coverage of VaR.

Johnson SU VaR			
Significant level	**Coverage test**	**Normal**	**Student-t**
	N	4	4
	Rate	0.016	0.016
1%	LRuc	0.78136	0.78136
	LRind	NA	NA
	LRcc	NA	NA
	N	10	9
	Rate	0.04	0.036
5%	LRuc	0.54257	1.10915
	LRind	NA	NA
	LRcc	NA	NA
	N	20	19
	Rate	0.08	0.076
10%	LRuc	1.14094	1.67769
	LRind	1.37955	1.80681
	LRcc	2.52049	3.48449

independence test, across all significance levels. Overall, the normal models perform slightly better than the student-t model for the Cornish-Fisher expansion and the opposite for the Johnson SU distribution.

The results obtained by combining the same GARCH model with different approximation methods are slightly better with the Johnson SU approximation in cases. Actually, the only GARCH model which yields better results when coupled with the Cornish-Fisher expansion than with the Johnson SU distribution is α = 1%.

Conclusion

In his paper, we intend to achieve two objectives. First, discuss and make a comparison in crisis periods of analytical results obtained with MG strategy on the Moroccan financial market where MV strategy is not expected to be appropriate because of its strict distributional requirements on asset returns. Second, demonstrate empirically that quasi-analytic GARCH VaR forecasts can be accurately constructed using analytic formula for higher moments of aggregated GARCH returns by using Cornish-Fisher expansion and the Johnson SU distribution.

Results show that the composition of MG method have the great accuracy for all significance levels (10, 5 and 1%); we considered GARCH VaR for forecasting. Our results are even more remarkable when we consider that the analysis is entirely out-of-sample and that the testing period (2011-2014) contains several years of excessively turbulent financial markets. At the end of this study, we identified the following findings:

Firstly, returns are not normal because their distribution are positively or negatively skewed and leptokurtic or platykurtic as in the case of our data. Second, the volatility is not stable in time for that the use of GARCH models to take into account the volatility dynamics is crucial in predicting the value-at-risk. Finally, in financial crisis environment, it may be of critical importance to implement the best strategy which fits the investor's preferences as good as possible. In fact, MG strategy is the appropriate strategy for portfolio analysis and risk management.

Conflict of Interests

The authors have not declared any conflict of interest.

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