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Population growth and gender time distribution in a small-open growth model

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This study proposes an endogenous population economic growth model with gender time distribution between work, children fostering and leisure. It deals with nonlinear dynamic interdependence between the birth rate, the mortality rate, the population, wealth accumulation, and time distribution. We model the production side and wealth accumulation on the basis of the Solow model. The population dynamics is influenced by the Haavelmo population model and the Barro-Becker fertility choice model. The different growth mechanisms are integrated in a compact framework by applying the utility function proposed by Zhang. We simulate the model to demonstrate existence of equilibrium points and motion of the dynamic system and examine the effects of changes in the effects of changes in the propensity to have children, woman's propensity to pursue leisure activities, and woman's human capital.

Key words: Endogenous population, small-open economy, propensity to have children; birth and mortality rate, gender time distribution.

INTRODUCTION

The purpose of this study is to examine dynamic interdependence between endogenous population growth, wealth accumulation, and gender time distribution among work, children fostering and leisure activities. The model is a synthesis of some well-known approaches in economic growth theory and population dynamics. We deal with dynamic interactions between wealth and population dynamics for a small-open economy.

The economic aspects of this paper are strongly influenced by the neoclassical growth theory (Solow, 1956). We follow the Solow model in modeling economic production and wealth accumulation. We deviate from the Solow model in modeling behavior of households. We analyze household behavior by the approach proposed by Zhang (1993). Since Malthus published his *An Essay on the Principle of Population* in 1798, economists have made great efforts in revealing dynamic complexity of population change. Modern economies have experiences un-precedented population dynamics (such as aging and declining fertility rates in developed economies). This research treats birth rate and mortality rate as endogenous variables. Barro and Becker (1989) examine an interaction between endogenous fertility and economic growth in an overlapping generation model.

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Acemoglu and Johnson (2007) try to analyze the impact of life expectancy on economic growth. Aging become a great concern in many modern economies. Given the population structure, aging is closely related to mortality rate. Lancia and Prarolo (2012) propose a model of the longevity of life and economic development. Our study is strongly influenced by the literature of the neoclassical growth theory and the literature of population growth and economic developed. A unique contribution of this paper is to model population growth in the framework of the Solow growth model with endogenous wealth accumulation and gender time distribution. The physical capital accumulation is built on the Solow growth model. The birth rate and mortality rate dynamics are influenced by the Haavelmo population model and the Barro-Becker fertility choice model. We synthesize these dynamic mechanisms in a compact framework, applying an alternative utility function proposed by Zhang (1993). The model is actually a synthesis of Zhang's two models. Zhang (2012) develops a growth model of a small-open economy. Zhang (2014) develops a growth model with endogenous population without considering gender issues. The paper is organized as follows. Section 2 introduces the basic model with wealth accumulation and human capital accumulation with government subsidy on education. Section 3 simulates the model. Section 4 carries out comparative dynamic analysis with regard to some parameters. Section 5 concludes the study.

The basic model

Like in the Solow model, we consider a one-sector economy. It has a single commodity for consumption and investment. There is a single internationally tradable good, called industrial good, in the world economy and the price of the industrial good is unity fixed in global markets. Capital depreciates at a constant exponential rate, δ_k, which is independent of the manner of use. We assume that the economy is too small to affect the world rate of interest, r^*. All the markets are perfectly competitive. Factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production. Households own assets of the economy and distribute their incomes to consumption, child bearing, and wealth accumulation. The population of each gender is homogeneous. We assume that each family consists of husband, wife and children. All the families are identical. We use subscripts $q=1$ and $q=2$ to stand for man and woman respectively. We use $N(t)$ to stand for the population of each gender. Let $T_q(t)$ and $\overline{T}_q(t)$ stand for work time and time spent on taking care of children of gender q and $\overline{N}(t)$ for the flow of labor services used in time t for production. We have $\overline{N}(t)$

$$\overline{N}(t) = [h_1 T_1(t) + h_2 T_2(t)]N(t), \tag{1}$$

where h_q is the level of human capital of gender q.

The economic production

Let $K(t)$ stand for the capital stock used by the economy at time t. We use $F(t)$ to represent the output level. The production function is

$$F(t) = AK^{\alpha}(t)\overline{N}^{\beta}(t), \ \ \alpha, \beta > 0, \ \ \alpha + \beta = 1, \tag{2}$$

where $A, \alpha,$ and β are parameters. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. Let the wage rate per unit of time be denoted by $w(t)$. The marginal conditions are

$$r^* + \delta_k = \frac{\alpha F(t)}{K(t)}, \ \ w(t) = \frac{\beta F(t)}{\overline{N}(t)}, \ \ w_q(t) = h_q\, w(t). \tag{3}$$

From (2) and (3), we solve

$$\frac{K(t)}{\overline{N}(t)} = \bar{\delta}, \ \ w = \beta\,\bar{\delta}^{\alpha}, \ \ w_q = h_q\, w, \ \ \bar{\delta} \equiv \left(\frac{\alpha A}{r^* + \delta_k}\right)^{1/\beta}. \tag{4}$$

Consumer behaviors

We now use an alternative approach to household proposed by Zhang (1993). To describe behavior of consumers, we denote per family wealth by $\bar{k}(t)$. Per family current income from the interest payment and the wage payments is

$$y(t) = r^*\,\bar{k}(t) + w h_1 T_1(t) + w h_2 T_2(t). \tag{5}$$

The disposable income per family is given by

$$\hat{y}(t) = y(t) + \bar{k}(t) = (1 + r^*)\bar{k}(t) + w h_1 T_1(t) + w h_2 T_2(t). \tag{6}$$

Let $n(t)$ and $p_b(t)$ stand for the birth rate and the cost of birth at time. Following Zhang (2014), we assume that children will have the same level of wealth as that of the parent. In addition to the time spent on children, the cost of the parent is given by

$$p_b(t) = n(t)\bar{k}(t). \tag{7}$$

The relation between fertility rate and the parent's time on raising children is

$$\bar{T}_q(t) = \theta_q\, n(t), \quad \theta_q \geq 0. \tag{8}$$

The household distributes the total available budget between saving, $s(t)$, consumption of goods, $c(t)$, and bearing children, $p_b(t)$. The budget constraint is

$$p(t)c(t) + s(t) + \bar{k}(t)n(t) = \hat{y}(t). \tag{9}$$

We consider that except work and child caring, parents also have their leisure. We denote the leisure time of gender q by $\tilde{T}_q(t)$. The time constraint is

$$T_q(t) + \bar{T}_q(t) + \tilde{T}_q(t) = T_0\,, \tag{10}$$

where T_0 is the available time for leisure, work and children caring. Insert (10) and (8) in (9)

$$c(t) + s(t) + \tilde{w}(t)n(t) + h_1\, w(t)\tilde{T}_1(t) + h_2\, w(t)\tilde{T}_2(t) = \bar{y}(t), \tag{11}$$

where

$$\tilde{w}(t) \equiv \bar{k}(t) + \bar{\theta}\,, \;\; \bar{y}(t) \equiv \left(1 + r^*\right)\bar{k}(t) + \bar{w}\,, \;\; \bar{\theta} \equiv h_1\,\theta_1\, w + h_2\,\theta_2\, w, \;\; \bar{w} \equiv \left(wh_1 + wh_2\right)T_0\,.$$

We assume that the utility is dependent on $c(t),\ s(t),\ \tilde{T}_q(t),$ and $n(t)$ as

$$U(t) = c^{\xi_0}(t)s^{\lambda_0}(t)\tilde{T}_1^{\sigma_{01}}(t)\tilde{T}_2^{\sigma_{02}}(t)n^{\upsilon_0}(t),$$

where ξ_0 is called the propensity to consume, λ_0 the propensity to own wealth, σ_{0q} the gender $q's$ propensity to use leisure time, and υ_0 the propensity to have children. The first-order condition of maximizing $U(t)$ subject to (11) yields

$$c(t) = \xi\,\bar{y}(t), \;\; s(t) = \lambda\,\bar{y}(t), \;\; \tilde{T}_q(t) = \sigma_q\,\bar{y}(t), \;\; n(t) = \frac{\upsilon\,\bar{y}(t)}{\tilde{w}(t)}, \tag{12}$$

where

$$\xi \equiv \rho\,\xi_0\,, \;\; \lambda \equiv \rho\,\lambda_0\,, \;\; \sigma_q \equiv \frac{\rho\,\sigma_{q0}}{wh_q}, \;\; \upsilon \equiv \rho\,\upsilon_0\,, \;\; \rho \equiv \frac{1}{\xi_0 + \lambda_0 + \sigma_{10} + \sigma_{20} + \upsilon_0}.$$

According to the definitions, the population change follows

$$\dot{N}(t) = \left(n(t) - d(t)\right)N(t), \tag{13}$$

where $n(t)$ and $d(t)$ are respectively the birth rate and mortality rate. In this study we assume that the mortality rate is negatively related to the disposable income as follows

$$d(t) = \frac{\bar{\upsilon}\, N^{b_0}(t)}{\bar{y}^{a_0}(t)}, \tag{14}$$

where $\bar{\upsilon} > 0$ is the mortality rate parameter and $a_0 \geq 0.$

It should be noted that in the literature of population and economic growth economists identify many factors for explaining population dynamics. These factors include, for instance, changes in gender gap in wages (Galor and Weil, 1996), labor market frictions (Adsera, 2005), and age structure (Hock and Weil, 2012). Barro and Becker (1989) propose an endogenous fertility in an overlapping generation model with exogenous economic growth. Recently Bosi and Seegmuller (2012) extend the model by taking account of the heterogeneity of households in terms of capital endowments, mortality, and costs per surviving child. The model is built on the quantity-quality trade-off of having children, summarized by the adjustment of the average rearing cost of a surviving child. They show that a rise in mortality increases the time cost per surviving child and enhances economic growth, while reducing demographic growth. We may make our model more robust in explaining complexity of population dynamics by extending and generalizing our model on the basis of these studies in the future.

Insert (12) and (14) in (13)

$$\dot{N}(t) = \left(\frac{\upsilon\,\bar{y}(t)}{\tilde{w}(t)} - \frac{\bar{\upsilon}\, N^{b_0}(t)}{\bar{y}^{a_0}(t)}\right)N(t). \tag{15}$$

We now find dynamics of wealth accumulation. According to the definition of $s(t)$, the change in the household's wealth is given by

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t) = \lambda\bar{y}(t) - \bar{k}(t). \tag{16}$$

The national saving is the sum of the households' saving. We have

$$S(t) + C(t) + n(t)\bar{k}(t)N(t) - K(t) + \delta_k\, K(t) = F(t), \tag{17}$$

Where $S(t) = s(t)N(t), C(t) = c(t)N(t),$ and $K(t) = \bar{k}(t)N(t).$ We built the model.

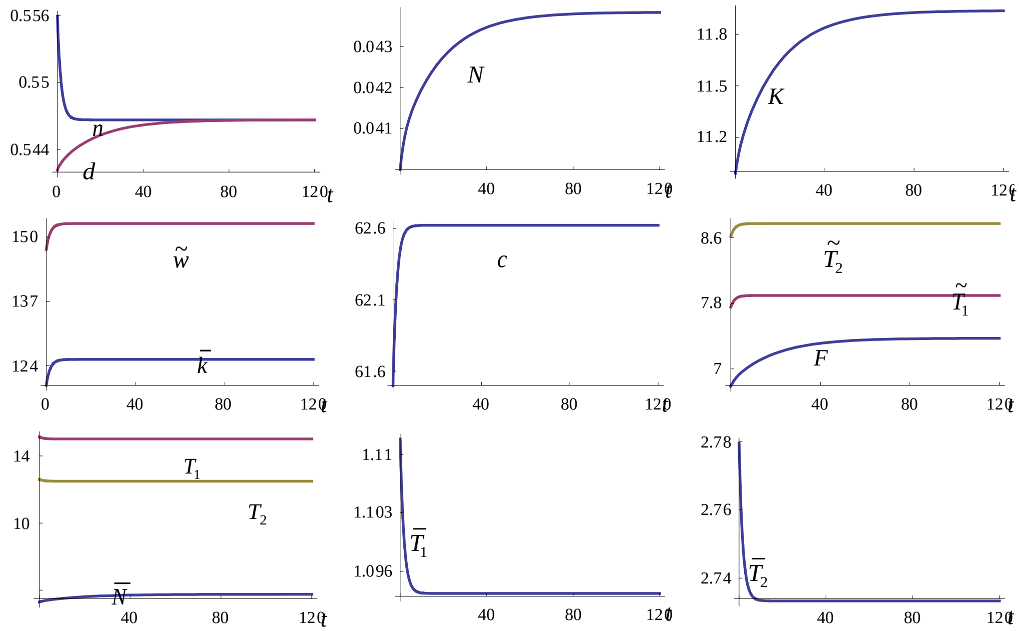

Figure 1. The motion of the economic system

The dynamics and its properties

The dynamics are expressed by differential equations with $\bar{k}(t)$ and $N(t)$ as the variables.

Lemma: All the variables are determined as functions of $\bar{k}(t)$ and $N(t)$ at any point of time by the following procedure: $\bar{y}(t) = (1 + r^*)\bar{k}(t) + \bar{w} \rightarrow c(t)$, $s(t)$, $\tilde{T}_q(t)$, and $n(t)$ by (12) $\rightarrow$ $\bar{T}_q(t)$ by (8) $\rightarrow$ $T_q(t)$ by (10) $\rightarrow$ $\bar{N}(t)$ by (1) $\rightarrow$ $K(t)$ by (4) $\rightarrow$ $F(t)$ by (2). The motion of $\bar{k}(t)$ and $N(t)$ is given by the following two differential equations

$$\dot{\bar{k}}(t) = \lambda \bar{y}(t) - \bar{k}(t),$$
$$\dot{N}(t) = \tilde{\Omega}(\bar{k}(t), N(t)) \equiv \left(\frac{\upsilon \bar{y}(t)}{\tilde{w}(t)} - \frac{\bar{\upsilon} N^{b_0}(t)}{\bar{y}^{a_0}(t)} \right) N(t). \qquad (18)$$

The expressions are complicated. It is difficult to explicitly interpret economic implications of the two equations. For illustration, we simulate the model to illustrate behavior of the system. In the remainder of this study, we specify the depreciation rate by $\delta_k = 0.05$, and let $T_0 = 24$. We specify the other parameters as follows

$$\alpha = 0.34, \; a = 0.05, \; b = 0.05, \; \lambda_0 = 0.6, \; \xi_0 = 0.3, \; v_0 = 0.4, \; \sigma_{10} = 0.16, \; \sigma_{10} = 0.16,$$
$$A = 1, \; a_0 = 0.2, \; b_0 = 0.2, \; h_1 = 5, \; h_2 = 4.5, \; \theta_1 = 2, \; \theta_2 = 5, \; \bar{\upsilon} = 1. \qquad (19)$$

We specify the initial conditions: $\bar{k}(0) = 120$ and $N(0) = 0.04$.

We plot the simulation result in Figure 1. The population grows from its low initial condition. The mortality rate and the labor force are augmented. The opportunity cost of children fostering and man's and woman's time of children fostering are increased.

The equilibrium values of the variables are as follows

$$N = 0.044, \; K = 11.94, \; \bar{N} = 5.75, \; F = 7.38, \; n = d = 0.55, \; w_1 = 4.23, \; w_2 = 3.81,$$
$$\tilde{w} = 152.74, \; \bar{k} = 125.24, \; T_1 = 15.01, \; T_2 = 12.49, \; \tilde{T}_1 = 7.9, \; \tilde{T}_2 = 8.8, \; \bar{T}_1 = 1.09,$$
$$\bar{T}_2 = 2.73, \; c = 62.62.$$

We calculate the two eigenvalues: -0.57 and -0.06.

COMPARATIVE DYNAMIC ANALYSIS

We use a variable $\bar{\Delta} x_j(t)$ to stand for the change rate of the variable, $x_j(t)$, in percentage due to changes in the parameter value.

A rise in the propensity to have children

First, we assume: $\upsilon_0 : 0.4 \Rightarrow 0.42$. The simulation results are plotted in Figure 2. In order to examine how each variable is affected over time, we should follow the motion of the entire system as each variable is related to the others in the dynamic system. As shown in Figure 2, the birth rate is increased, which also results in a rise in the population. The

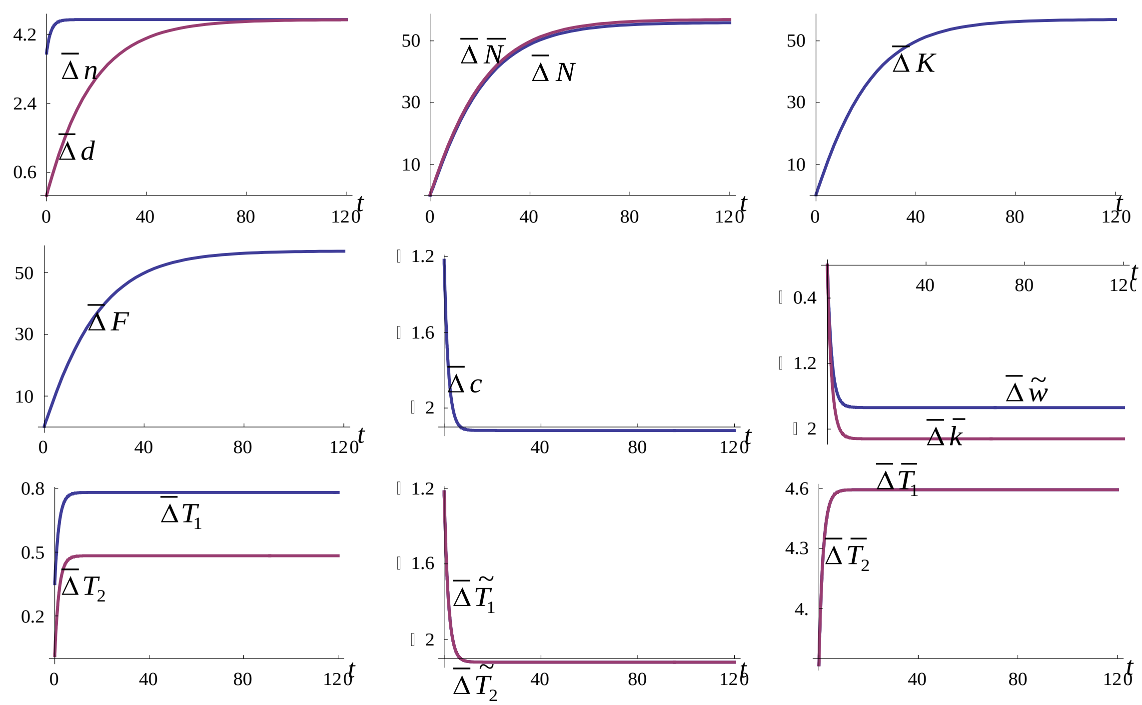

Figure 2. A rise in the propensity to have children

mortality rate is increased. The capital, total labor input and output level are all enhanced. The values of these variables are increased in association with population growth. Both the opportunity cost of children $\tilde{w}$ and the wealth per household are reduced.

A rise in woman's propensity to pursue leisure activities

We have: $\sigma_{02}: 0.16 \Rightarrow 0.17$. The simulation results are plotted in Figure 3. An immediate consequence of the preference change is that the wife spends more time on leisure and the husband has less leisure hours. The husband works more and the wife works less. Both the husband and wife reduce their time of children fostering. The consumption level, opportunity cost of children fostering and wealth are reduced. In association with the net impact of falling opportunity cost and reduction in the relative propensity to have children, the birth rate falls and the mortality rate rises. The population, total labor, wealth and output are all reduced.

Woman's human capital being improved

We now consider: $h_2: 4.5 \Rightarrow 4.7$. The results are plotted in Figure 4. The woman's wage rate rises. As the mother earns more per unit time, she works more. The opportunity cost of child fostering is increased in association with the mother's wage rising. The birth rate falls. The parents spend less time on children caring. The mother works more and the father works less. The father has more leisure time and the mother less. The family consumes more and has more wealth. The mortality rate falls in association with improved living conditions. The net impact of falling birth and mortality rates reduces the population in the long term. The capital, total labor input and output are increased.

Concluding remarks

This paper introduced endogenous population growth model into the Solow one sector growth model. The study proposed a dynamic interdependence between the birth rate, the mortality rate, the population, wealth accumulation, and time distribution between work, leisure and children caring. We emphasized the role of human capital, technological and preference changes on the birth and mortality rates and time distribution. The model is influenced by many traditional ideas about growth and population change. We took account of gender differences in human capital, the propensity to use leisure time, and children caring efficiency. We simulated the model to show the motion of the economic growth and population change and identified the existence of equilibrium points. We also examined the effects of changes in the propensity to

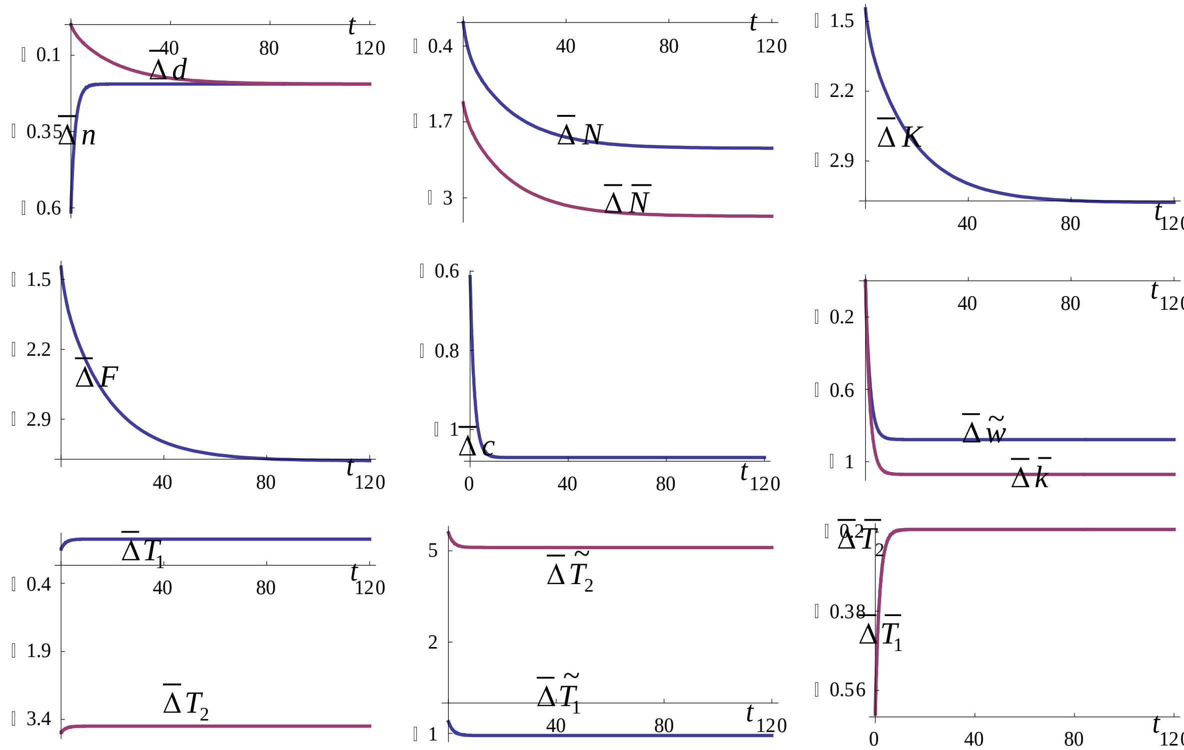

Figure 3. A rise in woman's propensity to pursuing leisure activities.

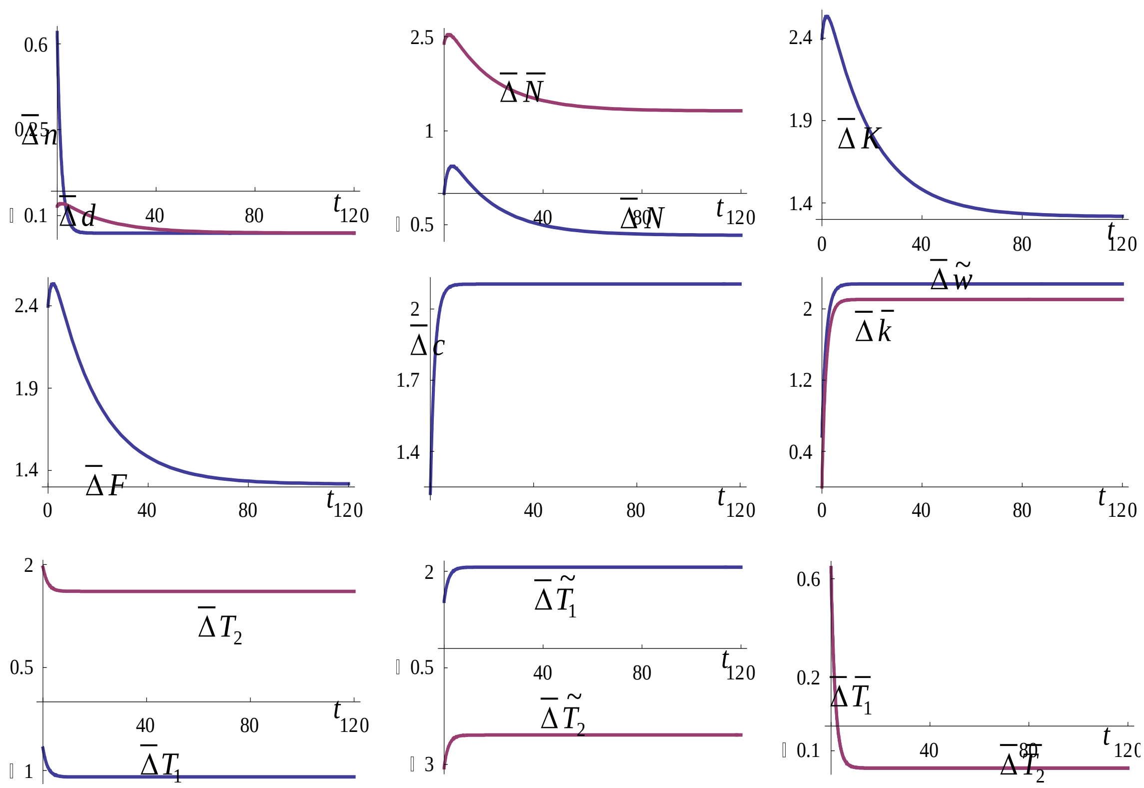

Figure 4. A rise in the mother's human capital.

have children, woman's propensity to use leisure, and woman's human capital.

Conflict of Interests

The author has not declared any conflict of interests.

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