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Full Length Research Paper

Non-axisymmetric dynamic response of imperfectly bonded buried fluid-filled orthotropic thin cylindrical shell due to incident compressional wave

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The main aim of this paper is to assess and compare the relative importance of the effects of considering the fluid presence and the bond imperfection while evaluating the non-axisymmetric dynamic response of an imperfectly bonded empty as well as fluid filled orthotropic thin cylindrical shell buried under soil and excited by compressional wave (P-wave). While applying thin shell theory, the effect of shear deformation and rotary inertia need not to be considered. The pipeline had been modeled as an infinite cylindrical shell imperfectly bonded to surrounding. A thin layer is assumed between the shell and the surrounding medium (soil) such that this layer possesses the properties of stiffness and damping both. The effects of the fluid presence on the shell displacement have been studied for different soil conditions and at various angles of incidence of the longitudinal wave. It is observed that magnitude of the dynamic response of fluid filled pipeline is more than that of an empty pipeline. Axial and radial deflection of thin pipe is considerable even under hard soil conditions under imperfect bonding of pipe with soil. Numerical results have been presented for the longitudinal compressional wave (P- wave) only.

Key words: Orthotropic, Imperfect bond, seismic wave, non-axisymmetric, dynamic response, buried pipelines, thin shell.

INTRODUCTION

Growing urbanization has created congestion and problem of space for providing above ground utility services. In recent years, the use of underground power cabling, lying down of optic fiber communications line and water supply lines have been finding increasing use of thin shell pipes made of different types of orthotropic materials. After arrival of reinforced plastic mortar (RPM) pipes and its increasing use in providing utility services to ever-growing urban population, need was felt to analyze the pipe of orthotropic materials under static and dynamic conditions. The behavior of buried pipeline is observed to be significantly different from above ground pipes. Response of these buried pipes under seismic or other dynamic conditions requires to be analyzed.

During past few years, number of papers like Cole et al. (1979), and Singh et al. (1987) has appeared on the axisymmetric dynamic response of buried orthotropic pipe/shells. Later Chonan (1981), Dwivedi and Upadhyay (1989, 1990, 1991) and Dwivedi et al. (1991) have analyzed the axisymmetric problems of imperfectly bonded shell for the pipes made of orthotropic materials. Upadhyay and Mishra (1988) have presented a good account of work on non-axisymmetric response of buried thick orthotropic pipelines under seismic excitation. Results show that that there is negligible axial and radial

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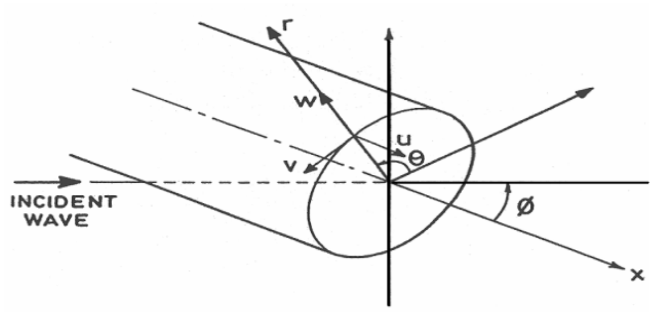


Figure 1. Geometry of the problem.

deflection of empty as well as fluid filled thick shell. Again Dwivedi et al. (1992a, 1992b), Dwivedi et al. (1993a, 1993b, 1996), and Dwivedi et al. (1998) have analyzed the non-axisymmetric problem of imperfectly bonded buried thick orthotropic cylindrical shells. Kouretzis et al. (2007) have presented analytical calculation of blast-induced strains on buried pipe lines. Hasheminajad and Kazemirad (2008) dynamic response of an eccentrically lined circular tunnel in poroelastic soil under seismic excitation. Lee et al. (2009) in their paper had done the risk analysis of buried pipelines using probabilistic method. But in all these analyses, pipeline had been modeled as thick shell. Rajput et al (2010) have reported comparison of non-axisymmetric dynamic response of imperfectly bonded buried orthotropic thick and thin fluid filled cylindrical shell due to incident shear wave (SH Wave) and have also presented non-axisymmetric dynamic response of imperfectly bonded buried orthotropic thin fluid empty cylindrical shell due to incident compressional wave.

As far as the non-axisymmetric dynamic response of thin shell is concerned, no work had been reported so far. Therefore, present paper attempts to analyze the effect of imperfect bond between pipe and surrounding medium on the non- axisymmetric dynamic response of buried orthotropic thin pipelines. A theoretical analysis of the non-axisymmetric steady state dynamic response of buried fluid-filled pipelines excited by seismic waves travelling in the surrounding infinite medium (soil) is presented. An infinite cylindrical shell model had been used for the thin pipeline. Comparisons of the numerical results for a fluid-filled shell with those for an empty shell have been presented and discussed.

BASIC EQUATIONS AND FORMULATIONS

The pipeline had been modeled as an infinitely long cylindrical shell of mean radius R and thickness h. It is considered to be buried in a linearly elastic, homogeneous and isotropic medium of infinite extent. Basic approach of the formulation is to obtain the mid plane displacements of the shell by solving the equations of motion of the orthotropic shell. Traction terms in the equations of motion are obtained by solving the three-dimensional wave equation in the surrounding medium. Appropriate boundary conditions are applied at the shell surfaces. Equations arising out of boundary conditions along with the equations of motion of the shell are simplified to yield a response equation in matrix form.

Equation governing the non axis-symmetric motion of an infinitely long orthotropic cylinder had been derived following the approach of Herrman and Mirsky (1957), Displacement at a particular point in the shell is taken in the form:

$u_x(z, \theta,x,t) = u(\theta,x,t) + z_{\psi x} (\theta,x,t)$

$u_\theta(z, \theta,x,t) = v(\theta,x,t) + z_{\psi\theta} (\theta,x,t)$

$u_z(z, \theta,x,t) = w(\theta,x,t)$,

where u_z, u_θ, u_x are displacement component of a point in the shell.

Considering an infinitely long cylindrical shell of mean radius R and thickness h buried in a linearly elastic, homogeneous and isotropic medium of infinite medium, a thin layer is assumed between the shell and the surrounding medium (soil). The degree of imperfection of the bond is varied by changing the stiffness and the damping parameters of this layer. The shell is excited by a longitudinal wave (p-wave). A wave of wavelength Λ (=2Π/ξ) is considered to strike the shell at an angle Φ with the axis of theshell (as shown in Figure 1). Let a cylindrical polar co-ordinate system (r, θ, x) is defined such that x coincides with the axis of the shell and, in addition, z is measured normal to the shell middle surface, which is given as:

$$z = r - R, \qquad -h/2 \le z \le h/2 \tag{1}$$

The basic equations which describe the dynamic behavior of cylindrical shells with bending resistance under arbitrary loads are derived from the system of equations which had been presented by Upadhyay and Mishra (1988). But in the thin shell theory, effect of shear deformation and rotary inertia is not considered. After equating all the inertial and moments term equal to zero, the equilibrium equations of thick shell in stress form (from above reference) reduces to:

$$\frac{1}{R}\frac{\partial Q_\theta}{\partial \theta} + \frac{\partial Q_x}{\partial x} - \frac{N_{\theta\theta}}{R} + P_1^* = \rho h \frac{\partial^2 w}{\partial t^2}; \tag{2a}$$

$$\frac{1}{R}\frac{\partial N_{\theta\theta}}{\partial \theta} + \frac{\partial N_{x\theta}}{\partial x} + \frac{Q_\theta}{R} + P_2^* = \rho h \frac{\partial^2 v}{\partial t^2}; \tag{2b}$$

$$\frac{1}{R}\frac{\partial M_{\theta\theta}}{\partial \theta} + \frac{\partial M_{x\theta}}{\partial x} - Q_\theta = 0; \tag{2c}$$

$$\frac{1}{R}\frac{\partial N_{xx}}{\partial x} + \frac{1}{R}\frac{N_{\theta x}}{\partial \theta} + P_4^* = \rho h [\frac{\partial^2 u}{\partial t^2}]; \tag{2d}$$

$$\frac{\partial M_{xx}}{\partial x}+\frac{1}{R}\frac{\partial M_{\theta x}}{\partial \theta}-Q_x=0 \quad (2e)$$

Where $N_{xx};N_{\theta\theta};N_{x\theta};N_{\theta x}$ and $M_{xx};M_{\theta\theta};M_{x\theta};M_{\theta x}$ are stress resultants and moments respectively.

In connection with the equation of equilibrium, it can be argued that transverse shearing force Q_θ makes a negligible contribution to equilibrium of forces in circumferential direction. So after making Q_θ equal to zero in Equation 2(b), the values of Q_θ and Qx are determined from Equation 2(c) and (e) and putting it into Equation 2(b) and (d), above equations reduces to:

$$\frac{\partial^2 M_{xx}}{\partial x^2}+\frac{\partial^2 M_{x\theta}}{R\partial\theta\partial x}+\frac{\partial^2 M_{\theta x}}{R\partial\theta\partial x}+\frac{1}{R^2}\frac{\partial^2 M_{\theta\theta}}{\partial\theta^2}-\frac{N_{\theta\theta}}{R}+P_1^*=\rho h\frac{\partial^2 w}{\partial t^2}; \quad (3a)$$

$$\frac{1}{R}\frac{\partial N_{\theta\theta}}{\partial\theta}+\frac{\partial N_{x\theta}}{\partial x}+P_2^*=\rho h\frac{\partial^2 v}{\partial t^2}; \quad (3b)$$

$$\frac{\partial N_{xx}}{\partial x}+\frac{1}{R}\frac{\partial N_{\theta x}}{\partial\theta}+P_4^*=\rho h\{\frac{\partial^2 u}{\partial t^2}\}; \quad (3c)$$

For thin shell theory, shear deformation is not considered due to negligible thickness. So the shear strain components according to Herrman and Mirsky (1857) about z-axis in r-θ and r-x plane γ_{xz} and $\gamma_{z\theta}$ will be zero (no coupling is there due to negligible thickness) but at the same time shear stress component would be there due to Kirchhoff's hypothesis. So according to Herrman and Mirsky (1957)

$$\gamma_{xz}=\frac{\partial w}{\partial x}+\psi_x=0;$$

$$\gamma_{z\theta}=\frac{1}{R+z}\frac{\partial w}{\partial\theta}+\psi_\theta-\frac{1}{R+z}(v+z\psi_\theta)=0$$

So from the above equations:

$$\psi_x=-\frac{\partial w}{\partial x};$$
$$\psi_\theta=\frac{1}{R}(v-\frac{\partial w}{\partial\theta});$$

Here ψ_x and ψ_θ are angle of rotation in r-x and r-θ plane but in the r-θ plane the tangential deflection is negligible compared to component of radial deflection in that direction. So:

$$\psi_x=-\frac{\partial w}{\partial x};$$
$$\psi_\theta=-\frac{1}{R}(\frac{\partial w}{\partial\theta}); \quad (4)$$

From the above, stress resultants come out to be:

$$N_{xx}=E_p\frac{\partial u}{\partial x}-\frac{D}{R}\frac{\partial^2 w}{\partial x^2}+\frac{\nu_{\theta x}E_p}{R}(w+\frac{\partial v}{\partial\theta});$$

$$N_{\theta x}=G_{\theta x}[h\frac{\partial v}{\partial x}+\frac{1}{R}(h+I/R^2)\frac{\partial u}{\partial\theta}+(I/R^2)\frac{\partial^2 w}{\partial\theta\partial x}];$$

$$M_{xx}=\frac{D}{R}[\frac{\partial u}{\partial x}-R\frac{\partial^2 w}{\partial x^2}-\frac{\nu_{\theta x}}{R}\frac{\partial^2 w}{\partial\theta^2});$$

$$M_{\theta x}=G_{x\theta}[-2(I/R)\frac{\partial^2 w}{\partial\theta\partial x}-(I/R^2)\frac{\partial u}{\partial\theta}];$$

$$N_{x\theta}=G_{x\theta}[h\frac{\partial v}{\partial x}-(I/R^2)\frac{\partial^2 w}{\partial\theta\partial x}+(h/R)\frac{\partial u}{\partial\theta}];$$

$$N_{\theta\theta}=(\frac{E_p^{'}}{R}+\frac{D^{'}}{R^3})(w+\frac{\partial v}{\partial\theta})+\frac{D^{'}}{R^3}\frac{\partial^2 w}{\partial\theta^2}+\nu_{\theta x}E_p\frac{\partial u}{\partial x};$$

$$M_{x\theta}=G_{x\theta}(I/R)[\frac{\partial v}{\partial x}-2\frac{\partial^2 w}{\partial\theta\partial x}];$$

$$M_{\theta\theta}=-\frac{D^{'}}{R^2}\frac{\partial^2 w}{\partial\theta^2}-\frac{D^{'}}{R^2}(w+\frac{\partial v}{\partial\theta})-\nu_{\theta x}D\frac{\partial^2 w}{\partial x^2}; \quad (5)$$

Here, $G_{x\theta}$, G_{xz}, $G_{z\theta}$ are shear moduli of the shell Material. When these values of stress resultants are placed into above equations of equilibrium, it results in the required equation of motion of shell in the matrix form as:

$$[\{L\}\ \{U\}]+\{P^*\}=0 \quad (6)$$

where [L] is a 3×3 matrix operator and terms {P*} is column matrix

$$L_{11}=D\frac{\partial^4}{\partial x^4}+\frac{D^{'}}{R^4}\frac{\partial^2}{\partial\theta^2}+\frac{D^{'}}{R^4}\frac{\partial^4}{\partial\theta^4}+\frac{2\nu_{\theta x}D}{R^2}\frac{\partial^2}{\partial\theta^2\partial x^2}+4G_{x\theta}\left(\frac{I}{R^2}\right)\frac{\partial^4}{\partial\theta^2\partial x^2}+\left(\frac{E_p^{'}}{R^2}+\frac{D^{'}}{R^4}\right)+\rho h\frac{\partial^2}{\partial t^2};$$

$$L_{12}=\frac{D^{'}}{R^4}\frac{\partial^3}{\partial\theta^3}-G_{x\theta}\left(\frac{I}{R^2}\right)\frac{\partial^4}{\partial\theta^2\partial x^2}+\left(\frac{E_p^{'}}{R^2}+\frac{D^{'}}{R^4}\right)\frac{\partial}{\partial\theta};$$

$$L_{13} = -\frac{D}{R}\frac{\partial^3}{\partial x^3} + \frac{\nu_{\theta x}E_p}{R}\frac{\partial}{\partial x} + G_{x\theta}\left(\frac{I}{R^3}\right)\frac{\partial^3}{\partial\theta^2\partial x};$$

$$L_{21} = L_{12};$$

$$L_{22} = G_{x\theta}h\frac{\partial^2}{\partial x^2} + \left(\frac{E_p^{'}}{R^2} + \frac{D^{'}}{R^4}\right)\frac{\partial^2}{\partial\theta^2} - \rho h\frac{\partial^2}{\partial t^2};$$

$$L_{23} = \frac{G_{x\theta}h}{R}\frac{\partial^2}{\partial\theta\partial x} + \frac{\nu_{\theta x}E_p}{R}\frac{\partial^2}{\partial\theta\partial x};$$

$$L_{31} = L_{13};$$

$$L_{32} = L_{23};$$

$$L_{33} = E_p\frac{\partial^2}{\partial x^2} + \left(\frac{G_{x\theta}}{R^2}\right)\left(h + \frac{I}{R^2}\right)\frac{\partial^2}{\partial\theta^2};$$

and

$\{U\} = [w \quad v \quad u]^T$

Where, w, v and u are the displacement components of the middle surface of the shell in the radial, tangential and axial directions respectively. The elements of column matrix $\{P^*\}$ are given by Herrman and Mirsky (1957) as:

$$P_1^* = \left(1 + \frac{z}{R}\right)\sigma_{zz}\Big|_{-h/2}^{h/2}, \quad P_2^* = \left(1 + \frac{z}{R}\right)\sigma_{z\theta}\Big|_{-h/2}^{h/2},$$

$$P_3^* = z\left(1 + \frac{z}{R}\right)\sigma_{z\theta}\Big|_{-h/2}^{h/2}, \quad P_4^* = \left(1 + \frac{z}{R}\right)\sigma_{zx}\Big|_{-h/2}^{h/2},$$

$$P_5^* = z\left(1 + \frac{z}{R}\right)\sigma_{zx}\Big|_{-h/2}^{h/2}$$

where, σ_{ij} denotes the stresses with their usual meaning, but for thin shell P_3^* and P_5^* are zero. Different constants appearing in the expressions for L_{ij} are defined as:

$$E_p = \frac{E_x h}{1-\nu_{x\theta}\nu_{\theta x}}, \quad E_p^{'} = \frac{E_\theta h}{1-\nu_{x\theta}\nu_{\theta x}}, \quad D = E_p\frac{h^2}{12}, \quad D' = E_p^{'}\frac{h^2}{12},$$

Where, moment of inertia, $I = h^3/12$ and E_x, E_θ are elastic moduli, $\nu_{x\theta}$, $\nu_{\theta x}$ the Poisson ratio and ρ is the density of the shell material. 'n' indicate the mode in circumferential direction; n = 0 represents the axisymmetric mode.

For the evaluation of $\{P^*\}$, stress σ_{ij} at $z = \pm$ (h/2) must be determined in the terms of incident and scattered field in the surrounding ground. The total displacement field in the ground is written as:

$$d = d^{(i)} + d^{(s)}$$

Where, superscripts i and s represents the incident and scattered parts of deflection respectively. By solving the wave equation in the surrounding infinite medium, the components of incident and scattered fields can be written as (Chonan, 1981):

$$d_r^{(i)} = \left[\begin{matrix}\left\{\gamma I_n^{'}\frac{\gamma r}{R}\right\}B_1 + \left\{-1\beta_1\delta I_n^{'}\frac{\delta r}{R}\right\}B_3 \\ +\left\{n\frac{R}{r}I_n\frac{\delta r}{R}\right\}B_5\end{matrix}\right]\cos n\theta\exp[i\xi(x-ct)]$$

$$d_\theta^{(i)} = \left[\begin{matrix}\left\{-n\frac{Rr}{r}I_n\frac{\gamma r}{R}\right\}B_1 + \left\{in\frac{R}{r}\beta_1 I_n(\frac{\delta r}{R})\right\}B_3 \\ +\left\{-\delta I_n^{'}\frac{\delta r}{R}\right\}B_5\end{matrix}\right]\sin n\theta\exp[i\xi(x-ct)]$$

and

$$d_x^{(i)} = \left[\left\{i\beta_1 I_n\frac{\gamma r}{R}\right\}B_1 + \left\{\delta^2 I_n\frac{\delta r}{R}\right\}B_3\right]x\cos n\theta\exp[i\xi(x-ct)] \tag{7}$$

where, $B_1 = B_1^{'}/R$, $B_3 = B_3^{'}/R^2$ $B_5 = B_5^{'}/R$ and ($'$) denotes differentiation with respect to the argument of the Bessel functions. The constants B_1, B_3 and B_5 depend on the parameters of the incident wave and may be expressed as:

$$B_1 = (-1)^{n+1}\left(i\chi\frac{A_1}{\varepsilon_1}\right), \; B_3 = (-1)^n\left(i\chi\frac{A_2}{\delta\varepsilon_2}\right), \; B_5 = (-1)^n\left(\chi\frac{A_3}{\delta}\right) \tag{8}$$

$$d_r^{(s)} = \left[\left\{\gamma K_n^{'}\left(\frac{\gamma r}{R}\right)\right\}B_2 + \left\{-i\beta_1\delta K_n^{'}\left(\frac{\delta r}{R}\right)\right\}B_4 + \left\{n\left(\frac{R}{r}\right)K_n\left(\frac{\delta r}{R}\right)\right\}B_6\right]\cos n\theta\exp[i\xi(x-ct)]$$

$$d_\theta^{(s)} = \left[\begin{matrix}\left\{-n\left(\frac{R}{r}\right)K_n\left(\frac{\gamma r}{R}\right)\right\}B_2 + \left\{in\left(\frac{R}{r}\right)\beta_1 K_n\left(\frac{\delta r}{R}\right)\right\}B_4 \\ +\left\{-\delta K_n^{'}\left(\frac{\delta r}{R}\right)\right\}B_6\end{matrix}\right]\sin n\theta\exp[i\xi(x-ct)]$$

$$d_x^{(s)} = \left[\left\{i\beta_1 K_n\left(\frac{\gamma r}{R}\right)\right\}B_2 + \left\{\delta^2 K_n\left(\frac{\delta r}{R}\right)\right\}B_4\right]\cos n\theta\exp[i\xi(x-ct)] \tag{9}$$

Where, d_r, d_θ, d_x components of displacement vector, $A_1; A_2; A_3$ are amplitudes of P, SV, SH waves respectively and $B_2 = B_2^{'}/R, B_4 = B_4^{'}/R^2$ and $B_6 = B_6^{'}/R$. $B_1^{'}.......B_6^{'}$ are arbitrary constants. Stress field due to the incident wave can be obtained by plugging above equations into the stress-displacement relations of the medium, and is given by:

$$\sigma_{rr}^{(i)} = \frac{\mu}{R}\begin{bmatrix} \left\{\left(2\varepsilon_1^2 - \varepsilon_2^2\right)I_n\left(\frac{\gamma r}{R}\right) + 2\gamma^2 I_n''\left(\frac{\gamma r}{R}\right)\right\}B_1 \\ + \left\{-2i\beta_1\delta^2 I_n''\left(\frac{\gamma r}{R}\right)\right\}B_3 \\ + 2n\left(\frac{R}{r}\right)\left\{\delta I_n'\left(\frac{\delta r}{R}\right) - \left(\frac{R}{r}\right)I_n\left(\frac{\delta r}{R}\right)\right\}B_5 \end{bmatrix}\cos n\theta \exp\left[i\xi(x-ct)\right]$$

$$\sigma_{r\theta}^{(i)} = \frac{\mu}{R}\begin{bmatrix} 2n\left(\frac{R}{r}\right)\left\{\left(\frac{R}{r}\right)I_n\left(\frac{\gamma r}{R}\right) - \gamma I_n'\left(\frac{\gamma r}{R}\right)\right\}B_1 \\ + 2in\frac{R}{r}\beta_1\left\{\delta I_n'\left(\frac{\delta r}{R}\right) - \frac{R}{r}I_n\left(\frac{\delta r}{R}\right)\right\}B_3 \\ + \left\{-\delta^2 I_n''\left(\frac{\delta r}{R}\right) + \delta\left(\frac{R}{r}\right)I_n'\left(\frac{\delta r}{R}\right) - \left(\frac{nR}{r}\right)^2 I_n\left(\frac{\delta r}{R}\right)\right\}B_5 \end{bmatrix}\sin n\theta \exp\left[i\xi(x-ct)\right]$$

$$\sigma_{rx}^{(i)} = \frac{\mu}{R}\begin{bmatrix} \left\{2i\beta_1\gamma I_n'\left(\frac{\gamma r}{R}\right)\right\}B_1 + \left\{\delta\left(2\beta_1^2 - \varepsilon_2^2\right)I_n'\left(\frac{\delta r}{R}\right)\right\} \\ + \left\{in\left(\frac{R}{r}\right)\beta_1 I_n\left(\frac{\delta r}{R}\right)\right\}B_5 \end{bmatrix}\cos n\theta \exp\left[i\xi(x-ct)\right]$$

$$\sigma_{r\theta}^{(s)} = \frac{\mu}{R}\begin{bmatrix} \left\{\left(2\varepsilon_1^2 - \varepsilon_2^2\right)K_n\left(\frac{\gamma r}{R}\right) + 2\gamma^2 K_n''\left(\frac{\gamma r}{R}\right)\right\}B_2 + \\ \left\{-2i\beta_1\delta^2 K_n''\left(\frac{\gamma r}{R}\right)\right\}B_4 + \\ 2n\left(\frac{R}{r}\right)\left\{\delta K_n'\left(\frac{\delta r}{R}\right) - \left(\frac{R}{r}\right)K_n\left(\frac{\delta r}{R}\right)\right\}B_6 \end{bmatrix}\cos n\theta \exp\left[i\xi(x-ct)\right]$$

$$\sigma_{r\theta}^{(s)} = \frac{\mu}{R}\begin{bmatrix} 2n\left(\frac{R}{r}\right)\left\{\left(\frac{R}{r}\right)K_n\left(\frac{\gamma r}{R}\right) - \gamma K_n'\left(\frac{\gamma r}{R}\right)\right\}B_2 + \\ 2in\frac{R}{r}\beta_1\left\{\delta K_n'\left(\frac{\delta r}{R}\right) - \frac{R}{r}K_n\left(\frac{\delta r}{R}\right)\right\}B_4 + \\ \left\{-\delta^2 K_n''\left(\frac{\delta r}{R}\right) + \delta\left(\frac{R}{r}\right)K_n'\left(\frac{\delta r}{R}\right) - \left(\frac{nR}{r}\right)^2 K_n\left(\frac{\delta r}{R}\right)\right\}B_6 \end{bmatrix}\sin n\theta \exp\left[i\xi(x-ct)\right]$$

$$\sigma_{rx}^{(s)} = \frac{\mu}{R}\begin{bmatrix} \left\{2i\beta_1\gamma K_n'\left(\frac{\gamma r}{R}\right)\right\}B_2 + \left\{\delta\left(2\beta_1^2 - \varepsilon_2^2\right)K_n'\left(\frac{\delta r}{R}\right)\right\}B_4 \\ + \left\{in\left(\frac{R}{r}\right)\beta_1 K_n\left(\frac{\delta r}{R}\right)\right\}B_6 \end{bmatrix}\cos n\theta \exp\left[i\xi(x-ct)\right] \tag{10}$$

where, I_n ( ) are modified Bessel functions of first kind, J_n ( ) are Bessel function of first kind and K_n ( ) are modified Bessel functions of second kind

With the help of above equations, the stresses at the outer surface of the shell (z = h/2 or r = R + h/2) can be obtained. Thus {P^*} in Equation (2) can be determined. For any disturbance propagating in the fluid governing linear acoustic equations are the continuity equation and the Euler equation of motion. These are given as follows:

$$\frac{\partial \rho_f}{\partial t} + \overline{\nabla}.(\overline{V}_f . \nabla)\overline{V}_f = \frac{1}{\rho_f}\overline{\nabla} p$$

Displacement d(r,θ, x, t) at any point, satisfied the equation of motion:

$$c_1^2 \underline{\nabla}\left(\underline{\nabla}.\underline{d}\right) - c_2^2 \underline{\nabla}\Lambda\underline{\nabla}\Lambda\underline{d} = \frac{\partial^2}{\partial t^2}(\underline{d}) \tag{11}$$

where, $c_1 = \left\{\frac{(\lambda + 2\mu)}{\rho_m}\right\}^{1/2}$ and $c_2 = \left\{\frac{\mu}{\rho_m}\right\}^{1/2}$ are the speeds of dilatational and shear waves respectively in the infinite medium. Further, λ and μ are the Lame's constant, and ρ_m is the density of the medium.

Now the mid plane displacement and slopes are assumed to be of the form:

$$w = w_0 \cos n\theta \exp[i\xi(x-ct)]$$

$$v = v_0 \sin n\theta \exp[i\xi(x-ct)]$$

$$u = u_0 \cos n\theta \exp[i\xi(x-ct)] \tag{12}$$

Plugging Equation (12) in Equation (2) and (11) along with the expression for {P*}, a set of three simultaneous algebraic equations were obtained. Four more equations were obtained by imposing the boundary conditions at the inner and outer surfaces of the shell, that is:

$$w = (d_r^{(i)} + d_r^{(s)})_{r=R+h/2}$$

$$v + (h/2)\Psi_\theta = (d_\theta^{(i)} + d_\theta^{(s)})_{r=R+h/2}$$

$$u + (h/2)\psi_x = (d_x^{(i)} + d_x^{(s)})_{r=R+h/2} \tag{13}$$

Boundary conditions at the outer surface of the shell (r = R + h/2) are obtained by assuming that the shell and the continuum are joined together by a bond which is thin, elastic and inertia less. This implies that the stress at the shell-soil interface is continuous. To take the elasticity of the bond into account, the stresses in the bond are assumed proportional to relative displacements between the shell and continuum. μ shear modulus of medium and ρ density of shell material

The inner surface of the shell continuity of the radial displacement had been assumed, that is,

$$\frac{\partial w}{\partial t} = \left[\frac{\partial d_r^f}{\partial t}\right]_{r=R-h/2}$$

$$(\sigma_{rx})_{r=R+h/2} = [(S_x + Z_x\frac{\partial}{\partial t})(\mu_x^i + \mu_x^s - u - (r-R)\psi_x]_{r=R+h/2}$$

$$(\sigma_{rr})_{r=R+h/2} = [(S_r + Z_r \frac{\partial}{\partial t})(\mu_r^i + \mu_r^s - w)]_{r=R+h/2}$$

$$(\sigma_{r\theta})_{r=R+h/2} = [(S_\theta + Z_\theta \frac{\partial}{\partial t})(\mu_\theta^i + \mu_\theta^s - u(r-R)\psi_\theta]_{r=R+h/2} \quad (14)$$

Where, $\zeta_R = \frac{\mu}{S_r.R}$, $\zeta_\theta = \frac{\mu}{S_\theta.R}$, and $\zeta_x = \frac{\mu}{S_x.R}$, are the non stiffness coefficient of the bond in radial, and axial direction, respectively; $\Gamma_r = \frac{\mu}{Z_r c_1}$, $\Gamma_\theta = \frac{\mu}{Z_\theta c_1}$ and $\Gamma_x = \frac{\mu}{Z_x c_1}$ are the non damping coefficient of the bond in radial, tangential and axial direction, respectively.

Thus, in-all seven algebraic equations are obtained. These seven equations when simplified give the final dynamic response equation, which may be put into the form

$$\{Q\}\{U_0\} = \mathrm{B}_1\{\mathrm{F}^1\} + \mathrm{B}_3\{F^2\} + \mathrm{B}_5\{F^3\} \quad (15)$$

Where [Q] is a (7×7)) matrix and $\{F^1\}$, $\{F^2\}$ and $\{F^3\}$ are (7×1) matrices. But for the response of longitudinal wave, the amplitudes due to shear waves B_3 and B_5 would be zero so the effect of $\{F^2\}$ and $\{F^3\}$ matrices would be eliminated. After putting values of $\mathrm{B}_3 = \mathrm{B}_5 = 0$ and substituting values of B_1 from Eq Equation (8), Equation (15) becomes

$$\{Q\}\{U_0\} = (-1)^{n+1}\left(i\chi \frac{A_1}{\varepsilon_1}\right)\{\mathrm{F}^1\} \quad (16)$$

Now if the unknown matrix $\{U_0\}$ is non-dimensionalized with respect to the amplitude of the incident wave (A_1), the elements of above Q and F matrix are as follows:

$$Q_{11} = \frac{\bar{h}^3}{12\eta_2}[n^4\eta_1 N + 4n^2\beta_1^2\eta_3 + N\beta_1^4 + \eta_1 N - 2n^2\eta_1 N + 2\nu_{\theta x} Nn^2\beta_1^2] + \frac{\bar{h}\eta_1 N}{\eta_2} - \Omega^2;$$

$$Q_{12} = \frac{\bar{h}^3}{12\eta_2}[-n^3\eta_1 N + n\beta_1^2\eta_3 + \eta_1 Nn] + \frac{\bar{h}\eta_1 Nn}{\eta_2};$$

$$Q_{13} = \frac{i\bar{h}^3}{12\eta_2}[-n^2\eta_3\beta_1 + N\beta_1^3] + \frac{i\bar{h}\nu_{\theta x} Nn\beta_1}{\eta_2};$$

$$Q_{14} = -\left(1 + \frac{\bar{h}}{2}\right)\bar{\mu}\left[\left(2\in_1^2 - \in_2^2\right)K_n(\alpha_1) + 2\gamma^2 K_n''(\alpha_1)\right];$$

$$Q_{15} = \left(1 + \frac{\bar{h}}{2}\right)\bar{\mu}\left[2i\beta_1\delta^2 K_n''(\alpha_2)\right];$$

$$Q_{16} = -\left(1 + \frac{\bar{h}}{2}\right)\bar{\mu}\left[\frac{2n\{\alpha_2 K_n'(\alpha_2) - K_n(\alpha_2)\}}{\left(\frac{1+\bar{h}}{2}\right)^2}\right];$$

$$Q_{21} = -Q_{12};$$

$$Q_{22} = \frac{\bar{h}^3}{12\eta_2}[-n^2\eta_1 N] + \frac{\bar{h}}{\eta_2}[-n^2\eta_1 N - \beta_1^2\eta_3] + \Omega^2$$

$$Q_{23} = -\frac{i\bar{h}}{\eta_2}[-n\eta_3\beta_1 - 2\nu_{\theta x} Nn\beta_1];$$

$$Q_{24} = \left(1 + \frac{\bar{h}}{2}\right)\bar{\mu}\left[2n\frac{\{K_n(\alpha_1) - \alpha_1 K_n'(\alpha_1)\}}{\left(1 + \frac{\bar{h}}{2}\right)^2}\right];$$

$$Q_{25} = \left(1 + \frac{\bar{h}}{2}\right)\bar{\mu}\left[2in\beta_1\frac{\{\alpha_2 K_n'(\alpha_2) - K_n(\alpha_2)\}}{\left(1 + \frac{\bar{h}}{2}\right)^2}\right];$$

$$Q_{26} = \left(1 + \frac{\bar{h}}{2}\right)\bar{\mu}\left[2n\left\{\frac{\delta}{\left(1 + \frac{\bar{h}}{2}\right)^2}\right\}K_n'(\alpha_2) - \delta^2 K_n''(\alpha_2)\left\{\frac{n^2}{\left(1 + \frac{\bar{h}}{2}\right)^2}\right\}K_n(\alpha_2)\right];$$

$$Q_{31} = Q_{13};$$

$$Q_{32} = -Q_{23};$$

$$Q_{33} = \frac{\bar{h}}{\eta_2}[-N\beta_1^3 - n_3 n^2(\frac{1+\bar{\mathrm{h}}^2}{12})] + \Omega^2;$$

$$Q_{34} = \left(1 + \frac{\bar{\mathrm{h}}}{2}\right)\bar{\mu}\left[2i\beta_1\gamma K_n'(\alpha_1)\right];$$

$$Q_{35} = \left(1 + \frac{\bar{h}}{2}\right)\bar{\mu}\left[\delta\left(2\beta_1^2 - \in_2^2\right)K_n'(\alpha_2)\right];$$

$$Q_{36}=\left(1+\frac{\bar{h}}{2}\right)\bar{\mu}\left[\frac{in\beta_1 K_n(\alpha_2)}{\left(1+\frac{\bar{h}}{2}\right)}\right];$$

$Q_{41}=1$, $Q_{42}=Q_{43}=0$,

$$Q_{44}=-\gamma K_n'(\alpha_1),\ Q_{45}=-i\beta_1\delta K_n'(\alpha_2),$$

$$Q_{46}=\frac{-n\ K_n(\alpha_1)}{\left(1+\frac{\bar{h}}{2}\right)};\ Q1_{51}=0,\quad Q_{52}=Q_{53}=1;$$

$$Q_{54}=\frac{n\ K_n(\alpha_1)}{\left(1+\frac{\bar{h}}{2}\right)};\quad Q_{55}=\frac{-in\beta_1 K_n(\alpha_2)}{\left(1+\frac{\bar{h}}{2}\right)};$$

$$Q_{56}=\delta K_n'(\alpha_2);\quad Q_{57}=Q_{47},\ ;Q_{58}=Q_{48}\ ;Q_{59}=0;$$

$Q_{61}=Q_{62}=Q_{63}=0$;

$$Q_{64}=-i\beta_1\ K_n(\alpha_1);\ Q_{65}=-\delta^2 K_n(\alpha_2);$$

$$Q_{66}=-\gamma K_n^{'}(\alpha_1)+\frac{\zeta_r\Gamma_r}{\Gamma_r-i\varepsilon_1\zeta_r}[(2\varepsilon_1^2-\varepsilon_2^2)K_n(\alpha_1)+2\gamma K_n^{''}(\alpha_1)]$$

$$Q_{67}=i\beta\delta K_n^{'}(\alpha_2)-\frac{\zeta_r\Gamma_r}{\Gamma_r-i\varepsilon_1\zeta_r}[(2i\beta\delta^2 K_n^{''}(\alpha_2)]$$

$$Q_{68}=\{-nK_n(\alpha_2)/(1+\bar{h}/2)\}+\frac{\zeta_r\Gamma_r}{\Gamma_r-i\varepsilon_1\zeta_r}[(2n\{\alpha_2 K_n^{'}(\alpha_2)-K_n(\alpha_2)\}/(1+\bar{h}/2)^2]$$

$$Q_{69}=0, Q_{71}=0, Q_{72}=Q_{73}=1, Q_{74}=Q_{75}=0$$

$$Q_{76}=\{nK_n(\alpha_1)/(1+\bar{h}/2)\}+\frac{\zeta_\theta\Gamma_\theta}{\Gamma_\theta-i\varepsilon_1\zeta_\theta}[(2n\{K_n(\alpha_1)-\alpha_1 K_n^{'}(\alpha_1)\}/(1+\bar{h}/2)^2]$$

$$Q_{77}=\{-in\beta K_n(\alpha_2)/(1+\bar{h}/2)]$$

$$F_1^1=\left(1+\frac{\bar{h}}{2}\right)\bar{\mu}\left[\left(2\epsilon_1^2-\epsilon_2^2\right)I_n(\alpha_1)+2\gamma^2 I_n''(\alpha_1)\right];$$

$$F_2^1=-\left(1+\frac{\bar{h}}{2}\right)\bar{\mu}\left[\frac{2n\ \{I_n(\alpha_1)-\alpha_1 I_n'(\alpha_1)\}}{\left(1+\frac{\bar{h}}{2}\right)^2}\right];$$

$$F_3^1=-\left(1+\frac{\bar{h}}{2}\right)\bar{\mu}\ [2i\ \beta_1\ \gamma\ I_n'(\alpha_1)];$$

$$F_4^1=\gamma\ I_n'(\alpha_1),\quad F_5^1=\frac{-n\ I_n(\alpha_1)}{\left(1+\frac{\bar{h}}{2}\right)};$$

$$F_6^1=-\{\gamma I_n^{'}(\alpha_1)-\frac{\zeta_r\Gamma_r}{\Gamma_r-i\varepsilon_1\zeta_r}[2\varepsilon_1^2-\varepsilon_2^2)\{I_n(\alpha_1)+2\gamma^2 I_n^{''}(\alpha_1)]$$

$$F_7^1=-\{nI_n(\alpha_1)/(1+\bar{h}/2]-\frac{\zeta_\theta\Gamma_\theta}{\Gamma_\theta-i\varepsilon_1\zeta_\theta}[2n\{I_n(\alpha_1)-\alpha_1 I_n^{'}(\alpha_1)\}/(1+\bar{h}/2)^2]$$

$$\{U\}=\left\lfloor\frac{w_0}{A_1}\ \frac{v_0}{A_1}\ \frac{u_0}{A_1}\ \frac{B_2}{A_1}\ \frac{B_4}{A_1}\ \frac{B_6}{A_1}\ \frac{B_f}{c_1}\right\rfloor^T=\lfloor\overline{W}\ \overline{V}\ \overline{U}\ \overline{B_2}\ \overline{B_4}\ \overline{B_6}\overline{B_f}\rfloor^T;$$

where

$$\bar{h}=\frac{h}{R},\ \eta_1=\frac{E_\theta}{E_x},\ \eta_2=\frac{G_{xz}}{E_x},\ \eta_3=\frac{G_{x\theta}}{E_x},$$

$$\eta_4=\frac{G_{z\theta}}{E_x},\ N=\frac{1}{(1-\nu_{x\theta}\nu_{\theta x})},\ \bar{\mu}=\frac{\mu}{G_{xz}},$$

$$\Omega^2=\bar{h}\,\bar{\mu}\frac{\epsilon_2^2}{\bar{\rho}},\ \bar{\rho}=\frac{\rho_m}{\rho},\ \alpha_1=\left(1+\frac{\bar{h}}{2}\right)\gamma$$ and

$$\alpha_2=\left(1+\frac{\bar{h}}{2}\right)\delta.$$

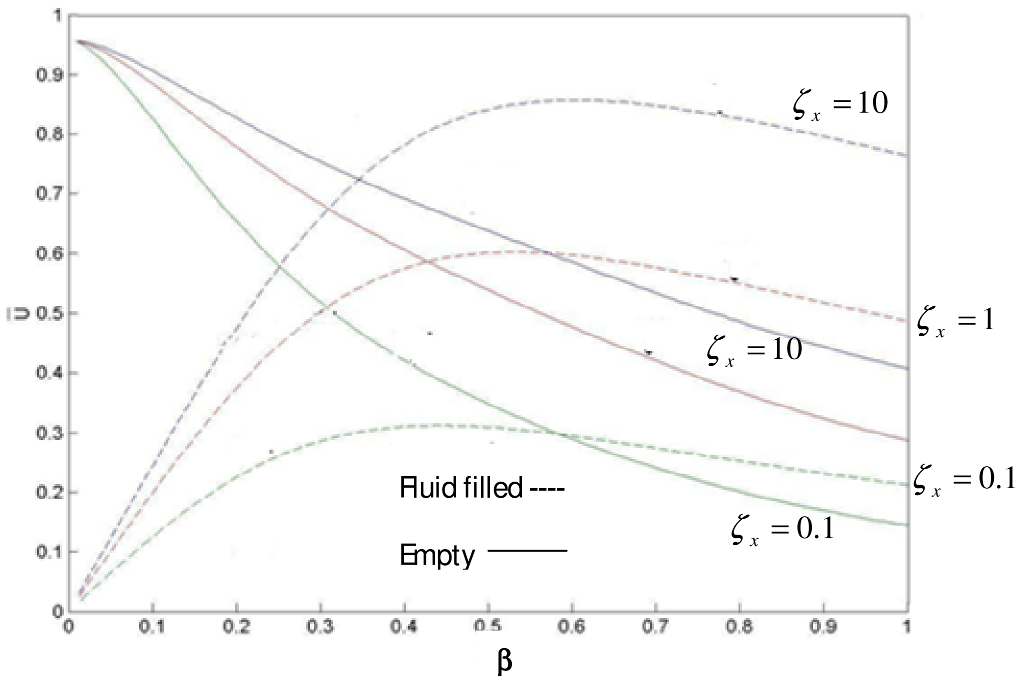
$$I_n'(\alpha_i),\ I_n''(\alpha_i),\ K_n'(\alpha_i)\ and\ K_n''(\alpha_i)$$

can be expressed as

$$I_n^{'}(\alpha_i)=\left(\frac{n}{\alpha_i}\right)I_n(\alpha_i)+I_{n+1}(\alpha_i),$$

$$I_n^{''}(\alpha_i)=\left[1+\left(\frac{n^2}{\alpha_i^2}\right)-\left(\frac{n}{\alpha_i^2}\right)\right]I_n(\alpha_i)-\left(\frac{1}{\alpha_i}\right)I_{n+1}(\alpha_i),$$

$$K_n^{'}(\alpha_i)=\left(\frac{n}{\alpha_i}\right)K_n(\alpha_i)-K_{n+1}(\alpha_i),$$



$$\phi = 5^\circ, \bar{\mu} = 0.01, \zeta_x = 0.1,1,10$$

Figure 2. Axial displacement $(\bar{U})$ vs. wave number (β) with soil stiffness ζ_x as parameter.

$$K_n^{''}(\alpha_i) = \left[1 + \left(\frac{n^2}{\alpha_i^2}\right) - \left(\frac{n}{\alpha_i^2}\right)\right] K_n(\alpha_i) + \left(\frac{1}{\alpha_i}\right) K_{n+1}(\alpha_i).$$

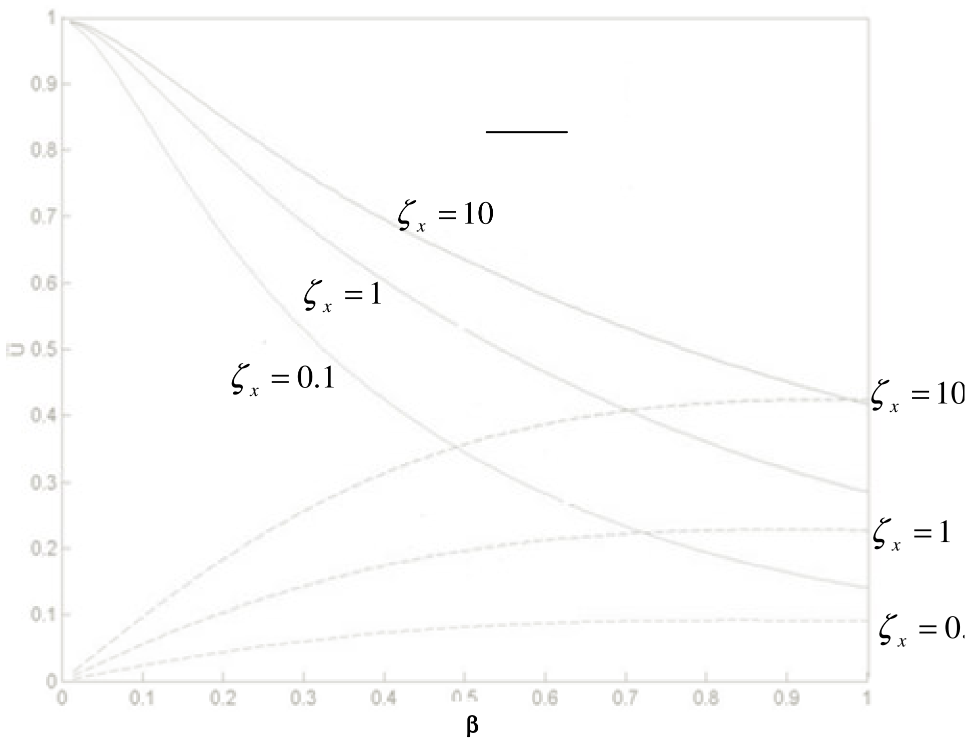
Here it must be pointed out that for an incident P-wave, strain $\in_1$ = β (a non-dimensional wave number of incident wave). Whereas, for an incident shear wave (SV-wave or SH-wave) $\in_2$ = β. In the present work, the non-dimensional wave number of the incident wave, that is, β (= 2π R/∧) has been given as input, so either $\in_1$ or $\in_2$ is always known. The other ∈ can be obtained by using the following relation:

$$\left(\frac{\in_2}{\in_1}\right)^2 = \frac{c_1^2}{c_2^2} = \frac{2(1 - \nu_m)}{(1 - 2\nu_m)} \tag{17}$$

where ν_m is the Poisson ratio of the medium.

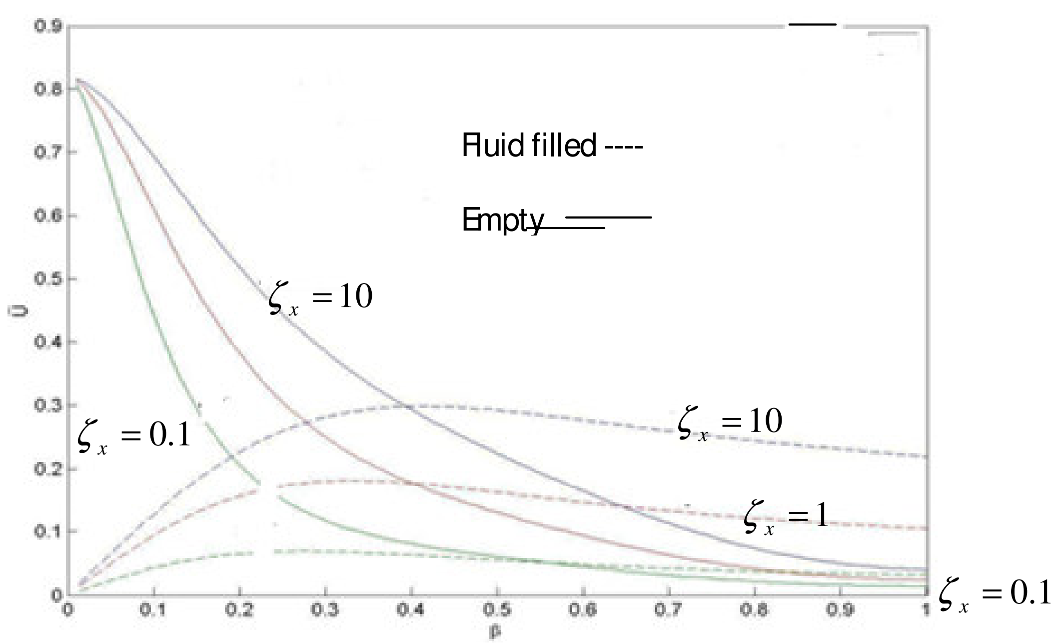
RESULTS AND DISCUSSION

Results are presented for a transversely isotropic shell with r-θ as the plane of isotropy. Consequently $E_\theta = E_z$, $G_{xz} = G_{x\theta}$, $\nu_{x\theta} = \nu_{xz}$, $\nu_{\theta z} = \nu_{z\theta}$, $G_{z\theta} = E_\theta / 2(1+\nu_{\theta z})$. Thus we have $\eta_3 = \eta_2$ and $\eta_4 = G_{z\theta}/E_x = \eta_1/2(1+\nu_{\theta z})$. In addition $\nu_{\theta Z} = \nu_{x\theta} = 0.3$ has been taken in the numerical calculations. Different values of shell orthotropy parameters η_1 and η_2 are used as 0.5, 0.01, 0.05 and 0.1, 0.05, 0.02, respectively. Soil parameter $\bar{\mu}$ had been varied from 0.1 to 10.0 to take into account different soil conditions around the pipe, representing soft to hard soil. For all the values of $\bar{\mu}$, ν_m = 0.25 had been assumed. Thickness to radius ratio of the shell ($\bar{h}$) had been taken as 0.01 and the density ratio of the surrounding medium to the shell ($\bar{\rho}$) had been taken as 0.75. Non-dimensional amplitude of the middle surface of the shell in the radial and axial directions ($\bar{W}$ and $\bar{U}$) have been plotted against the non-dimensional wave number of the incident P-wave (β=2πR/Λ). The shell response had been shown for empty and fluid filled shell for non-axisymmetric mode (flexural mode, n = 1) taking stiffness coefficients (ζ_x ζ_θ ζ_r) and damping coefficients (Γ_x Γ_θ Γ_r) as parameters. Figures 2 to 4 shows the effect of stiffness coefficient ζ_x on axial displacement $\bar{U}$ of the shell for soft, medium and hard types of soil respectively. At small angle of incident wave and for soft



$\phi = 5^{\circ}, \bar{\mu} = 0.1, \zeta_x = 0.1,1,10$

Figure 3. Axial displacement $(\overline{U})$ vs. wave number (β) with ζ_x as parameter.



$\phi = 60^{\circ}, \bar{\mu} = 10, \zeta_x = 0.1,1,10$

Figure 4. Axial displacement $(\overline{\mathrm{U}})$ vs. wave number (β) with ζ_x as parameter.

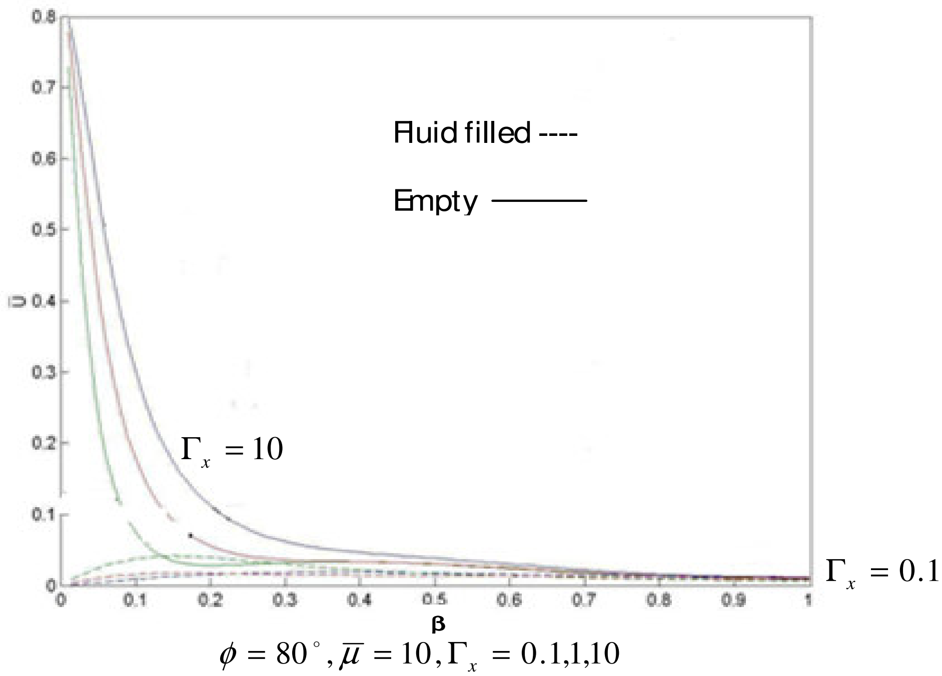


Figure 5. Axial displacement $(\overline{U})$ vs. wave number (β) with Γ_x as parameter.

soil, the effect of soil stiffness ζ_x is more in fluid filled shell as compare to empty shell, but at higher angle of incident wave and for hard soil, the effect of ζ_x on axial displacement is more in fluid filled shell as compare to empty shell as shown in Figure 4.

Figures 5 to 7 shows the effect of damping coefficient Γ_x on axial displacement $\overline{U}$ of the shell. At small angle of incident of the wave number and for soft soil the effect of Γ_x is more in fluid filled shell as compare to empty shell, but at higher angle of incident of the wave number and for hard soil the effect of Γ_x is more in fluid filled shell as compare to empty shell. Figure 5 shows that at higher wave number with higher angle of incidence under hard soil condition, the axial displacement is negligible both in the case of empty shell as well as fluid filled shell. The axial displacement is significant in fluid filled shell as compared to empty shell buried under soft soil.

Figures 8 to 9 shows the effect of stiffness coefficient ζ_x on radial displacement $\overline{W}$ of the shell with increasing wave number under different soil conditions. The radial displacement of fluid filled shell, first decreases then increases with increasing value of wave number. A reverse phenomenon can be seen in case of empty shell. Under imperfect bond conditions, radial displacement in empty shell is more predominant.

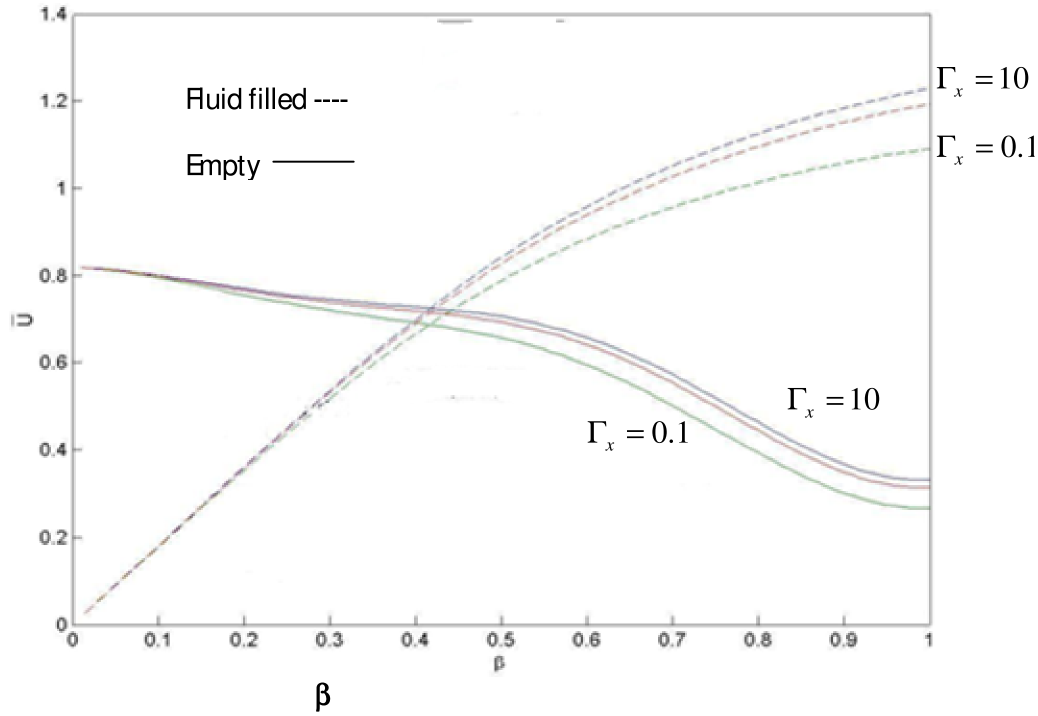
Figures 11 to 13 show the effect of damping coefficient Γ_x on radial displacement of the shell $\overline{W}$. As wave number increases radial displacement first decreases then increases with increasing value of Γ_r in medium soil in case of fluid filled shell but trend is reversed in empty shell at higher incidence angle.

Figure 14 shows the effect of orthotropy parameter η_2 on axial displacement of the shell with soil stiffness as another variable. Results show that orthotropy parameter η_2 has negligible effect on the response in the case of fluid filled and empty shell.

Figures 15 and 16 shows the effect of density of fluid on radial and axial displacement of the buried thin shell, respectively. Fluid density had been taken as variable and its value has been varied from 0.13 to 0.66. Results show that with increasing density of the fluid, radial displacement increases and axial displacement decreases.

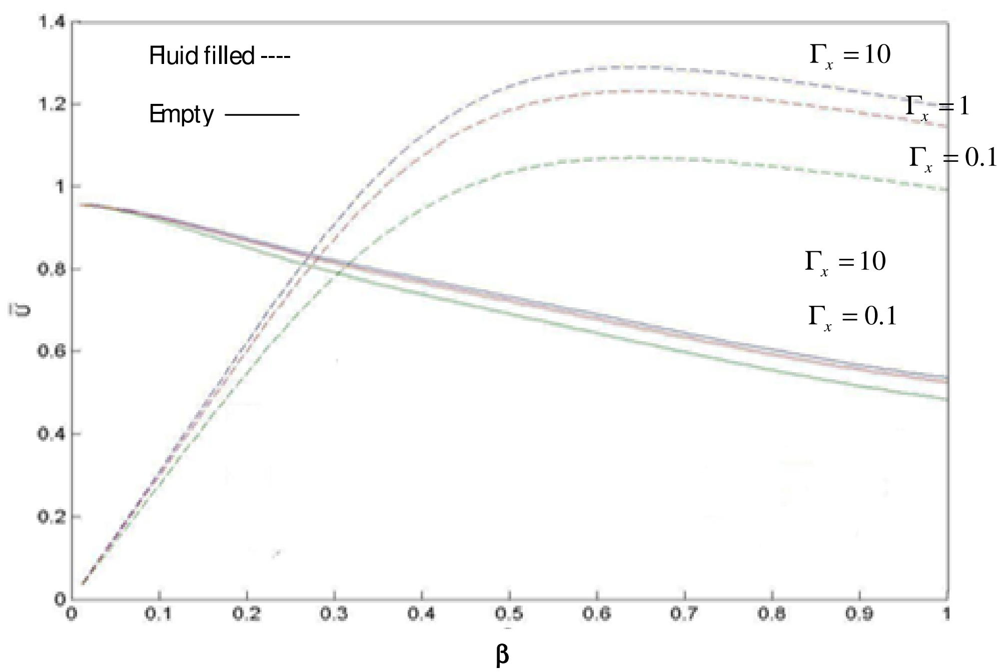
Conclusions

To study the effects of the fluid presence on the thin shell displacement, under different soil condition at various angles of incidence of the longitudinal wave under imperfect bonding, parametric results in graphical



$\phi = 5^\circ, \bar{\mu} = 0.01, \Gamma_x = 0.1,1,10$

Figure 6. Axial displacement $(\overline{\mathrm{U}})$ vs. wave number (β) with Γ_x as parameter.



$\phi = 60^\circ, \bar{\mu} = 0.1, \Gamma_x = 0.1,1,10$

Figure 7. Axial displacement $(\overline{\mathrm{U}})$ vs. wave number (β) with Γ_x as parameter.

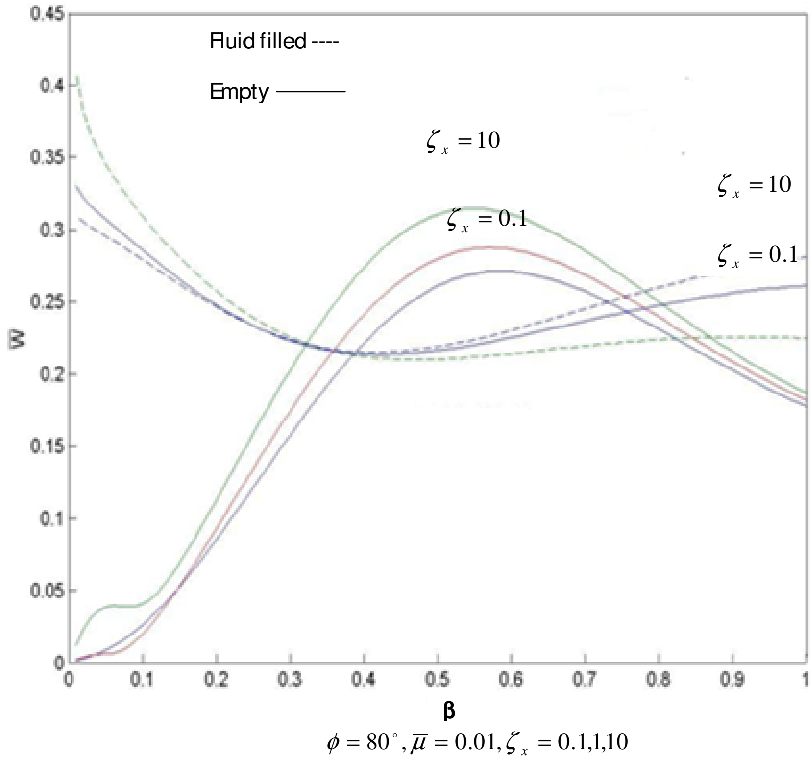


Figure 8. Radial displacement $(\overline{W})$ vs. wave number (β) with ζ_r as parameter.

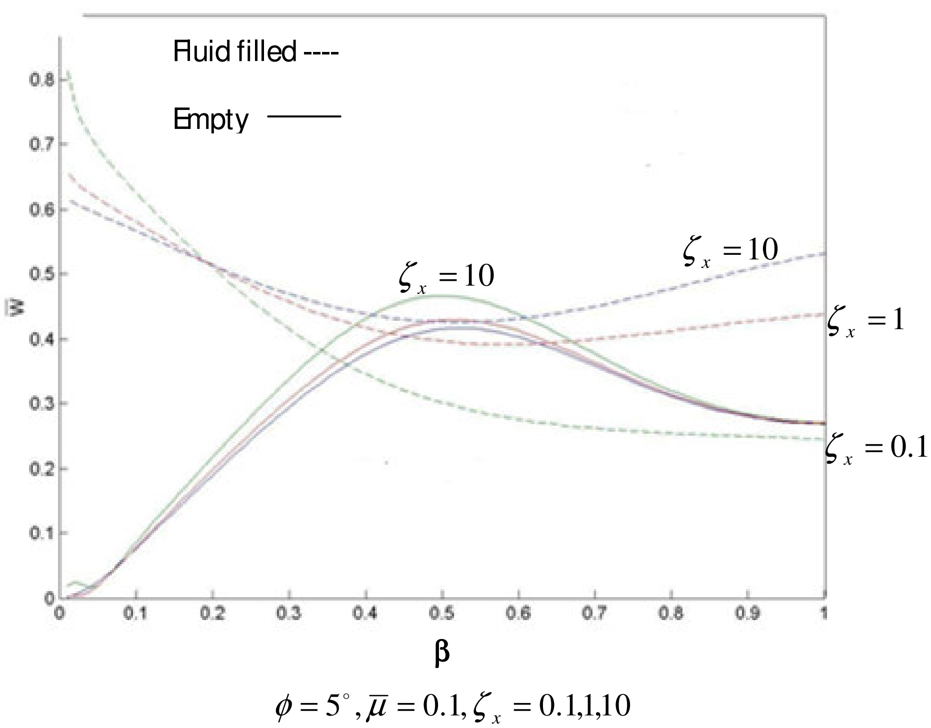
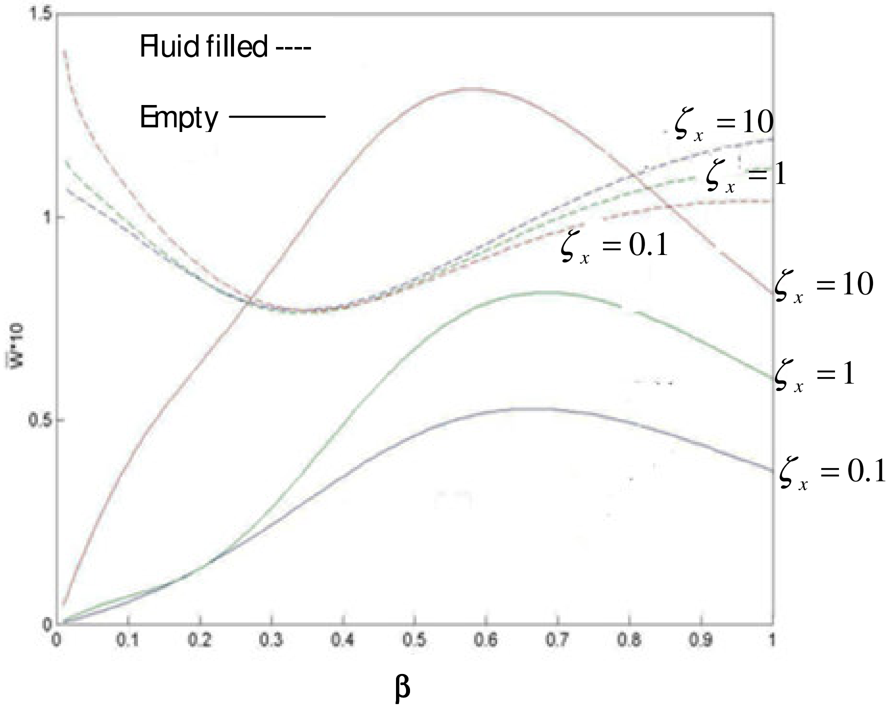
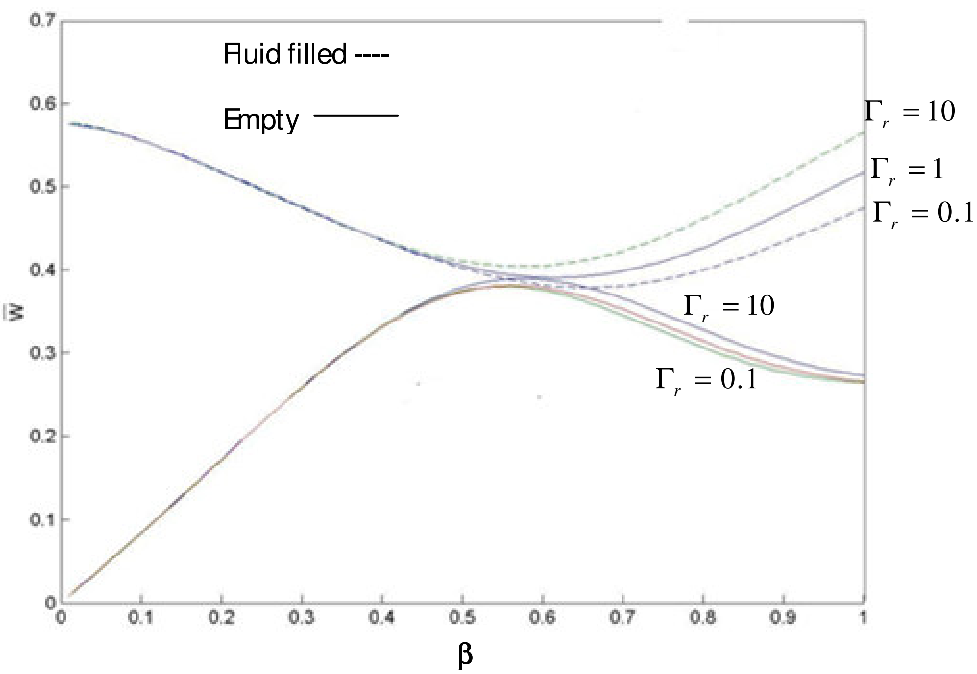


Figure 9. Radial displacement $(\overline{W})$ vs. wave number (β) with ζ_r as parameter.



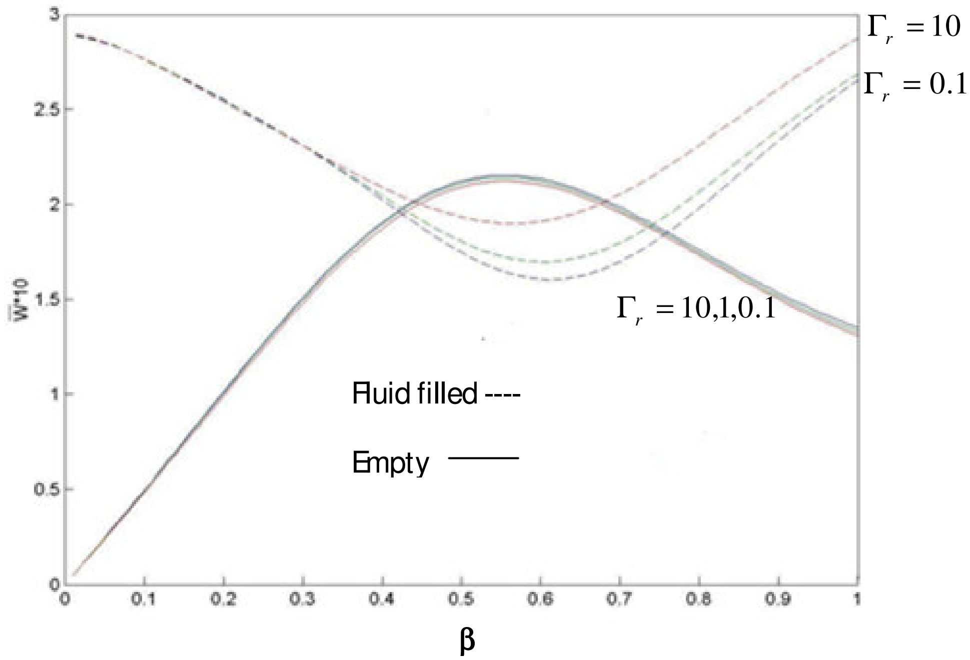
$\phi = 60^\circ, \overline{\mu} = 10, \zeta_x = 0.1, 1, 10$

Figure 10. Radial displacement $(\overline{W})$ vs. wave number (β) with ζ_r as parameter.



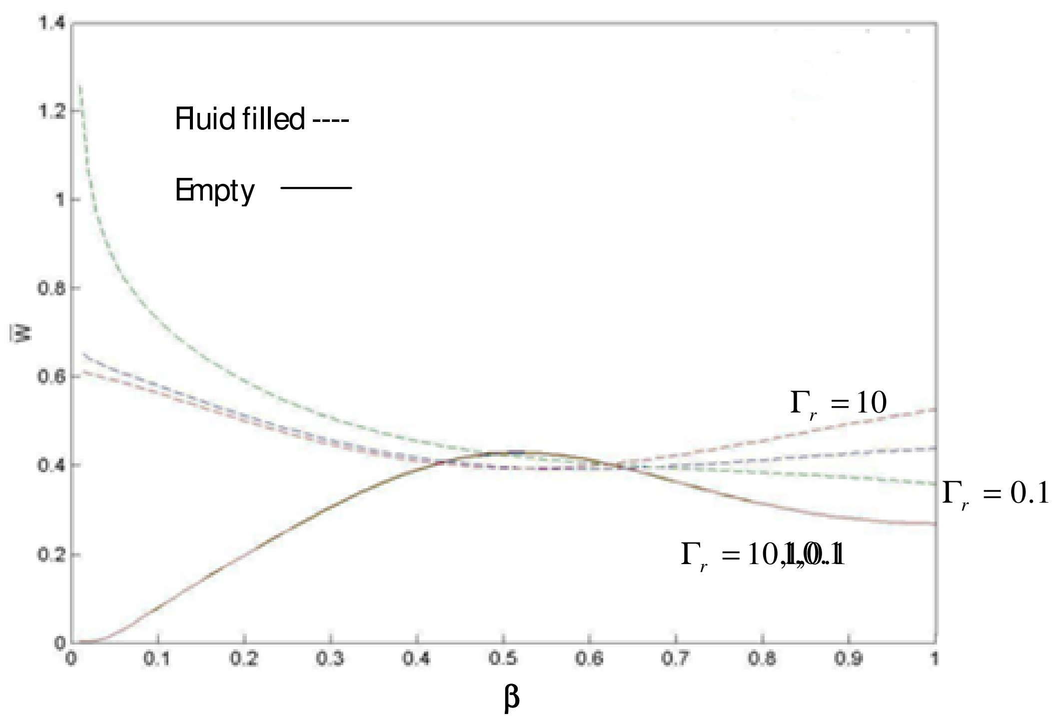
$\phi = 80^\circ, \overline{\mu} = 0.1, \Gamma_r = 0.1, 1, 10$

Figure 11. Radial displacement $(\overline{W})$ vs. wave number (β) with Γ_r as parameter.



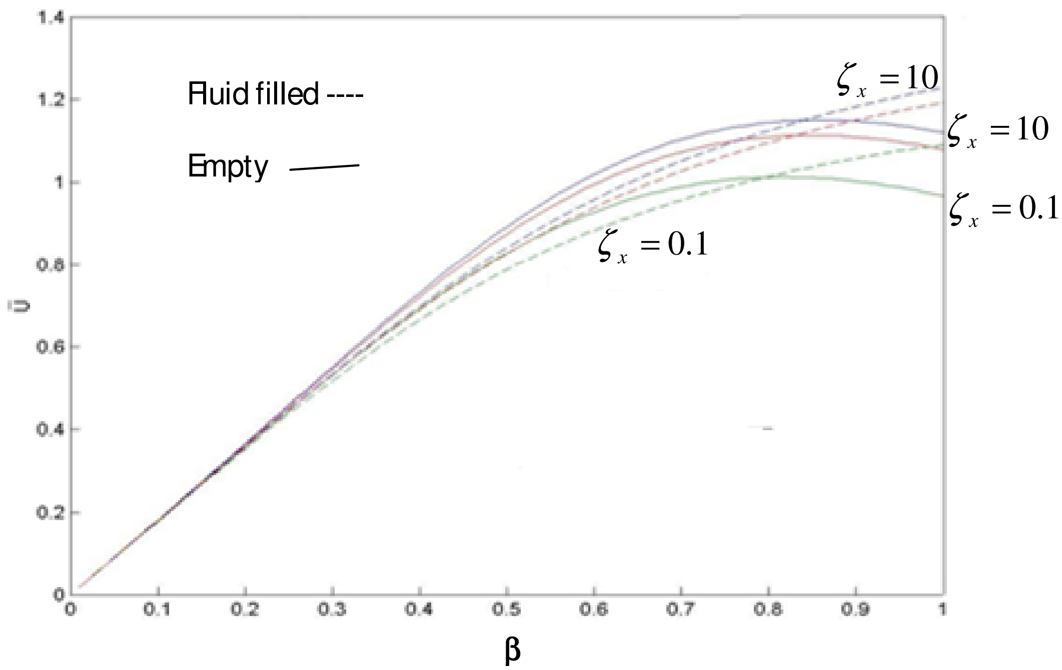
$\phi = 5^\circ, \overline{\mu} = 0.01, \Gamma_r = 0.1,1,10$

Figure 12. Radial displacement $(\overline{W})$ vs. wave number (β) with Γ_r as parameter.



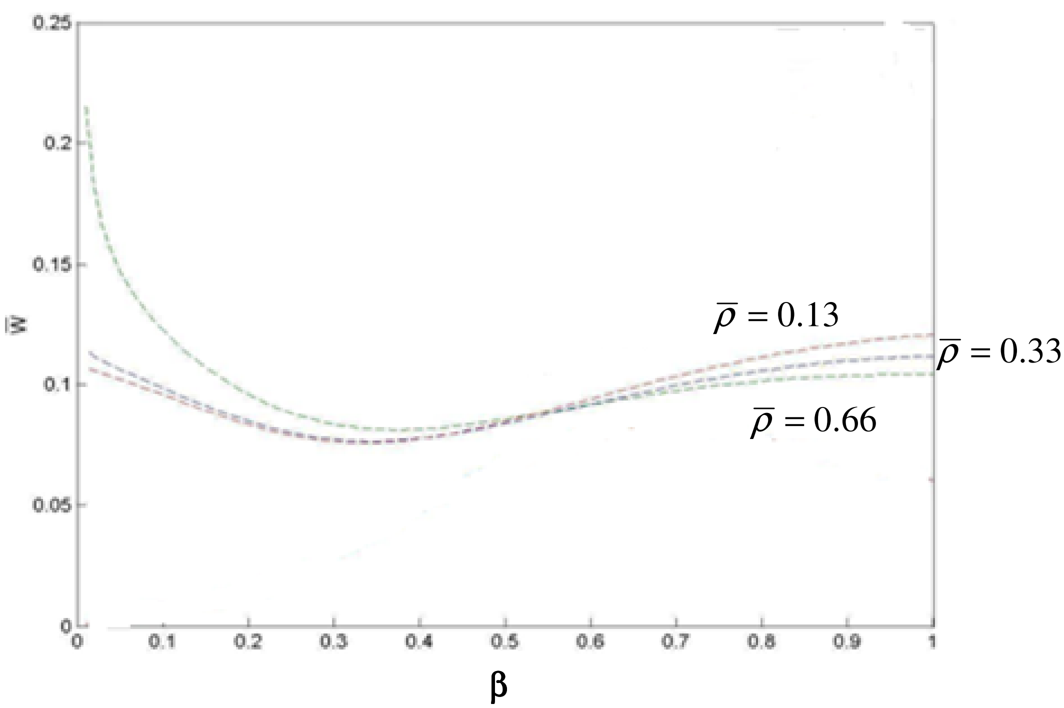
$\phi = 80^\circ, \overline{\mu} = 10, \Gamma_r = 0.1,1,10$

Figure 13. Radial displacement $(\overline{W})$ vs. wave number (β) with Γ_r as parameter.



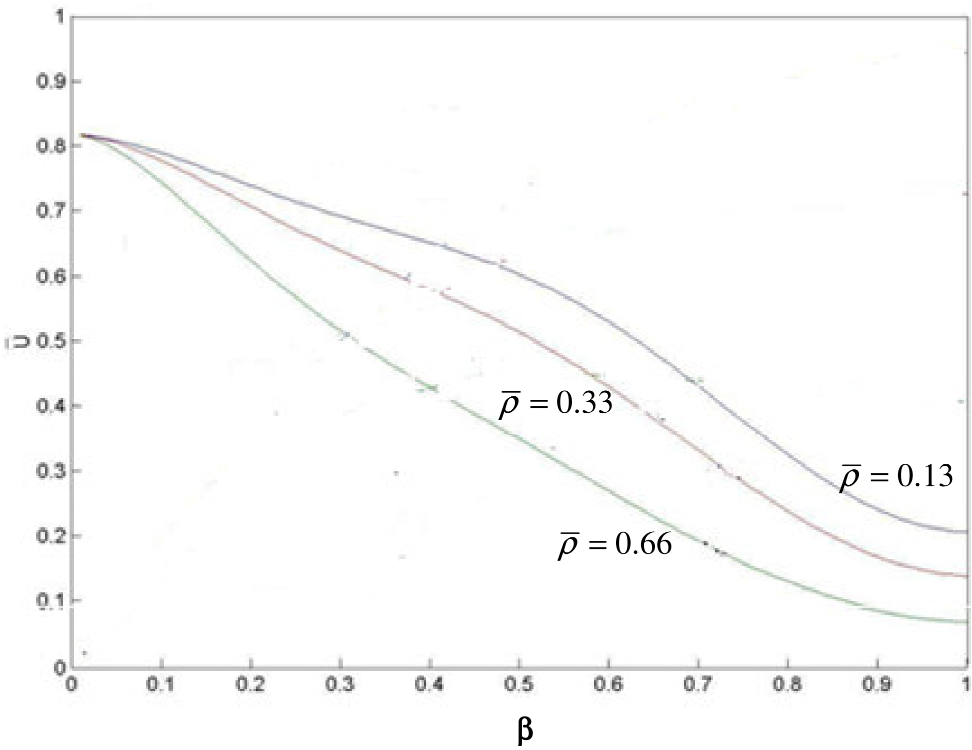
$$\phi = 5^{\circ}, \bar{\mu} = 0.1, \zeta_x = 1, \eta_2 = 0.1, 0.05, 0.02$$

Figure14. Axial displacement $(\overline{U})$ vs. wave number (β) with η_2 as parameter.



$$\phi = 60^{\circ}, \bar{\mu} = 0.1, \Gamma_r = 1, \eta_1 = 0.1, \eta_2 = 0.05, \bar{\rho} = 0.13, 0.33, 0.66$$

Figure 15. Radial displacement $(\overline{W})$ vs. wave number (β) with fluid density ($\bar{\rho}$) as parameter.



$$\phi = 5^\circ, \bar{\mu} = 0.1, \zeta_x = 1, \eta_1 = 0.1, \eta_2 = 0.05, \bar{\rho} = 0.13, 0.33, 0.66$$

Figure 16. Axial displacement $(\bar{U})$ vs. wave number (β) with fluid density ($\bar{\rho}$) as parameter.

form have been generated. Based on the results presented, following general conclusions could be drawn:

1. It is found that magnitude of the response of fluid filled pipeline can become even more than that of an empty pipeline, and hence, it cannot be assumed that a fluid filled pipeline will always furnish safe and conservative response.
2. Both the shell orthotropic parameters influence the radial displacement equally well but η_2 has a stronger influence on the axial displacement than η_1.
3. The density of the fluid becomes the important parameters in determining the shell response if incident longitudinal wave is of smaller wavelength.
4. The fluid filled pipeline response assumes considerable importance in soft soil condition and at higher apparent wave speed.
5. The fluid filled pipeline response due to incident longitudinal wave is significant only at large angle of incidence. Its response effect is small in hard shell.
6. For large angle of incidence, radial deflection is higher in fluid filled pipe as compared to empty shell. Thus for larger wavelength, empty pipe response is more important because the most common cause of pipeline failure is excessive axial deformation, while at smaller wavelength the fluid filled pipe has much importance for axial displacement.
7. Axial deflection and radial deflection both increase when the value of bonding parameter stiffness coefficient ($\zeta_x \quad \zeta_\theta \quad \zeta_r$) and damping coefficient ($\Gamma_x \, \Gamma_\theta \, \Gamma_r$) increase from zero to infinity (perfect to imperfect bonding) as variable.
8. The presence of fluid inside the shell, in general affects the radial displacement of the shell much more than the axial displacement, and in certain cases the change in radial displacement due to fluid presence is more prominent than that realized by variation of the bond parameter.

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