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Review Paper

Shortcomings in the standard continuum based implicit joint model of layered rocks

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Layered rock masses can be described efficiently using a continuum formulation. There are two distinctive continuum based formulations that are found in the published literatures e.g. conventional continuum formulation based models such as Ubiquitous Joint model and non conventional formulation based models such as Cosserat Continuum models. Such equivalent Continuum models may provide reasonably accurate predictions when joint slips are minimal that is. When the shearing is in the direction of layering and rock layer bending can be neglected. However, when joint slips are large and loading direction is not aligned with the direction of layering models based on conventional continuum theories may considerably overestimate the deformation since the bending rigidity of the rock layers are not incorporated in such model formulations. For the case of rock layers with bending stiffness, an Accurate Continuum model can be formulated successfully on the basis of Cosserat continuum theory. The accuracy of both the conventional and the Cosserat Continuum models to describe the load-deformation behaviour of the layered rocks is studied in this paper.

Key words: Ubiquitous Joint model, Cosserat model, layered rock, flexural toppling.

INTRODUCTION

Modelling the behaviour of rock masses consisting of a large number of layers is often necessary in mining applications (e.g. coal mining). Such a modelling can be carried out in a discontinuous manner by explicit introduction of joints using either the finite element or distinct element approach (Goodman et al., 1968; Cundall, 1987).

When the number of layers to be modelled is excessively large (that is when the layers are thin compared to the dimensions of the engineering structures) it is advantageous to devise a continuum-based method. A continuum description of a layered medium can be formulated as long as consistency and statistical homogeneity in joint properties and spacing can be established. Such a continuum model provides a large-scale (average) description of the material response to loading. The continuum model devised in such a manner is often known as smeared (implicit) joint model in a sense that the joints are implicit in the choice of the stress-strain relationship adopted for the equivalent continuum. A distinctive advantage of the smeared joint model is that in a numerical (e.g. finite element) solution the problem region can now be discretized with a coarser mesh (that is subdivided into fewer finite elements) than in the discrete models where the size of the finite elements cannot exceed the layer thickness. Thus, in smeared joint models, the size of the elements is solely dictated by computational needs rather than by the layer thickness.

In the models based on the conventional equivalent continuum approach (that is standard implicit joint model), the layered material is replaced with a homogeneous anisotropic medium characterised by the so called effective elastic moduli comprising the heterogeneity of the medium. The elastic standard implicit joint model has been extended for the layered materials exhibiting strength anisotropy along the layer interfaces (e.g. Ubiquitous Joint model in FLAC (Itasca, 2008)). Such equivalent continuum models may provide reasonably accurate predictions when joint slips are minimal that is, when the shearing is in the direction of layering and rock layer bending can be neglected. However, when joint slips are large and loading direction is not aligned with the direction of layering, rock layers do bend as they slip against each other. In such cases models based on conventional continuum theories may considerably overestimate the deformation since the bending rigidity of the rock layers are not incorporated in such model formulations.

For the case of rock layers with bending stiffness, such an implicit continuum model can be formulated successfully on the basis of Cosserat theory (Cosseart and Cosserat, 2009). The Cosserat model provides a large-scale (average) description of a layered medium. An important feature of the Cosserat model is that it incorporates bending rigidity of individual layers in its formulation and this makes it different from other conventional implicit models. Cosserat based equivalent continuum models were formulated in (Mühlhaus, 1993) and (Adhikary and Dyskin, 1998) where the rock layers were assumed to be elastic. In (Adhikary and Dyskin, 1998), provision was made for plastic deformation along the joints only. Adhikary and Guo (2002) further developed a model incorporating plastic deformation of both joints and rock layers.

The accuracy of both the Implicit Joint and the Cosserat Models to accurately describe the load-deformation behaviour of the layered rocks will be studied in this paper.

THEORETICAL FORMULATIONS

A full description of the Implicit (Ubiquitous Joint) model for strength anisotropy can be found in (Itasca, 2008). A full description of the two dimensional plane strain Cosserat model with elastic rock layers was previously presented in (Adhikary and Dyskin, 1998) and with elasto-plastic rock layers was presented in Adhikary and Guo, 2002). Hence, this research only concentrates on the major differences between the Implicit Joint model and the Cosserat model.

In the Cosserat model using the Cartesian coordinates (x_1, x_2), the material point displacement can be defined by a translational vector (u_1, u_2) and by a rotation Ω_3, whereas the material point displacement is defined only by a translational vector (u_1, u_2) in the Implicit Joint model. Here, axis 3 is aligned to the out of plane direction and axis 2 is perpendicular to the layers.

The two-dimensional Cosserat model has 4 non-symmetric stress components $\sigma_{11}, \sigma_{22}, \sigma_{21}, \sigma_{12}$ and two couple stresses m_{31}, m_{32}, whereas the two-dimensional implicit joint model has only three stress components $\sigma_{11}, \sigma_{22}, \tau = \sigma_{21} = \sigma_{12}$.When the rock layers are aligned in the 1-coordinate direction, the moment stress term m_{32} vanishes.

Cosserat model

The four stresses are conjugate to four deformation $\gamma_{11}, \gamma_{22}, \gamma_{21}, \gamma_{12}$ measures defined by:

$$\gamma_{ij} = \frac{\partial u_j}{\partial x_i} - \varepsilon_{3ij}\Omega_3 \tag{1}$$

And the couple stress m_{31} is conjugate to the respective curvature κ_1 defined by:

$$\kappa_1 = \frac{\partial \Omega_3}{\partial x_1} \tag{2}$$

The elastic stress strain relationships are described by:

$$\boldsymbol{\sigma} = [D_e]\boldsymbol{e}_e \tag{3}$$

Where $\boldsymbol{\sigma} = \{\sigma_{11}, \sigma_{22}, \sigma_{21}, \sigma_{12}, \mathrm{m}_{31}\}$,

$$\boldsymbol{e} = \{\gamma_{11}, \gamma_{22}, \gamma_{21}, \gamma_{12}, \kappa_1\} \tag{4}$$

$$D = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 \\ & A_{22} & 0 & 0 & 0 \\ & & G_{11} & G_{12} & 0 \\ & symm & & G_{22} & 0 \\ & & & & B_1 \end{bmatrix} \tag{5}$$

Here,

$$A_{11} = \frac{E}{1 - \nu^2 - \dfrac{\nu^2(1+\nu)^2}{1 - \nu^2 + \dfrac{E}{hk_n}}},$$

$$A_{22} = \frac{1}{\dfrac{1 - \nu - 2\nu^2}{E(1-\nu)} + \dfrac{1}{hk_n}},$$

$$A_{12} = \frac{\nu}{1-\nu} A_{22} \tag{6}$$

$$\frac{1}{G_{11}} = \frac{1}{G} + \frac{1}{hk_s}, \; G_{11} = G_{12} = G_{21}, \; G_{22} = G_{11} + G \tag{7}$$

And

$$B_1 = \frac{Eh^2}{12(1-\nu^2)}\left(\frac{G - G_{11}}{G + G_{11}}\right) \tag{8}$$

Where E is the Young's modulus of the intact layer, ν is the Poisson's ratio, h is the layer thickness, G is the shear modulus of the intact layer, k_n and k_s are the joint normal and shear stiffness. When the layer thickness h tends to zero (that is. B_1 tends to zero) the Cosserat model reduces to the standard Implicit Joint model.

Implicit joint model

The three stresses are conjugate to three deformation $\varepsilon_{11}, \varepsilon_{22}, \gamma$ measures defined by:

$$\varepsilon_{ij} = \frac{\partial u_j}{\partial x_i} \text{ for } i=j \tag{9}$$

$$\gamma = \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \tag{10}$$

The elastic stress strain relationships are described by:

$$\boldsymbol{\sigma} = [D_e]\boldsymbol{e}_e \tag{11}$$

Where $\boldsymbol{\sigma} = \{\sigma_{11}, \sigma_{22}, \tau\}$, $\boldsymbol{e} = \{\varepsilon_{11}, \varepsilon_{22}, \gamma\}$ and (12)

$$D = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & G \end{bmatrix} \tag{13}$$

Here,

$$A_{11} = \frac{E}{1-\nu^2 - \dfrac{\nu^2(1+\nu)^2}{1-\nu^2+\dfrac{E}{hk_n}}},$$

$$A_{22} = \frac{1}{\dfrac{1-\nu-2\nu^2}{E(1-\nu)} + \dfrac{1}{hk_n}},$$

$$A_{12} = \frac{\nu}{1-\nu}A_{22} \tag{14}$$

Where E is the Young's modulus of the intact layer, ν is the Poisson's ratio, h is the layer thickness, G is the shear modulus of the intact layer, k_n and k_s are the joint normal and shear stiffness.

The Ubiquitous Joint model described in FLAC (Itasca, 2008) is a strength anisotropy model and thus is assumed to have isotropic elastic properties with reduced strength in the direction of rock layering.

Figure 1 presents the stresses and volume forces acting on a Cosserat element and Ubiquitous Joint element representing a layered rock with layers oriented in the 1-direction.

DEFICIENCY INHERENT IN THE IMPLICIT JOINT MODEL

The Implicit Joint model works well as long as the rock layers are subjected to translational deformation without any bending (this may include slip along the layer interfaces). However when the rock layers undergo bending during loading the Implicit Joint model (such as incorporated in FLAC (Itasca, 2008)) may break down completely.

For simplicity let us assume that the rock layer interfaces (joints) have zero shear strength that is. both the cohesion and the friction angle along the layer interfaces are zero and the rock layer is elastic. Thus if the layered rock is subjected to loading such that the layers slip along the interfaces and at the same time undergo bending. Though the magnitude of the shear stress component along the layer interfaces will be zero, the shear stress component perpendicular to the layering direction dies not vanish and will increase in proportion to layer bending. However in the Implicit Joint model the magnitude of shear stress component σ_{21} (that is the shear stress component acting in the direction perpendicular to the layering direction) cannot increase as it is restricted to be equal to σ_{12} (that is the shear stress in the layering direction which is assumed to be zero; Figure 2a). This is due to the virtue of the fundamental assumptions of two equal shear components made in the standard continuum formulation to avoid the elemental rotation. A zero shear stress component in the direction perpendicular to layering implies essentially a weak rubber like material with no bending stiffness. Thus such implicit joint models may yield erroneous and excessively large deformations.

This could happen easily in the case of slopes excavated in layered rocks (Figure 2) where the rock layers slip against each other and bend into the excavation giving rise to so-called flexural toppling failure. Since the joint shear strength (that is the shear strength along the layering direction) is generally low, the shear strength in the Implicit Joint model will be reduced as soon as joint start to slip irrespective of intact rock layer strength. An attempt by the author to back analyse the centrifuge experiment of flexural toppling failures reported in (Adhikary and Guo, 2002) using Ubiquitous Joint model (FLAC - Itasca, 2008) yielded a very erroneous result mimicking circular failure mode similar to failures seen in homogeneous slopes but with reduced strength due to the presence of weak planes.

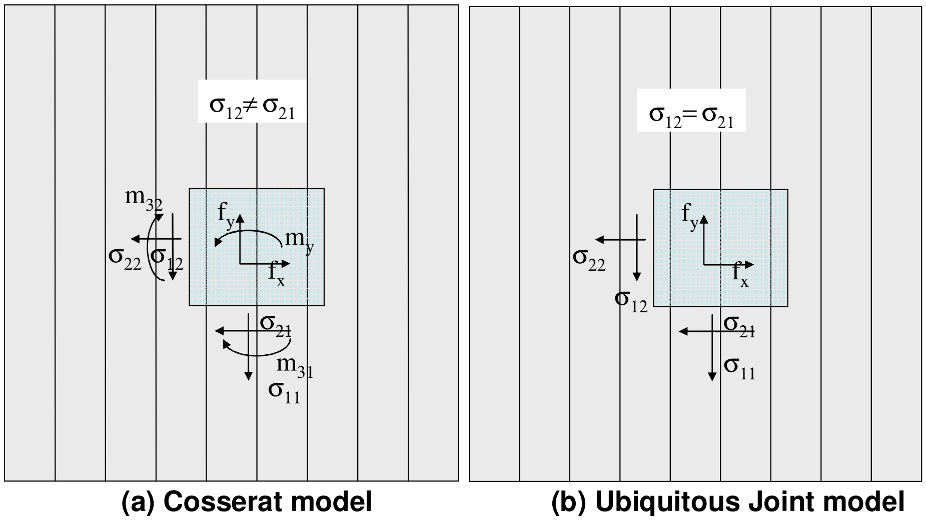

Figure 1. Stresses and volume forces acting on a Cosserat element and a Ubiquitous joint element.

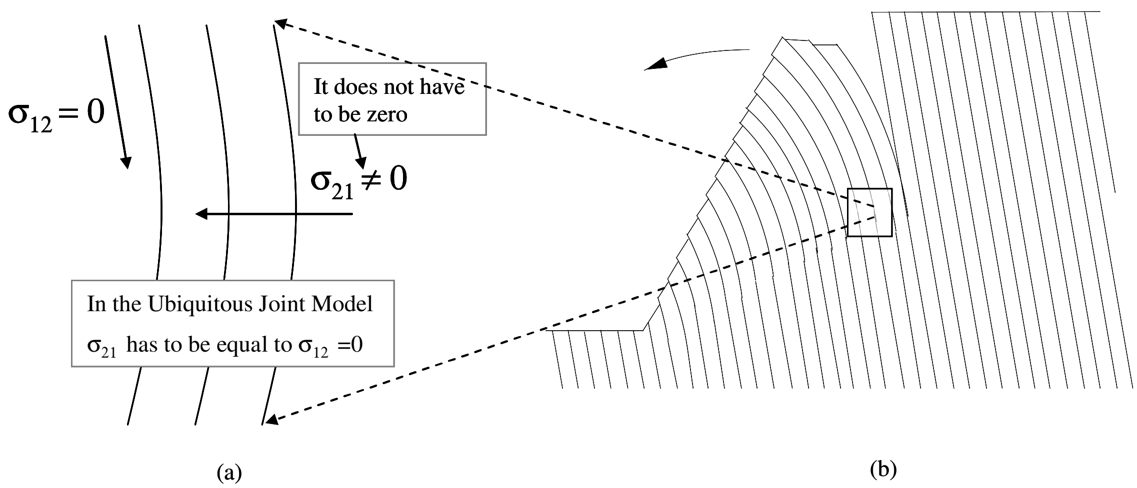

Figure 2. A schematic showing (a) erroneous shear stress that may arise in the Ubiquitous Joint model (b) flexural toppling failures.

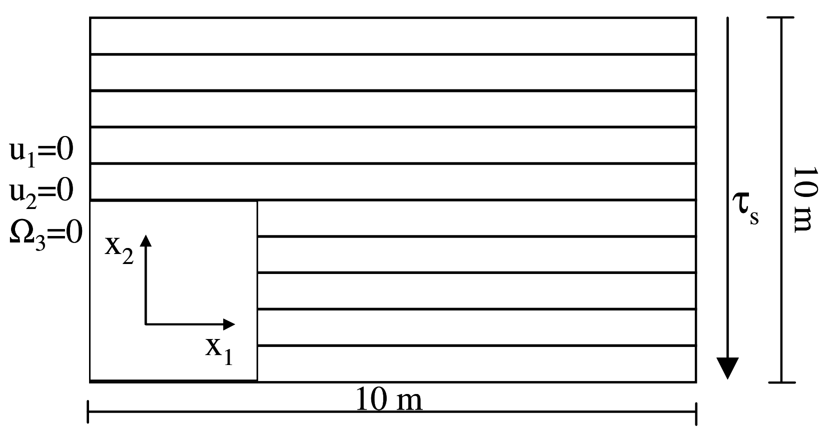

Figure 3. A schematic of the example used in the analytical verification.

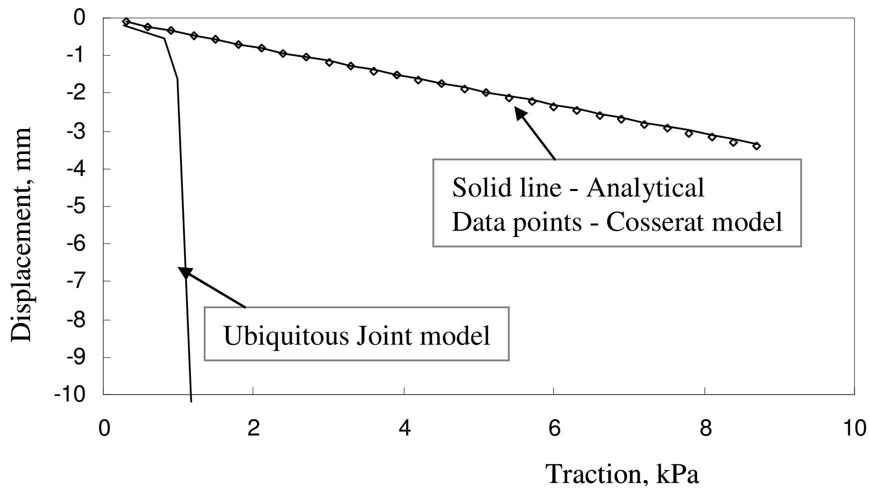

Figure 4. Comparison of the analytical and numerical results.

But the Cosserat model does not suffer from the same deficiency since it can have two different shear stress components. During loading if the layered rock deforms in such a way that the layer do slip against each other implying zero $\sigma_{12,}$ σ_{21} will remain non-zero and will increase with layer bending depending upon the bending rigidity of the rock layers. The couple stresses arising from layer bending will counter the rotation arising due to the differences in the two components of the shear stresses.

NUMERICAL VERIFICATION

A simple case as shown in Figure 3 will be considered in order to highlight the deficiency in the Implicit Joint model. Here 10 layers are perfectly clamped on the left-hand side and a traction τ_s is applied on the right hand side. The rock layers are assumed to have Young's modulus (*E*) of 10 GPa, Poisson's ratio of 0.20, thickness of 1m and length (*l*) of 10m. The joint normal and shear stiffness is assumed to be very big (that is 10^{10} GPa/m) implying no-elastic anisotropy. The strength anisotropy is introduced by assuming zero joint shear strength. Since the shear strength in the layer direction is zero, the deformation solution should remain independent of the x_2 direction, which allows analytical verification of the results on the basis of beam theory, which yields the elastic deflection of the beam as (Timoshenko and Goodier, 1970):

$$u_2(l) = \frac{4\tau_s l^3}{Eh^2}\left(1-\nu^2\right) \quad (15)$$

This problem is analysed with a plane strain Cosserat finite element code as well as Ubiquitous Joint model built in FLAC (Itasca, 2008). The problem domain is discretized into 400 isoparametric quadrilateral elements. Figure 4 shows the comparison of the analytical and the numerical calculations. The elastic deflection obtained from the Cosserat model agrees quite well with the analytical deflection. However, the Ubiquitous Joint model produces excessively large deflection indicating the bending of a rubber like material with no bending stiffness. Additional simulations with different rock layer Young's modulus *E* or layer thickness *h* did not make any difference in the Ubiquitous Joint model FLAC (Itasca, 2008) results, whereas the Cosserat model results agreed well with the analytical solution (Equation 15).

CONCLUSION

The analysis of the constitutive equations (that is. the requirement that two shear stress components in the Ubiquitous Joint model be the same) and the numerical simulation of bending of a package of layered rocks clearly demonstrate that the standard Implicit Joint models (e.g. Ubiquitous Joint model built in FLAC - Itasca, 2008) can completely break down when the rock layers undergo bending during loading and hence could lead to erroneous results. Use of standard implicit joint models should be limited to small deformation cases where possibility of rock layer bending is negligible. Any attempt to use such standard implicit joint model for the simulation of layered rock with the possibility of rock layer bending when shearing direction is not aligned with the direction of layering (e.g. flexural toppling failures of rock slopes, deformation of underground excavations in layered rocks) will provide erroneous results. Whereas the Implicit Joint models based on non-standard continuum (e.g. Cosserat models) can accurately simulate the load deformation behaviour of layered rocks.

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