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# Stability and accuracy of the finite difference time domain (FDTD) method to determine transmission line traveling wave voltages and currents: The lightning pulse

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**The numerical solution of the transmission line wave equation is examined using the transmission line model of a lightning return stroke (LRS) taken as a case study. The LRS is represented by a transverse electromagnetic wave travelling at a velocity close to the velocity of light along a highly ionized lightning leader channel. The leader channel is modeled as a lossy transmission line. Attempts to model the lightning return stroke by an electric circuit in order to determine the currents and radiated electromagnetic fields is categorized as the distributed transmission line model (DTLM). The DTLM method in turn may be subdivided into two categories: Distributed inductance (L), capacitance (C) and resistance (R) model (DLCRM) and the lumped LCR model (LCRM). In this brief communication, we indicate some pitfalls to watch out for in solving for the transmission line wave equationusing numerical methods, and how these may be avoided. Furthermore, we propose tests that may be applied, including the analytical solution of diffusion waves, to ensure the stability and accuracy of the numerical solution DLCRM simulations.**

**Key words:** Finite difference time domain method, numerical solution of transmission line wave equation, lightning return stroke model, lightning radiated electromagnetic fields.

## INTRODUCTION

The electric transmission line is used to model many electrical engineering phenomena in both man-made (e.g. overhead power line, underground cable, microelectronic circuit line) and natural (e.g. the nerve system of the human body and the thunder storm lightning channel) circuits or systems. Accurate, stable and fast solution of the transmission line wave equation is important to use the calculated currents and voltages of the wave in protection, shielding and filtering of the engineering systems threatened by high speed electromagnetic waves characterized by large transient currents that change within submicrosecond time intervals. Thus the accurate solution of both electric current and voltages using numerical methods such as the finite difference time domain methodis of great interest. In this brief communication, we explore the solution of the transmission line wave equation representing the LRS wave along the cloud-to-ground (earth flash) lightning channel. Earth flash LRS currents and radiated lightning electromagnetic pulses (LEMP) are electrically the most severe aspects of a thunder storm. The magnitude of its severity, quantified by LRS currents and radiated electromagnetic fields, and its mitigation are critical aspects of design, installation, operation and protection of power transmission and distribution systems, wind turbine generators with long turbine blades, aircraft and rockets. But direct measurements of natural LRS currents or LEMP at ground level and well above ground are not easily measured. Hence there is a need to correctly model the LRS channel in order to determine the currents and the magnitudes of the LEMP.

We do not in this paper discuss the physics of LRS, or

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compare the DLCRM (Hoole and Hoole, 1988, 1993; Hoole, 1993) and LCRM (Pierce and Price, 1977; Little, 1978) models. Our concern here is to alert and propose methods to avoid errors in computing the currents or the LEMP using the DLCRM. The search for a self consistent lightning return stroke (LRS) model has reached a fascinating and challenging point where many different approaches are explored to correctly model it: Price and Pierce (1999), Eyathi and Cooray (2005), and Baba and Rakov (2008) to mention a few. Lightning measurement and analysis groups - such as those in Europe (Berger, 1977) and in the USA (Baba and Rakov, 2009), have provided a massive data base of lightning measurements. Those who are working on the characterization of lightning and developing powerful software codes to simulate and study lightning need to exercise great care in the mathematical modeling and computational work done. Some aspects of the care needed to be exercised in modeling and simulating the LRS currents are presented herein. The distributed transmission line model (DLCRM) where the time-invariant line elements are calculated using basic electromagnetic theory, provides stable results for the voltage and time impulses which agree with current measurements at ground level (Berger, 1977), and yield computed LEMP that agree with measured LEMP (Baba and Rakov, 2009: 7; Little, 1978; Hoole, 1993; Moosave et al., 2009; Deindorfer and Uman, 1990).

## DISTRIBUTED INDUCTANCE, CAPACITANCE AND RESISTANCE TRANSMISSION LINE MODEL (DLCRM)

Although the seed thought for the DLCRMwas planted by the lumped LCR model of Bruce-Golde followed by the work of many others on lightning engineering (Golde, 1977), it was most rigorously worked out and reported by Pierce and Price (1977), and Little (1978). In this earlier modeling work, however, lumped circuit parameters for each line segment (lumped-LCR) were used instead of distributed LCR parameters. Amongst others, Hoole and Hoole (1988, 1993), and Hoole (1993) reported a self consistent DLCRM, correcting some of the short comings of the earlier works. In an attempt to establish the scientific basis for the DLCRM (Hoole and Hoole, 1988), the electric plasma nature of the lightning channel was examined and it was shown that the LRS wave satisfies the essential characteristics of a quasi-transverse electromagnetic (quasi-TEM, not simply a TEM) wave along an un-magnetized plasma channel. The quasi-TEM wave is what is exactly approximated by a DLCRM transmission line travelling wave.

The FDTD approximation of the quasi-TEM wave equation is given by (Hoole and Hoole, 1993; Hoole, 1993)

$$\left(\frac{R\Delta t}{2}+L\right)V_n(z,t-\Delta t)+\ 2\left(L-\frac{\Delta t^2}{C\Delta z^2}\right)V_n(z,t)$$

$$+\frac{\Delta t^2}{C\Delta z^2}\left(V_{n+1}(z+\Delta z,t)+\ V_{n-1}(z-\Delta z,t)\right. \quad \ldots \ldots\ldots\ldots\ldots(1)$$

for the voltages $V_n$ along the DLCRM line (n= 1,2,…. N where N is the number of segments into which the channel has been divided into), with per unit length inductance, capacitance and resistance of L, C and R respectively. The time step is $\Delta$t and the distance step is $\Delta$z. With an initial current (e.g. lightning leader current) $I_o$ assumed to be flowing along the entire length of the line, the current equation is approximated by (Hoole, 1993)

$$I_n(z,t)=I_0+\sum_{i=1}^{n}\left(\frac{V_i(z,t+\Delta t)-V_i(z,t-\Delta t)}{2\Delta t}\right)C_i \quad \ldots\ldots\ldots(2)$$

where, as noted previously, $I_0$ is the leader current that flows along the lightning channel before the initiation of the return stroke.

The line is subdivided into *n* segments of equal length. Each segment *i* has a capacitance of $C_i$ per unit length, and the summation in Equation (2) is over the entire length of the lightning channel, representing integration over the whole length of the channel. Using L, C and R values that are representative of the lightning channel, it was shown that the DLCRM yields both currents and radiated electromagnetic fields (LEMP) that agree well with measured lightning currents and radiated LEMP (Hoole and Hoole, 1993).

## TESTING THE ACCURACY OR RELIABILITY OF THE COMPUTER SIMULATED LIGHTNING

To start the time-stepping computations, initial values are required. The initial potential $V_0$ and current $I_0$ along the line are known (Golde, 1977). In solving the FDTD approximation, DLCRM transmission line wave, the so-called magic time step ($\Delta t = \Delta z/c$, where c is the velocity of light) and the Courant stability criterion (Taflove and hagness, 2005) alone are not adequate. In the DLCRM travelling wave, the L, C and R per unit length values play an explicit role in determining the stability and accuracy of the FDTD solution, as seen in (1) and (2). In the LCRM, there is this inherent weakness where the distributed line elements are lumped together for each segment: thus the L, C and R elements that appear in the simulation program as the per unit length elements multiplied by the length of each segment. The length of each segment does not play a direct role in the LCRM circuit solution, where as in DLCRM, it plays an explicit role, as for instance in (1) where it appears as ($\Delta z^2$).

It is our experience that the Equations (1) and (2) could be rapidly solved using finite difference time domain (FDTD) method in one spatial dimension (*z*), where the time step is kept small compared to $\sqrt{LC}$ and $L/2R$ in order to obtain stable solutions with sufficient number of calculated points in the wave front. The distance step is chosen so to keep it longer than $\sqrt{2\Delta t/RC}$. Whence, to ensure stable solutions, the following two steps are adopted: (1) Choose $\Delta t$ such that it is small compared to $L/2R$, and (2) choose $\Delta z$ so that it is greater than both $\Delta t\sqrt{LC}$ and $\sqrt{2\Delta t/RC}$. These conditions ensure the stability of the time stepping and spatial stepping used in FDTD method to solve (1) and (2), whatever the magnitude of *R*, *L*, and *C* may be in (1). This is roughly verified by considering the ratios $\Delta z^2 : 2\Delta t/RC$ : $\Delta t^2/LC$.

In the original (Price and Pierce, 1977) and more recent work on lightning modeling (Moosavi et al., 2009; Deindoffer and Uman, 1990; Theeyathi and Cooray, 2005), self consistent tests to ensure correct and stable solutions of Equations (1) and (2) are not discussed. It is rather uncertain that some of the complicated models used for the line elements such as using atime-varying LRS channel resistance, inductance or capacitance (Moosavi et al., 2009; Theeyathi and Cooray, 2005; Baba and Rakov, 2009), give better results. Recent work has reconfirmed that the simpler form of the DLCRM reported in Hoole and Hoole (1993) and Hoole (1993) is a reliable and an adequate model of cloud-to-ground lightning flash (Hoole et al., 2010).

## RESULTS

In Figure 1a is given the lightning return stroke currents calculated using the DLCRM reported in Hoole and Hoole (1993) and Hoole (1993). The peak lightning return stroke current and the maximum rate of rise of current over the convex shaped part of the current wavefront, clearly and correctly captured in the DLCRM computation as seen in Figure 1a, are important parameters which determine the adverse effects such asheating, high pressure, electromagnetic interference and induced voltages due tothe lightning return stroke on aircraft structure, communication and control electronics, and power systems. In Figure 1b is shown theelectric field calculated at a distance of 10 km from thelightning flash, using the integral method (Hoole and Hoole, 1987). A knowledge of themagnitude and submicrosencond changes of lightning radiated electromagnetic pulse (LEMP) at various points in space is crucial when designing filters and shields to protect mission critical systems such as the military aircraft control system. The LEMP can cause changes in digital data and control signals that could result in destabilizing the control system sending the fast flying military, fighter aircraft spinning out of control.

The electric field is calculated from the LRS currents calculated using the DLCRM (Hoole and Hoole, 1993). The initial, sharp peak of the electric field is due to the second, convex section of the return stroke current wavefront. If the current versus time characteristics of the return stroke was correctly obtained from the DLCRM computer simulation, then the electromagnetic fields calculated from those current yields very helpful, lightning parameters above the ground and far from the LRS flash which are very difficult or impossible to measure.

The hump following the initial sharp peak in electric fields, as shown in Figure 1b, also matches with measured electric fields (Baba and Rakov, 2009; Hoole and Hoole, 1993). The computer simulations were limited to the first 20 μs, the LEMP period of interest in engineering design, the period over which very good agreement was found for the following parameters of the lightning: LRS current waveform at ground level, electric and magnetic field waveforms, LRS velocity and rate of rise of currents. In the future, we intend to examine the long term characteristics and computation of the LRS.

When solving Equations (1) and (2), the following three factors must be noted:

1. The L, C and R segment at the earth end must be rigorously worked out, and the values should be reasonable when compared to values obtained by using electromagnetic and plasma theory (Price and Pierce, 1977; Little, 1978; Hoole and Hoole, 1993). For instance, if the upward leader is assumed to be a resistive element, or if it is represented by C and R only, the numerical solution of the DLCR model will result in a singularity point at the earth end. When sketching the current waveform, this may get inadvertently suppressed by the computer plot of the current waveform. Thus it is important to ensure that points on the wave-front of the earth end segment were calculated by the FDTD computer code.
2. The length of eachLCR segment must explicitly appear in the numerical computation of the traveling wave. It ought not to be multiplied with the per unit length values of the R, L and C elements to form a lumped LCR network.
3. Care should be exercised in the time steps used. The time steps should be kept to a value well below the ratio L/R. Moreover, the time step should also be less than or equal to the length of each segment divided by the velocity of the wave (Taflove and Hagness, 2005). This ensures that at each point in time at which the computations of Equations (1) or (2) are done, the wave has not shot over into an adjacent segment of the DLCRM line. A figure for the time step may be worked out assuming a reasonable value for the velocity of the return stroke, say c/3 where c is the velocity of light.

Once the solution is done, take a look at the current wavefront to ensure that the solution obtained is not for aR $>>2\pi f L$diffusion problem, where f is the significant and highest frequency of the LEMP spectrum (e.g. f = 1 to 6 MHz (Berger, 1977; Hoole and Hoole, 1996). In the diffusion case, the final current waveforms look rather smooth and restrained, and the velocity is about two orders less than the velocity of light (Hoole, 1993).

In order to check the proper working of the computer-coded DLCRM and to ensure reliable solutions of V and I, two helpful, analytical tests may be employed. First, set the values of the L, C and R elements to reflect a diffusion problem, for which there is an analytical solution (Hoole and Hoole, 1988). Now compare the current pulses obtained from the numerical solution of the DLCRM and the analytical solution of the same problem (Hoole and Hoole, 1993). Second, set the values of L, C and R elements to that of the LRS parameters that are to form the DLCRM to be computer simulated (Hoole and Hoole, 1993). Do two sets of computer simulation of LRS using the DLCRM. First, let the length of the lightning channel (e.g. 1 km long) be divided, say, into 10 segments (thus the length of each segment is 100 m long for a 1 km long channel). Obtain the voltage changes and the current pulses in each segment. Now repeat the solution with the channel divided into, say, 30 segments. Repeat the DLCRM simulation to compute the current and voltage pulses. Do the V and I values agree with time

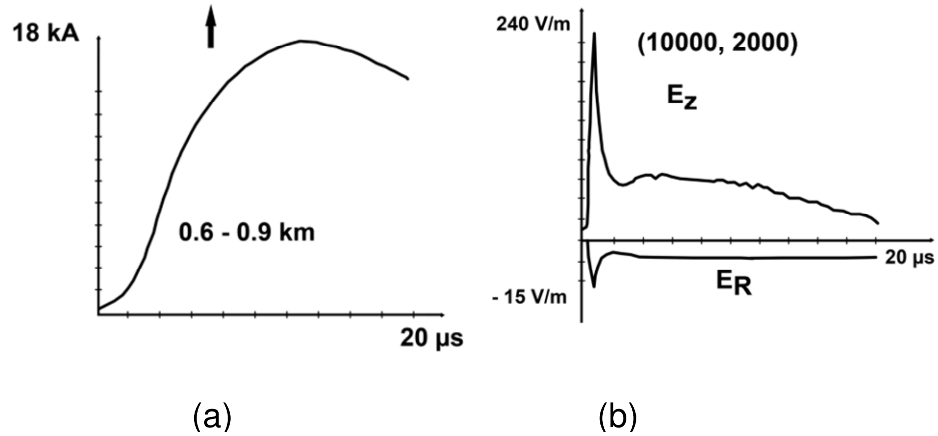


**Figure 1.** (a) Lightning return stroke current calculated using the DLCRM, at above-the-ground height of 0.75 km along the lightning return stroke channel. Observe the three parts of the return stroke current wavefront: First a ramp like increase, second a sharply increasing convex wavefront. Thirdly, a slow rate of rising towards the peak of the current waveform; (b) Lightning radiated electric field calculated 10 km horizontally away from the lightning flash, and at a height of 2 km from ground level. $E_z$ (peak at about 240 V/m) is the vertical, dominant component of the radiated electric field – a component that is important in engineering systems. $E_R$ (peak at about -15 V/m) is the horizontal component (parallel to the earth) of the radiated electric field at (10 and 2 km).

domain characteristics of the LRS at the same height of the lightning channel?

## SUMMARY

The transmission line wave equations that were numerically solved using Equations (1) and (2) is an approximation of the Maxwell's equations for time-varying electromagnetic fields. This brief communication has provided rules for discretizing the transmission line wave equation in time and spatial domains to obtain accurate and stable solutions of both voltages and currents along the distributed transmission line. Moreover, two specific methods to test the accuracy of the solutions have been proposed. On the solution of Maxwell's equations using a numerical method such as the finite difference method, we would like to sound the same note of caution as in (3) previously mentioned. It is pertinent that the time step should be so constrained that at each step, the wave remains within, say, a finite difference grid. This would result in a grid made up of a vast number of elements. Moreover, since the current (electric) wave travels along the LRS channel at a velocity (e.g. c/3) less than the velocity at which the radiated electric and magnetic waves travel out from the channel (at c, the velocity of light), care must be taken to account the existence of two waves at two different velocities: One wave moving at a velocity less than the velocity of light along the lightning leader channel modeled by a transmission line, and the second wave radiated out from the transmission line at the velocity of light.

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