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Full Length Research Paper

An investigation of the effect of hall current and rotational parameter on dissipative fluid flow past a vertical semi-infinite plate

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Magneto hydrodynamics (MHD) stokes problem for a vertical semi-infinite plate in a dissipative rotating fluid with hall current has been considered. The partial differential equations governing the problem are framed and then solved using numerical methods of implicit finite difference approximations. An analysis of effects of parameter on the velocity (both primary and secondary) profiles and temperature distribution profiles are shown graphically and the results are discussed.

Key words: Hall current, dissipative fluid, rotational parameter.

INTRODUCTION

Considerable progress has been made recently in the general theory of rotating fluids due to its various applications especially in the geological sciences. The application of rotating fluid are in oceanic, atmospheric and in cosmic fluid dynamics and solar physics (Kinyanjui and Uppal, 1998). Takhar and Soundalgekar (1976) did as study on the viscous dissipation effects on heat transfer in boundary layer flow past a semi-infinite horizontal flat plate and in 1977 they studied a forced and free convective flow past a semi- infinite vertical plate and also the MHD and heat transfer over a semi-infinite plate under a transverse magnetic field. Some of their other works include dissipation effects of MHD flow past a semi-infinite vertical plate, MHD free convection flow past a vertical semi-infinite plate with a uniform free-stream in 1985 and MHD free convection flow past a semi-infinite plate with uniform heat flux also in 1985.

Soundalgekar et al. (1979 a) studied free convection effects on MHD stokes problem for a vertical plate and they discovered that skin friction increased owing to a greater heating of the plate. Soundalgekar et al. (1979b) studied the finite difference analysis of free convection effect on stokes problem for a vertical plate in dissipative fluid. Soundalgekar et al. (1981) studied the MHD stokes problem for a vertical infinite plate with variable temperature. Chartuverdi (1996) studied the finite difference study of MHD stokes problem for a vertical infinite plate in a dissipative heat generating fluid with hall and Ion-slip current. Takhar and Soundalgekar, (1997) studied the forced and free convective flow past a semi-infinite vertical plate and MHD and heat transfer over a semi-infinite plate under a transverse magnetic field.

Kinyanjui and Uppal (1998) studied the MHD stokes problem for a vertical infinite plate in a dissipative rotating fluid with hall current and they later investigated the effect of both hall and Ion-slip currents on the flow of heat generating rotating fluid system. They observed that for an Eckert value of 0.02, there was a decrease in the primary velocity profile with an increase in rotational parameter but in the case of secondary velocity profiles, there was initially a decrease and as the distance from the plate increased, the secondary velocity profile increased. They also observed that an increase in hall parameter has no effect on the temperature profile but an increase in time causes an increase in the temperature profiles. Kinyanjui et al. (1999) studied the finite difference analysis of free convection effects on MHD problem for a vertical plate in a dissipative rotating fluid system with constant heat flux and hall current and also did a finite difference analysis of MHD stokes problem for a vertical infinite plate in a dissipative fluid with constant heat and hall current.

Kinyanjui et al. (2001) studied Magnetohydrodynamic free convection heat and mass transfer of a heat

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generating fluid past an impulsively started infinite vertical porous plate with hall current and radiation absorption MHD stokes free convection flow past an infinite vertical porous plate subjected to constant heat flux with ion-slip current and radiation absorption was studied (Kwanza and Uppal, 2003).

Chamkha (2004) studied unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. The presence of heat absorption (thermal sink) effects had the tendency to reduce the fluid temperature. This caused the thermal buoyancy effects to decrease resulting in a net reduction in the fluid velocity.

Hazem (2008) studied the effect of hall current on transient hydromagnetic Couette–Poiseuille flow of a viscoelastic fluid with heat transfer. Palani et al. (2009) studied MHD flow past a semi-infinite vertical plate with mass transfer. It observed that the magnetic parameter M has a retarding effect on velocity (Palani et al., 2009). Sweet et al. (2011) studied the analytical solution for the unsteady MHD flow of a viscous fluid between moving parallel plates. They discovered that an increase in the fluid density corresponded to a decrease in the velocity in the y-direction and that the initial velocity in the x-direction decreased with an increase in the fluid density.

In spite of all these studies, effect of hall current on a dissipative rotating fluid past a vertical semi-infinite plate has received little attention. Hence, the main objective of the present study is to investigate the MHD stokes problem for a vertical semi-infinite plate in a dissipative rotating fluid with hall current.

MATHEMATICAL ANALYSIS

We consider the flow of a viscous incompressible magneto hydrodynamics (MHD) free-convection heat generating fluid past an impulsively started semi-infinite vertical plate. The plate is jerked into motion in its own plane with constant velocity.

Since the motion of the boundary is in the x-direction then it may be assumed that the motion of the fluid will also be in that direction. Assume that a strong magnetic field of uniform strength H_0 is applied perpendicular to the direction of the flow as shown in Figure 1.

It is assumed that the induced magnetic field is negligible such that $H = (0,0,H_o)$. The assumption is justified because the magnetic Reynold's number of a partially ionized fluid is very small. The system is considered to be rotating with a uniform angular velocity Ω about the z-axis is taken normal to the plate. Since the plate is semi which -infinite in length, the variables are functions of z^+ and t^+ only. At time $t^+ > 0$, the plate start moving impulsively in its own plane with constant velocity v_0 and its temperature is instantaneously increased or decreased to $T^+{}_w$ which is constantly maintained later. The temperature of the fluid and the plate are assumed to be the same initially.

The equation of conservation of electric charge $\nabla . J = 0$ gives $j_{z^+} =$ constant where $J = (j_{x^+}, j_{y^+}, j_{z^+})$, this constant equal to zero, $j_{z^+} = 0$ at the plate which is non-conducting thus $j_{z^+} = 0$ everywhere in the flow. The generalized Ohm's law must be modified to include Hall current as follows:

$$\vec{J} + \frac{\varpi_e \tau_e (\vec{J} \times \vec{H})}{H_0} = \sigma\left(\vec{E} + \mu_e \vec{q} \times \vec{H} + \frac{1}{e\eta_e} \nabla p \right) \quad (1)$$

Where $\sigma, \mu_e, \tau_e, e, \eta_e, P_e$ are electrical conductivity, magnetic permeability, cyclotron frequency, collision time, electric charge, the number density of electrons and the electron pressure respectively.

It is assumed that $\varpi_e \tau_e \leq 1$. The applied magnetic field is assumed to be zero and the pressure gradient may be neglected. The effect of Ionslip and thermoelectric are neglected. Thus equation 1 becomes:

$$(j_x., j_y.) + \frac{\varpi_e \tau_e (j_y.H_0, -j_x.H_0)}{H_0} = \sigma\mu_e (v^* H_0, -u^* H_0) \quad (2)$$

Equating x^+ and y^+ component in the above equation yields:

$$j_x + \varpi_e \tau_e j_y = \sigma\mu_e v^* H_0 \quad (3)$$

$$j_y + \varpi_e \tau_e j_x = \sigma\mu_e u^* H_0 \quad (4)$$

Calculating j_{x^+} and j_{y^+} we have:

$$j_{x^+} = \frac{\sigma\mu_0 H_o [v^+ + mu^+]}{1+m^2} \quad (5)$$

$$j_{y^+} = \frac{\sigma\mu_0 H_o [mv^+ + u^+]}{1+m^2} \quad (6)$$

When the effect of rotation is considered, the coriolis force has to be included in the momentum equation. Considering a rotating frame of reference with a uniform angular velocity Ω, the equations of motion become:

$$\frac{\partial u^+}{\partial t} - 2\Omega v^+ = \frac{\upsilon \partial^2 u^+}{\partial z^2} + g\beta^* (T - T^+{}_\infty) + \frac{\mu_0 H_0}{\rho} j_{y^+} \quad (7)$$

$$\frac{\partial v^+}{\partial t} - 2\Omega v^+ = \frac{\upsilon \partial^2 u^+}{\partial z^2} - \frac{\mu_0 H_0}{\rho} j_{x^+} \quad (8)$$

$$\frac{\partial T^+}{\partial t} = \frac{K}{\rho C_p} \frac{\partial^2 T^+}{\partial z^{+2}} + \frac{v}{C_p}\left[\left(\frac{\partial V^+}{\partial z^+} \right)^2 + \left(\frac{\partial u^+}{\partial z^+} \right)^2 \right] \quad (9)$$

where g is the acceleration due to gravity, β^* is the volumetric

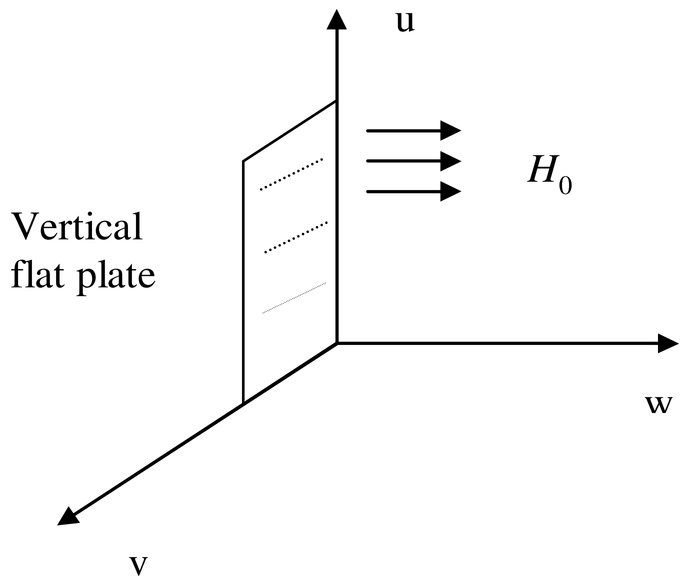

Figure 1. The flow configuration with co-ordinate system.

coefficient of thermal expansion, T^+, T_∞^+ are the temperature in the boundary layer and free- stream respectively,□ the fluid density , k is the thermal conductivity, C_p is the specific heat at a constant pressure, v is the kinematic viscosity, J_{x+}, J_{Y+} are the current density components and u^+, v^+ are the components in the X^+ and Y^+ directions.

The non-dimensional quantities are:

$$t = \frac{t^+ u_o^{\;2}}{\nu}, \quad z = \frac{z^+ u_0}{\nu}, \quad u = \frac{u^+}{u_o}, \quad v = \frac{v^+}{u_o},$$

$$x = \frac{x^+ u_0^{\;2}}{\nu}, \quad y = \frac{y^+ u_0^{\;2}}{\nu}, \quad Ec = \frac{U_o^{\;2}}{Cp\left(T^*_s - T^*_\infty\right)},$$

$$\theta = \frac{\left(T^+ - T^+_\infty\right)}{\left(T_s^{\;+} - T_\infty^{\;+}\right)}, \quad \Pr = \frac{\upsilon \rho C_p}{K} = \frac{\mu C_p}{K}, \quad Gr = \frac{\upsilon g \beta \left[T^*_s - T^*_\infty\right]}{U_o^{\;3}}$$

$$Er = \frac{\Omega \upsilon}{U^2_{\;0}}, \quad M_I = \sqrt{\frac{\sigma \mu^2_{\;e} H^2_{\;o} \upsilon}{\mu U^2_{\;o} / \upsilon}} = \sqrt{\frac{\text{Magnetic force}}{\text{inertia force}}}$$

Where

Gr- Grashof number
Pr- Prandtl number
Er- Rotational parameter
M^2- Magnetic parameter
Ec- Eckert number

Using the non-dimensional parameters, the governing equations of the fluid flow can be expressed in non-dimensional form as:

$$\frac{\partial \vec{q}}{\partial t} = \frac{\partial^2 \vec{q}}{\partial z^2} + Gr\theta - M^2 \vec{q} \qquad (10)$$

$$\Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial^2 t} + \Pr Ec \left[\left(\frac{\partial \vec{q}}{\partial z} \right) \left(\frac{\partial q}{\partial z} \right) \right] \qquad (11)$$

The initial conditions at t< 0 take the form

$$q(z,t) = 0 \qquad \theta(z,t) = 0$$

The initial conditions at $t \geq 0$ take the form

$$q(0,t) = 1 \qquad \theta(0,t) = 1$$

The boundary conditions as $z \longrightarrow \infty$ take the form

$$q(\infty,t) = 0 \qquad \theta(\infty,t) = 0$$

SOLUTION OF THE PROBLEM OF FINITE-DIFFERENCE METHOD

We solve our system of equations using the finite-difference method. In the finite-difference method, our Equations 10 and 11 take the form:

$$\frac{q(i,j+1) - q(i,j)}{\Delta t} = \frac{q(i+1,j) - 2q(i,j) + q(i-1,j)}{\Delta z^2} - m^2 q + Gr\theta(i,j) \qquad (12)$$

$$\Pr \frac{\theta(i,j+1) - \theta(i,j)}{\Delta t} = \frac{\theta(i+1,j) - 2\theta(i,j) + \theta(i-1,j)}{(\Delta z)^2} + \Pr Ec \left(\frac{q(i+1,j) - q(i,j)}{\Delta z} \right) \left(\frac{\vec{q}(i+1,j) - \vec{q}(i,j)}{\Delta z} \right) \qquad (13)$$

The index i refers to z and j to time t and $\Delta z = 0.1$

DISCUSSION

The results obtained are represented in form of graphs. The Prandtl number Pr is taken to be 0.71 (corresponding to air) and the magnetic parameter M^2 is taken to be 5.0 signifying a strong magnetic field. Gr > 0 (= 5) corresponds to cooling of the plate by free convection currents since the plate is at higher temperature than the surrounding and Gr < 0 (= -5) to heating of the plate by free convection currents since the plate is at a lower temperature that the surrounding.

In the presence of cooling of the plate by free convectional currents that is, when the Grashoff's number is greater than zero (equal to five) from Figures 2 and 3 we note that for Ec=0.01, an increase in the rotational parameter leads to a decrease in both the primary and secondary velocity profiles and for Ec=0.02, in Hall parameter for Ec = 0.01 and Ec = 0.02 leads to an increase both the primary velocity profiles and the secondary velocity profiles. We also observe that an increase in time for both Ec = 0.01 and Ec = 0.02 causes an increase in both the primary and secondary velocity. From Figure 4, the temperature velocity profiles for Gr >0, we observe that an increase in the rotational parameter

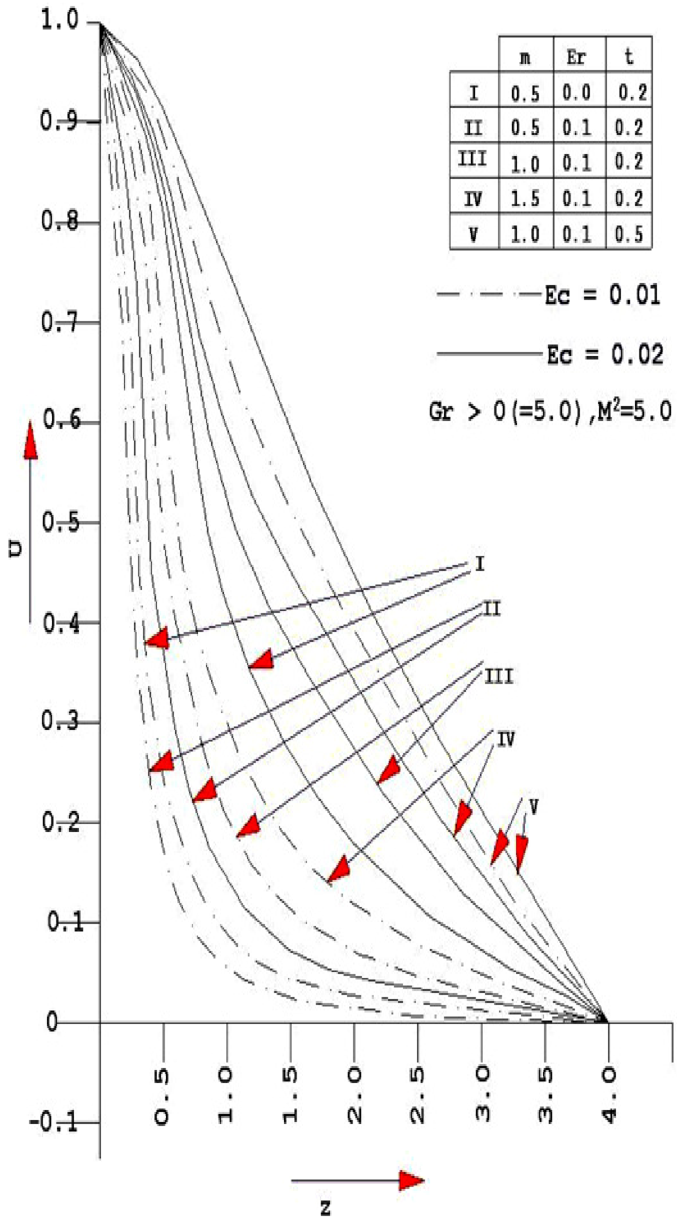

Figure 2. Primary velocity profiles for case of cooling plate by free convectional currents strong magnetic field with Eckert numbers of 0.01 and 0.02.

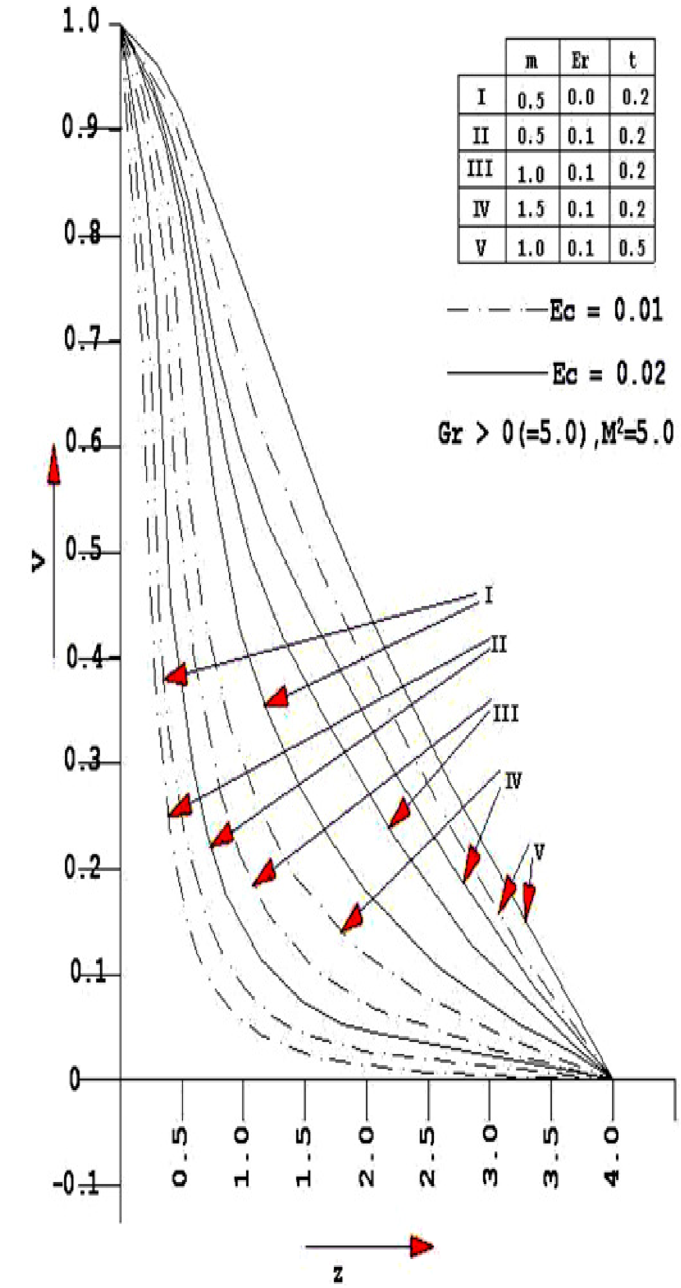

Figure 3. Secondary velocity profiles for case of cooling plate by free convectional currents strong magnetic field with Eckert numbers of 0.01 and 0.02.

Er, leads to an decrease in the temperature profiles for Ec = 0.01 but there is no change for Ec = 0.02 in the temperature profiles and an increase in Hall parameter m for both Ec = 0.01 and Ec = 0.02 causes no change in the temperature profiles. An increase in time causes an increase in the temperature profiles for both Ec = 0.01 and Ec = 0.02.

In case 2, in the presence of heating of plate by free-convection current for the case of Gr < 0 (= -5) from Figures 5 and 6 we observe that for Ec=0.01, an increase in the rotational parameter leads to a decrease in both the primary and secondary velocity profiles. For Ec=0.02, an increase in the rotational parameter leads to an increase in both the primary and secondary velocity profiles and an increase in hall parameter for Ec = 0.01 and Ec = 0.02 leads to an increase both the primary velocity profiles and the secondary velocity profiles. An increase in time for both Ec = 0.01 and Ec = 0.02 causes an increase in both the primary and secondary velocity.

From Figure 7, we observe that change in the rotational

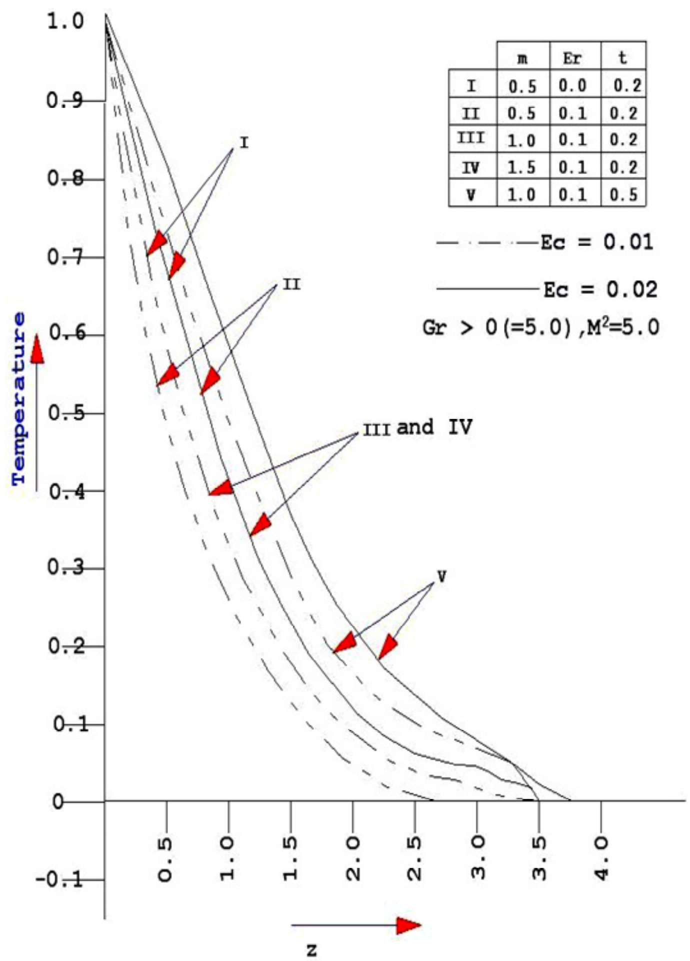

Figure 4. Temperature distribution profiles for case of cooling plate by free convectional currents strong magnetic field with Eckert numbers of 0.01 and 0.02.

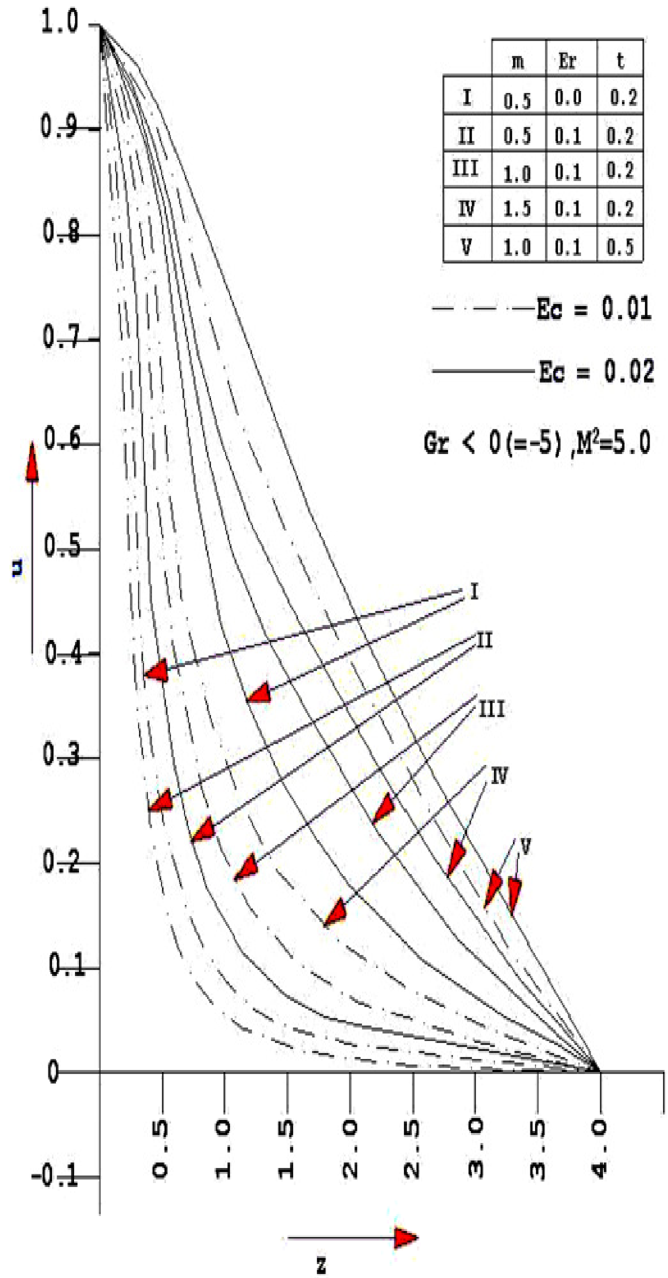

Figure 5. Primary velocity profiles for the case of heating plate by free convectional currents strong magnetic field with Eckert numbers of 0.01 and 0.02.

parameter causes no change in the temperature profiles. An increase in the Hall parameter causes no change in the temperature profile and also an increase in time causes a decrease in the temperature profiles.

Conclusion

This paper considered the flow of a viscous incompressible magneto hydrodynamics (MHD) free-convection heat generating fluid past an impulsively started semi-infinite vertical plate. The governing equations were non-dimensionalized and then transformed using implicit-finite difference method. The following points were concluded:

1) An increase in hall parameter for both cooling of the plate by free convection currents and heating of the plate by free convection currents led to an increase in the velocity profiles.

2) An increase in hall parameter for both cooling of the plate by free convection currents and heating of the plate by free convection currents has no effect on the temperature profiles.

3) An increase in rotational parameter led to a decrease in the velocity profiles when the Eckert number was 0.01 and an increase in velocity profiles when the Eckert number was 0.02.

4) An increase in rotational parameter had no effect on temperature profiles except in the case of cooling the

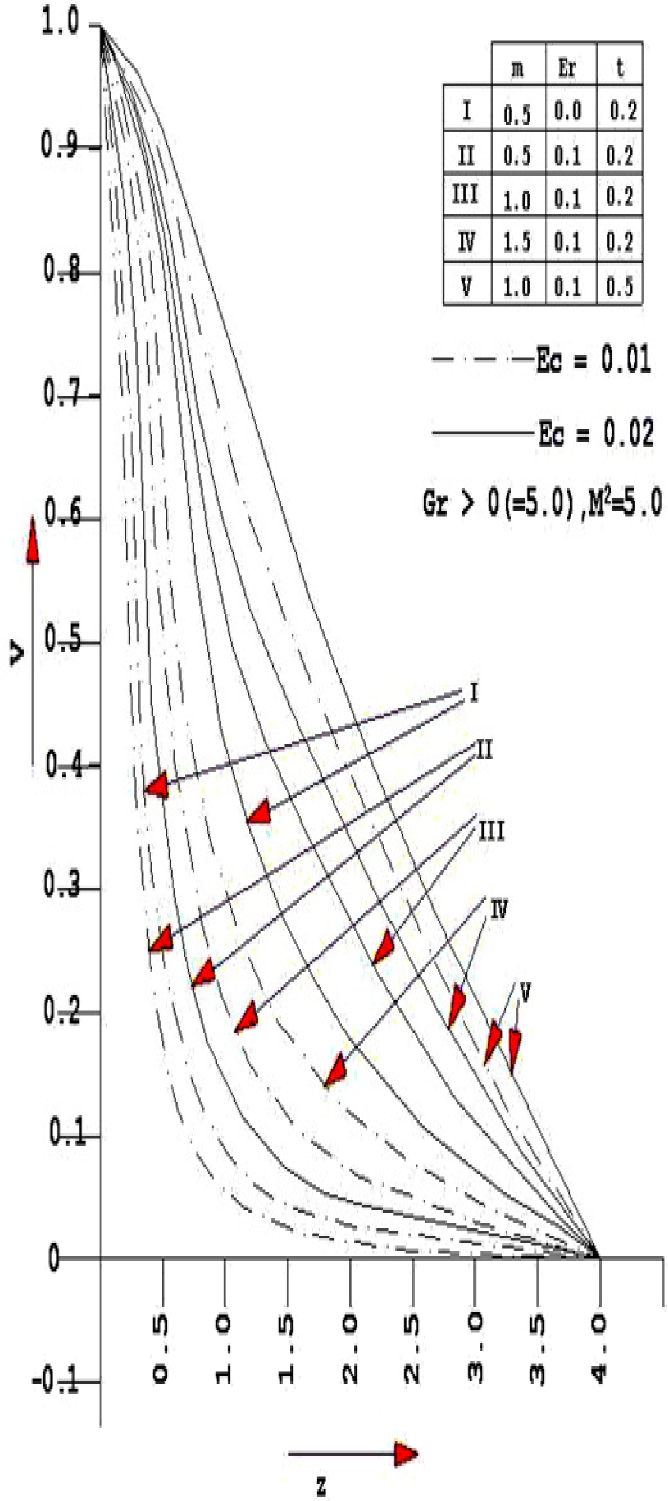

Figure 6. Secondary velocity profile for the case of heating plate by free convectional currents strong magnetic field with Eckert numbers of 0.01 and 0.02.

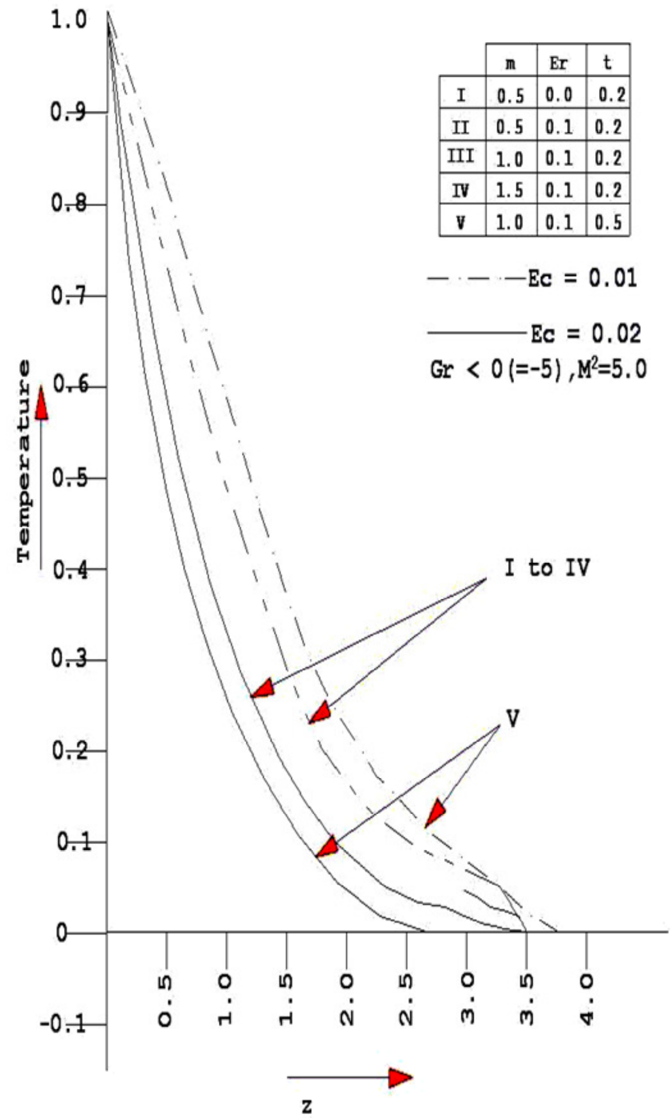

Figure 7. Temperature distribution profiles for the case of heating plate by free convectional currents strong magnetic field with Eckert numbers of 0.01 and 0.02.

plate when the Eckert number was 0.01. In this case, it caused a decrease in the temperature profiles.

5) An increase in the time led to an increase velocity (both primary and secondary) profiles in the case cooling the plate by free convection currents and led to a decrease in velocity profiles in the case of heating the plate by free convection currents.

It is hoped that the results will be useful for applications and that they can also be used for comparison with other problems dealing with Hall current and rotational parameter which might be more complicated .It is also hoped that the results can serve as a complement to other studies.

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