Review

An integral treatment for combined heat and mass transfer by mixed convection along vertical surface in a saturated porous medium

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Combined free and forced convection heat and mass transfer over an impermeable vertical surface embedded in a saturated porous medium is considered. A similarity transformation has been used to study the mixed convection boundary-layer flow over a semi-infinite vertical flat plate. Integral solutions are derived for the coupled nonlinear similarity equations of coupled heat and mass transfer in porous media for the case where the free stream velocity and temperature and concentration near the wall are kept constant. The governing parameters for the problem under consideration are the Lewis number Le, the buoyancy ratio N and mixed convection parameter ($\frac{Ra}{Pe}$). The results for the heat and mass transfer coefficients in terms of Nusselt and Sherwood numbers are represented graphically for the various values of governing parameters of the problem. The heat and mass transfer in the boundary layer region has been analyzed for aiding and opposing flows.

Keywords: Convection in porous media, heat and mass transfer, integral method, boundary layer.

INTRODUCTION

The mechanisms of thermal and solutal transport by porous matrix are a phenomenon of great interest from the theory and application point of view. This is primarily because of the numerous applications of flow porous media, such as storage of radioactive nuclear waste materials, transpiration cooling, separation processes in chemical industries, filtration, transport processes in aquifers, ground water pollution, etc. The motivation of the present study is the fact that, both free and forced convection exist simultaneously in many of these applications. This is particularly relevant in situations where the Grashof number is large. Theories of mixed convection heat and mass transfer in porous media have been prepared for vertical surfaces (Lai, 1991; Chamkha and Khaled, 1999; Chamkha and Khaled, 2000; Postelnicu, 2007; Hassan, 2009; Oladapo, 2010; Bansod and Jadhav, 2010; Tak et al. , 2010), horizontal surfaces

(Li and Lai, 1997; Bansod, 2003; Lakshmi and Murthy, 2008; Bansod and Jadhav, 2009) and inclined surfaces (Singh et al., 2002; Bansod, 2005; Shateyi, 2008; Beg et al., 2009). All these studies have been reported based on boundary layer analysis. The state of art concerning coupled heat and mass transfer by mixed convection in porous media has been summarized in the excellent monographs by Nield and Bejan (2006) and Ingham and Pop (2002).

In this chapter, an integral procedure is developed for combined heat and mass transfer by mixed convection from a vertical surface embedded in a saturated porous medium. The mixed convection flow is promoted by the uniform free stream and density variation due to the combination of temperature and concentration gradients. The Darcy model is considered and the porous medium porosity is assumed to be low. The heat and mass transfer in boundary layer region has been analyzed for aiding and opposing buoyancies for the aiding and opposing flows. The value of ($\frac{Ra}{Pe}$) is found to be the

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controlling parameter for the mixed convection. The results of heat and mass transfer in terms of Nusselt and Sherwood number, are presented for a wide range of governing parameters like the buoyancy ratio (N), -0.5 ≤ N ≤ 8, Lewis number (Le), 0.1 ≤ Le ≤ 100 and flow driving parameter ($\frac{Ra}{Pe}$), 0.1 ≤ $\frac{Ra}{Pe}$ ≤ 100.

Governing equations and transformation

Mixed convection heat and mass transfer from an impermeable vertical surface embedded in a fluid saturated porous medium was considered. The surface temperature and concentration are kept constant. At the same time, the temperature and concentration sufficiently far from the wall are T_∞ and C_∞, respectively. An external flow with a uniform velocity is introduced to the medium. For buoyancy induced by heat and mass transfer, the density is assumed constant everywhere, except in the body force term of Darcy's equation. Having invoked the Boussinesq and boundary layer approximation, the governing equations based on Darcy's law are given by:

$$\frac{\partial u}{\partial y} = \frac{Kg}{v}\left(\beta_T \frac{\partial T}{\partial y} + \beta_C \frac{\partial C}{\partial y}\right) \quad (1)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \quad (2)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \quad (3)$$

In the aforestated equations, (u,v) are the Darcian velocities in the (x,y) directions. β_T and β_C are the thermal and concentration expansion coefficients respectively, v is the kinematic viscosity, μ is the viscosity of the fluid, ρ is the density, g is acceleration due to gravity, K is the permeability of the porous medium, α and D are the equivalent thermal and mass diffusivities of the porous medium. T and C are the temperature and concentration. The boundary conditions at the wall are:

$$y = 0 \, ; v = 0 \, , T = T_W \, , C = C_W \quad (4)$$

and at infinity are

$$y \to \infty \, ; u = U_\infty \, , T \to T_\infty \, , C \to C_\infty \quad (5)$$

To solve the set of simultaneous equations defined previously, the following dimensionless variables are introduced:

$$\eta = \frac{y}{x}(Pe)^{1/2} \quad (6)$$

$$\Psi = \alpha(Pe)^{1/2} f(\eta) \quad (7)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty} \quad (8)$$

$$\phi(\eta) = \frac{C - C_\infty}{C_W - C_\infty} \quad (9)$$

In the aforestated equations, $Pe = \frac{U_\infty x}{\alpha}$ is Peclet number, ψ is stream function and f is its non-dimensional counterpart, θ is non-dimensional temperature distribution and ϕ is non-dimensional concentration distribution and η is similarity variable. After transformations, the resulting equations are

$$f'' = \frac{Ra}{Pe}(\theta' + N\phi') \quad (10)$$

$$\theta'' = -\frac{1}{2}f\theta' \quad (11)$$

$$\phi'' = -\frac{Le}{2}f\phi' \quad (12)$$

The parameter $N = \frac{\beta_C(C_W - C_\infty)}{\beta_T(T_W - T_\infty)}$ measures the relative strength of mass and thermal diffusion in the buoyancy-induced flow. It is clear that N = 0 for pure thermal buoyancy-induced flow, infinite for mass-driven flow, positive for aiding flow and negative for opposing flow. The diffusivity ratio, $Le = \frac{\alpha}{D}$ is nothing but the ratio of the Schmidt number $\left(\frac{v}{D}\right)$ and Prandtl number, $\left(\frac{v}{\alpha}\right)$ which denotes the relative rates of propagation of energy and mass within a system. The flow controlling parameters $\frac{Ra}{Pe} = \frac{Kg\beta_T(T_W - T_\infty)}{\mu U_\infty}$, measures the relative importance of the buoyancy effects and forced convection and is independent of x, the distance measured along the wall. $(\frac{Ra}{Pe}) \to 0$, represent the forced convection flow. The transformed boundary conditions are

$$\eta = 0; \quad f = 0, \quad \theta = 1, \quad \phi = 1 \tag{13}$$

$$\eta \to \infty; \quad f' = 1, \quad \theta = 0, \quad \phi = 0 \tag{14}$$

In the equations, the primes indicate the derivative with respect to the similarity variable η. '∞' denotes the thickness of the boundary layer.

INTEGRAL SOLUTION AND DISCUSSION

The transformed energy Equation (10) to (12) together with the boundary conditions (13) and (14) can be integrated with respect to η from $\eta = 0$ to $\eta = \infty$, we get:

$$-\theta'(0) = \frac{1}{2}\int_0^\infty f'\theta \, d\eta \tag{15}$$

$$-\phi'(0) = \frac{Le}{2}\int_0^\infty f'\phi \, d\eta \tag{16}$$

The infinity is boundary layer thickness for temperature and concentration. We assume exponential temperature and concentration profiles as follows: -

$$\theta(\eta) = \exp(-\frac{\eta}{\delta_T}) \tag{17}$$

$$\phi(\eta) = \exp(-\frac{\xi\eta}{\delta_T}) \tag{18}$$

Here, δ_T is arbitrary scale for the thermal boundary layer thickness whereas ξ is its ratio to the concentration boundary layer thickness δ_C. With the help of the aforestated profiles and using Equation (10), Equations (15) and (16) can be obtained in two distinct expressions for δ_T^2 as:

$$\frac{1}{\delta_T^2} = \left(\frac{Ra}{Pe}\right)\left[\frac{\xi+1+2N}{4(\xi+1)}\right] \tag{19}$$

$$\frac{1}{\delta_T^2} = \left(\frac{Ra}{Pe}\right)\left[\frac{2\xi+N(\xi+1)}{4\xi^2(\xi+1)}\right]Le \tag{20}$$

Equations (19) and (20) can be combined to give the following cubic algebraic equation for determining the boundary layer thickness ratio ξ as:

$$\xi^3 + (1+2N)\xi^2 - [(2+N)Le]\xi - N.Le = 0 \tag{21}$$

As ξ is determined using Newton-Raphson method from Equation (21), the results of practical interest in many applications are the heat and mass transfer coefficients, in terms of Nusselt and Sherwood numbers are given by equations

$$\frac{Nu}{(Pe)^{1/2}} = -\theta'(0)$$

$$= \frac{1}{\delta_T}$$

$$= 0.5\left[\frac{\xi+1+2N}{(\xi+1)}\right]^{1/2}\left(\frac{Ra}{Pe}\right)^{1/2} = Nu$$

$$\frac{Sh}{(Pe)^{1/2}} = -\phi'(0)$$

$$= \frac{\xi}{\delta_T}$$

$$= 0.5 \, \xi\left[\frac{2\xi+N(\xi+1)}{2\xi(\xi+1)}\right]^{1/2}Le^{1/2}\left(\frac{Ra}{Pe}\right)^{1/2} = Sh$$

The notations Nu and Sh are used for the Nusselt number and Sherwood number respectively. As an indication of proper formulation and accurate calculation, the results thus obtained have been compared with the data published earlier with the case of thermally induced

$$Nu = 0.4448\left[\frac{\xi+1+2N}{(\xi+1)}\right]^{1/2}\left(\frac{Ra}{Pe}\right)^{1/2} \tag{22}$$

and

$$Sh = 0.4448 \, \xi\left[\frac{2\xi+N(\xi+1)}{2\xi(\xi+1)}\right]^{1/2}Le^{1/2}\left(\frac{Ra}{Pe}\right)^{1/2} \tag{23}$$

The Nusselt and Sherwood numbers are plotted in Figures 1 (a, b and c) and 2 (a, b and c) respectively as a function of Lewis number. Approaching the forced convection dominated region (that is $\frac{Ra}{Pe} = 0.1$), the heat transfer coefficient shows little dependence on the Lewis number, while the mass transfer coefficient shows the

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b
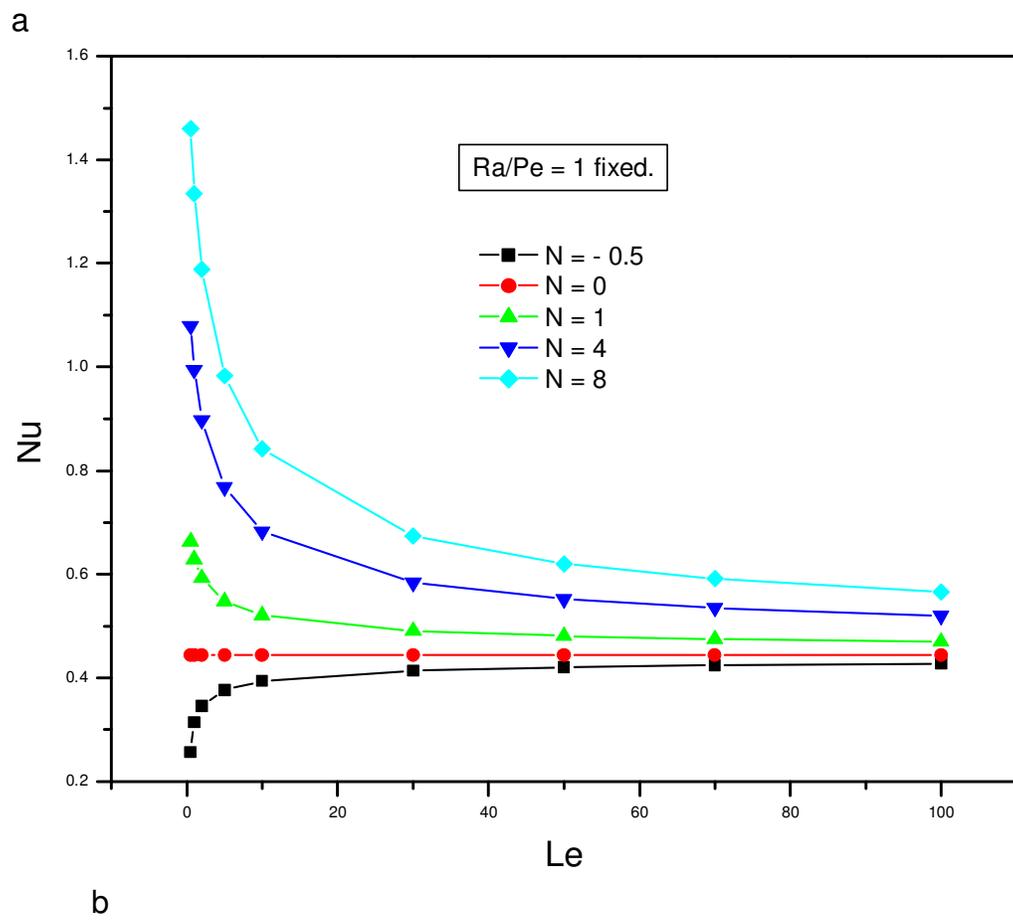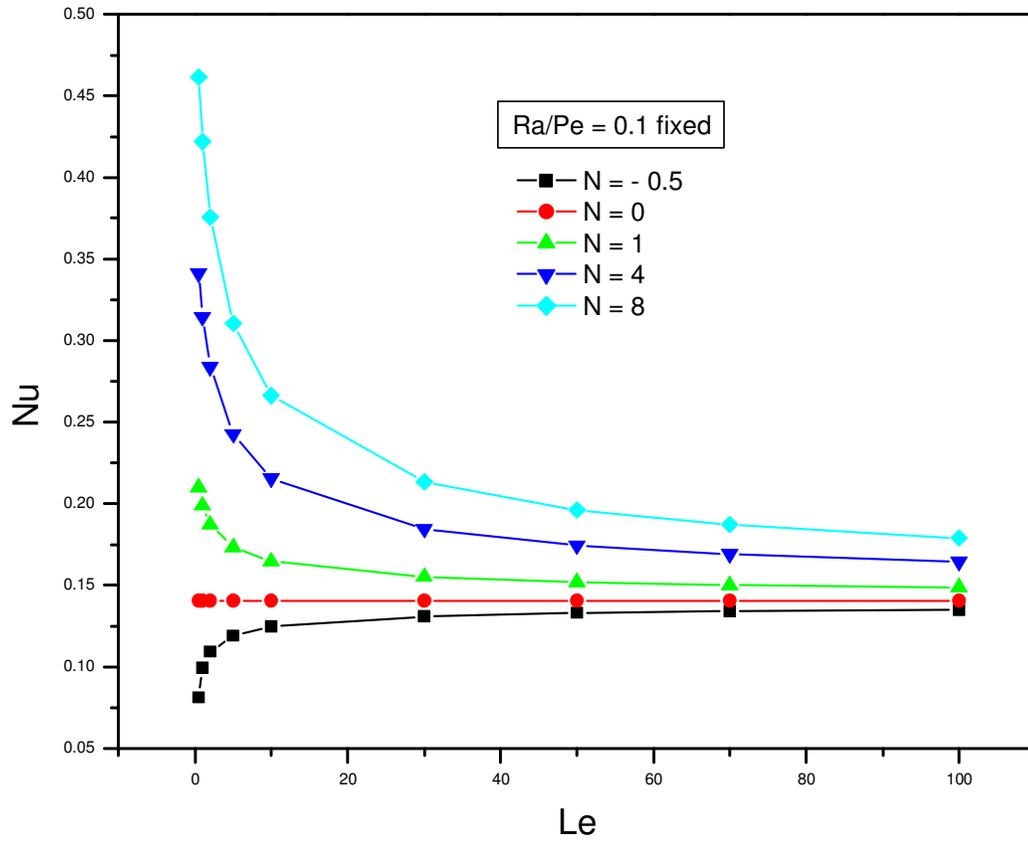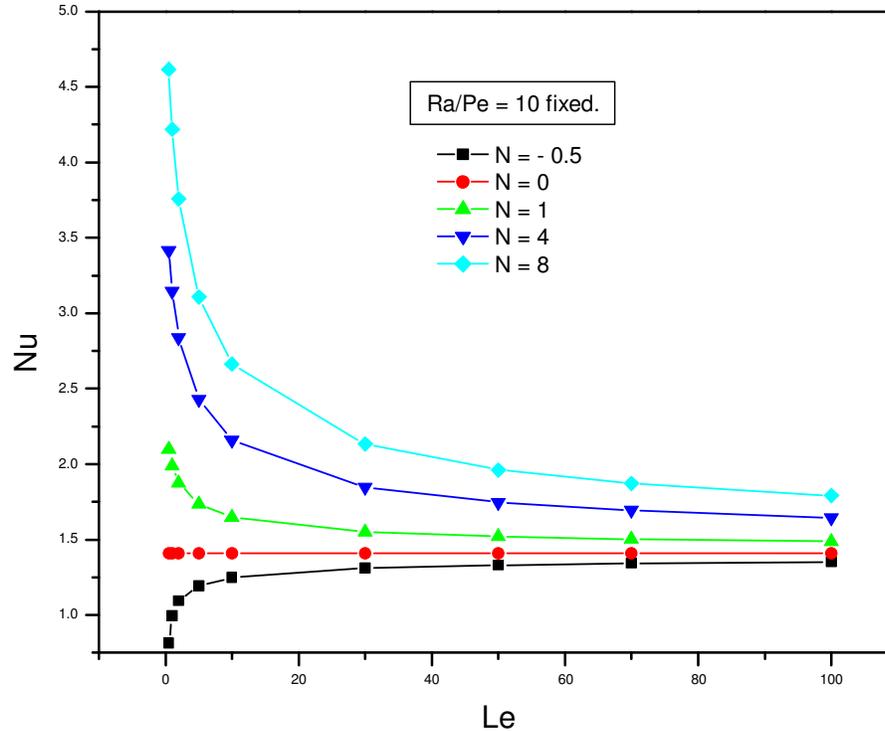
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Figure 1. (a) Heat transfer results as a function of Lewis number when (Ra / Pe) = 0.1 and N = - 0.5, 0, 1, 4, 8. (b) Heat transfer results as a function of Lewis number when (Ra / Pe) = 1 and N = - 0.5, 0, 1, 4, 8. (c) Heat transfer results as a function of Lewis number when (Ra / Pe) = 10 and N = - 0.5, 0, 1, 4, 8.

dependence of Lewis number to $Le^{1/2}$. As the value of mixed convection parameter increases, the flow approaches the buoyancy-dominated regime. For N > 0, it is observed that the heat transfer is greatly enhanced by the mass buoyancy effect, while it is considerably reduced for N < 0. It is important to note that, the mass transfer coefficient does not have any physical significant meaning for thermally induced flow that is N = 0. The curve N = 0 is included in Figure 1 (a, b and c) only for comparison.

To understand the dependence of the heat and mass transfer coefficients on N, Equations (22) and (23) are plotted in Figures 3(a, b and c) and 4 (a, b and c). For Le < 1, it is observed that the heat transfer coefficient is increased for N > 0 and decreased for N < 0. For Le >1, the situation is reversed and the flow reversal takes place at a larger value of N. In the forced convection dominated regime, both heat and mass transfer coefficients vary linearly with N. To show the mixed convection results, the heat and mass transfer coefficients are plotted in Figures 5 (a, b and c) and 6 (a, b and c), respectively as a function of $(\frac{Ra}{Pe})$. It is clear from these figures that the convection favoring effect of $(\frac{Ra}{Pe})$ is countered by the

parameters. For a given Le, it is observed that buoyancy-dominated regime extends toward a smaller value of $(\frac{Ra}{Pe})$ as N increases.

However, for a given N, the buoyancy-dominated regime retreats towards a larger value of $(\frac{Ra}{Pe})$ as Le increases. Therefore, for a given value of $(\frac{Ra}{Pe})$, the heat transfer regime under which the system operation depends on the value of N and Le. Selected values of Nu and Sh are listed in Table (1) for mixed convection. As expected, the heat transfer decreases with Le for opposing buoyancy whereas it increases with Le for aiding buoyancy.

Concluding remarks

Integral solution for mixed convection heat and mass transfer from a vertical surface embedded in a fluid saturated porous medium is reported. The heat and mass transfer in the boundary layer region has been analyzed
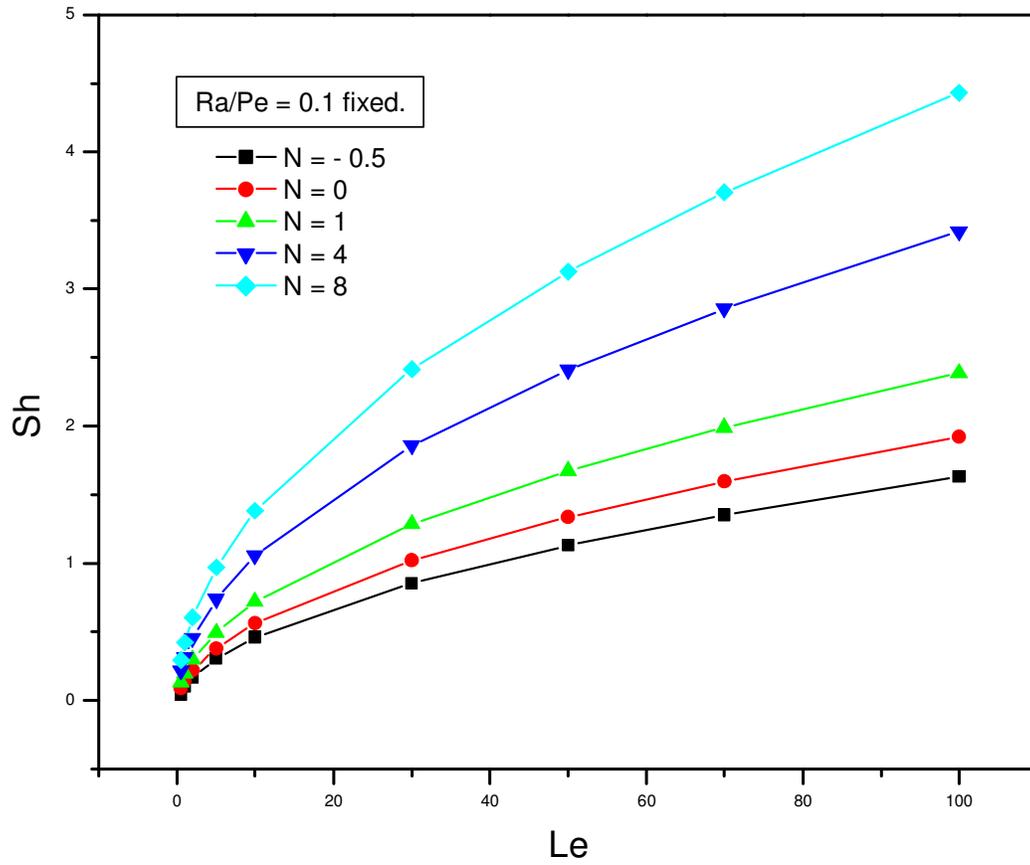
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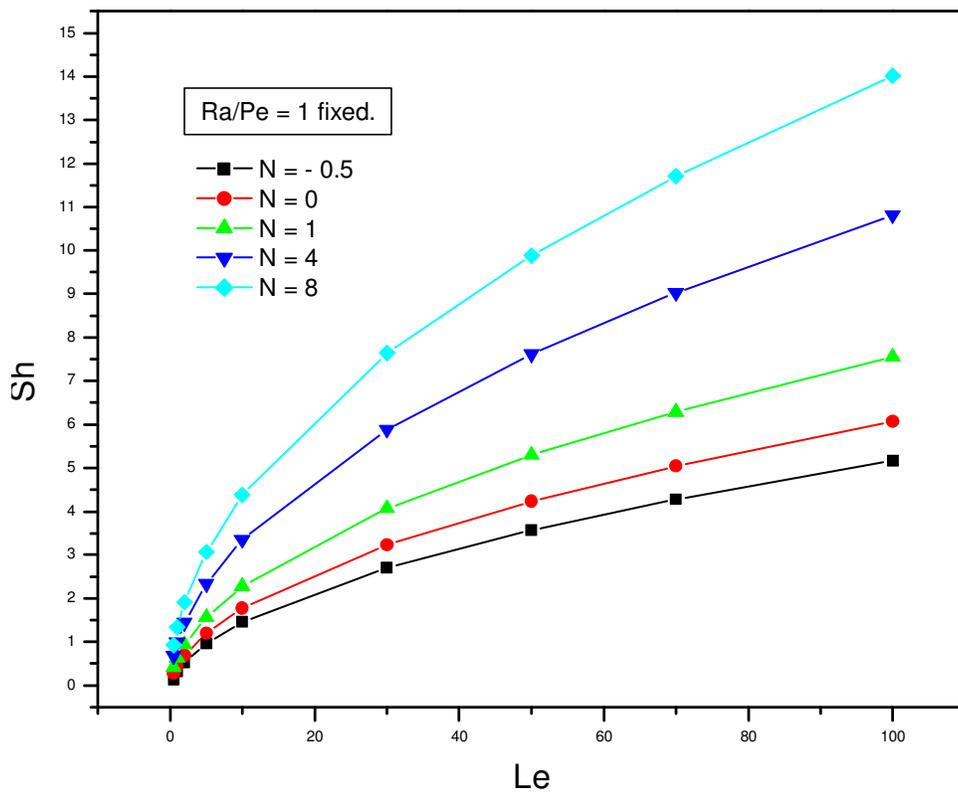
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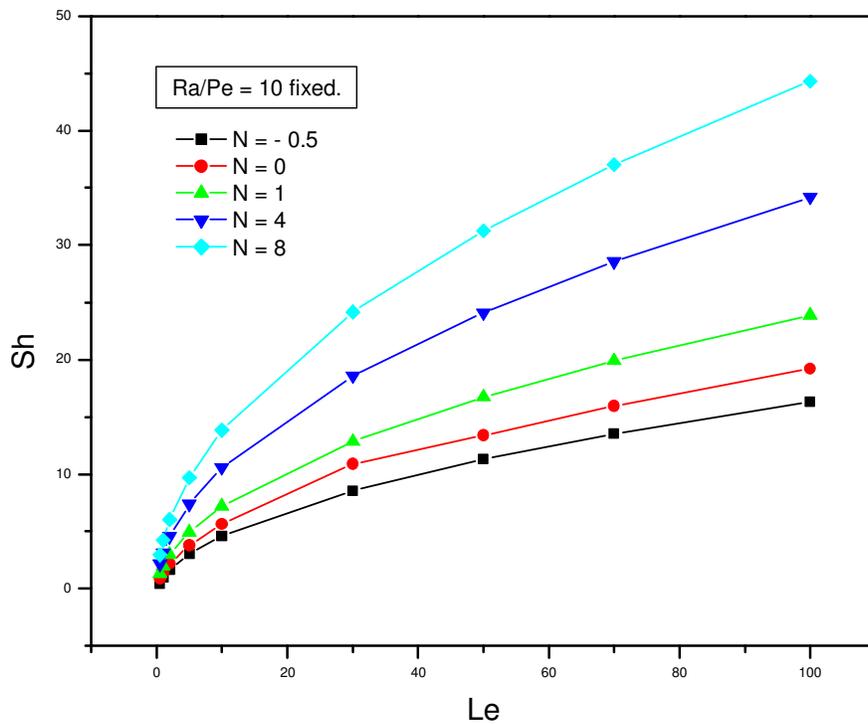
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Figure 2. (a) Mass transfer results as a function of Lewis number when (Ra / Pe) = 0.1 and N = - 0.5, 0, 1, 4, 8. (b) Mass transfer results as a function of Lewis number when (Ra / Pe) = 1 and N = - 0.5, 0, 1, 4, 8. (c) Mass transfer results as a function of Lewis number when (Ra / Pe) = 10 and N = - 0.5, 0, 1, 4, 8.
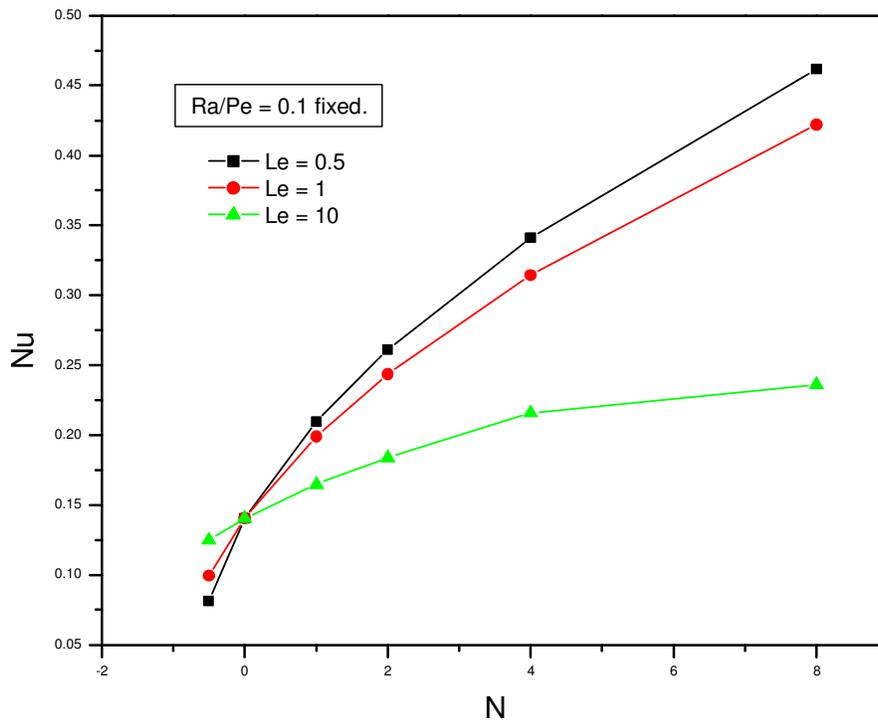
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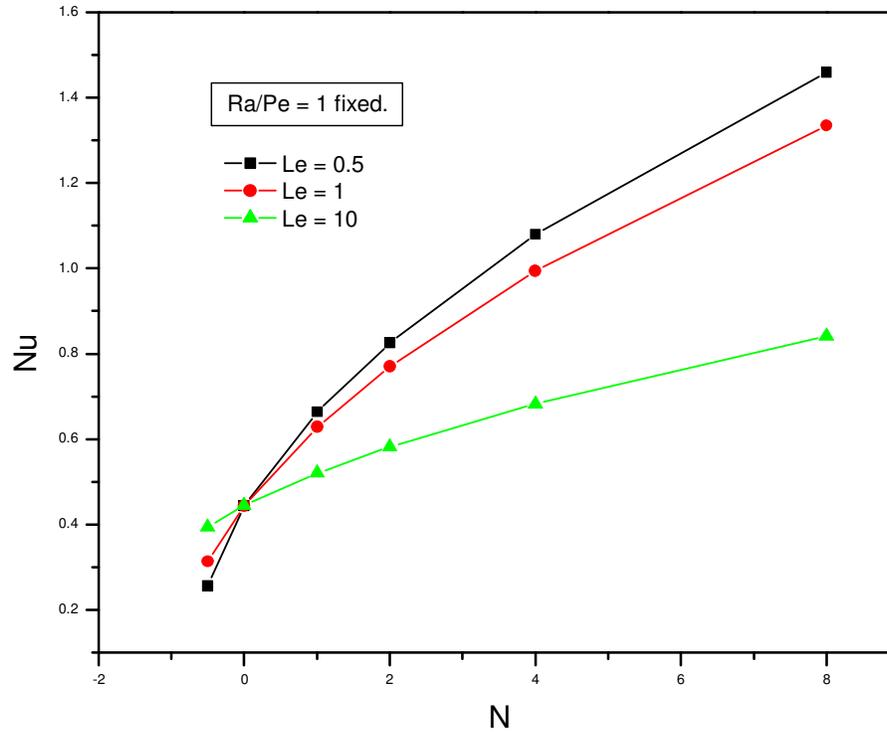
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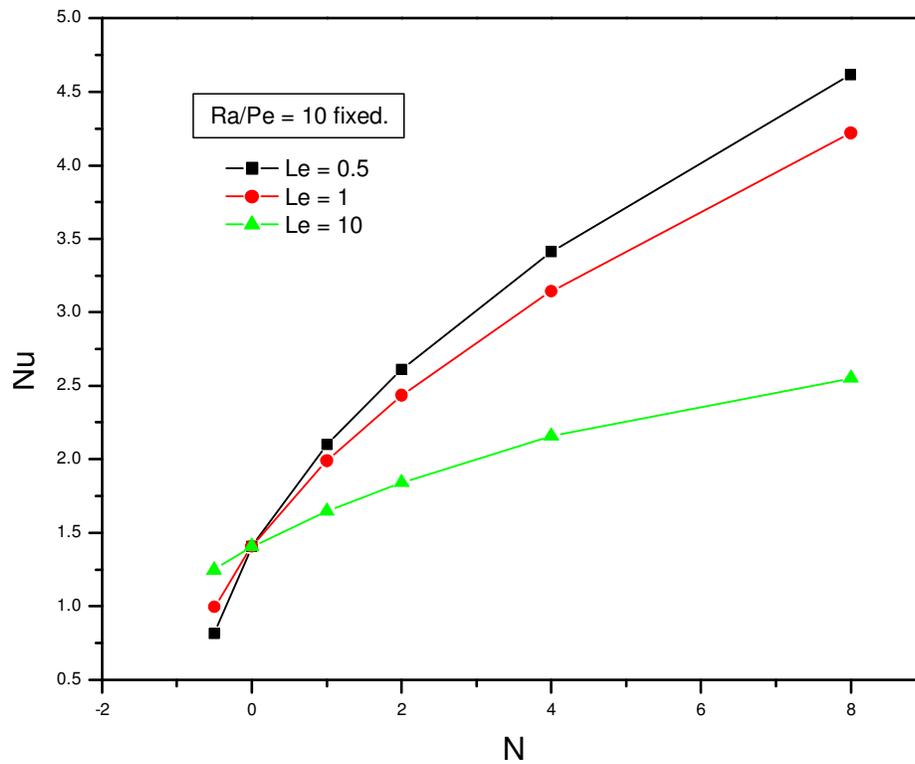
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Figure 3. (a) Heat transfer results as a function of buoyancy ratio when (Ra / Pe) = 0.1 and N = 0.5, 1, 10. (b) Heat transfer results as a function of buoyancy ratio when (Ra / Pe) = 1 and N = 0.5, 1, 10. (c) Heat transfer results as a function of buoyancy ratio when (Ra / Pe) = 10 and N = 0.5, 1, 10.
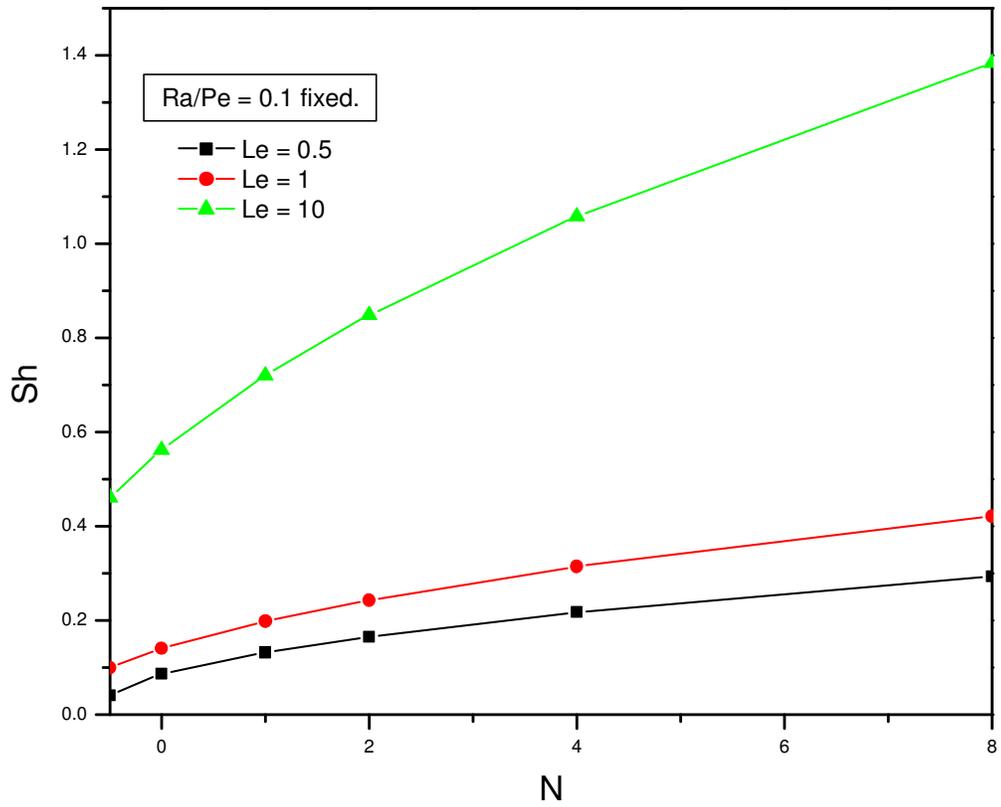
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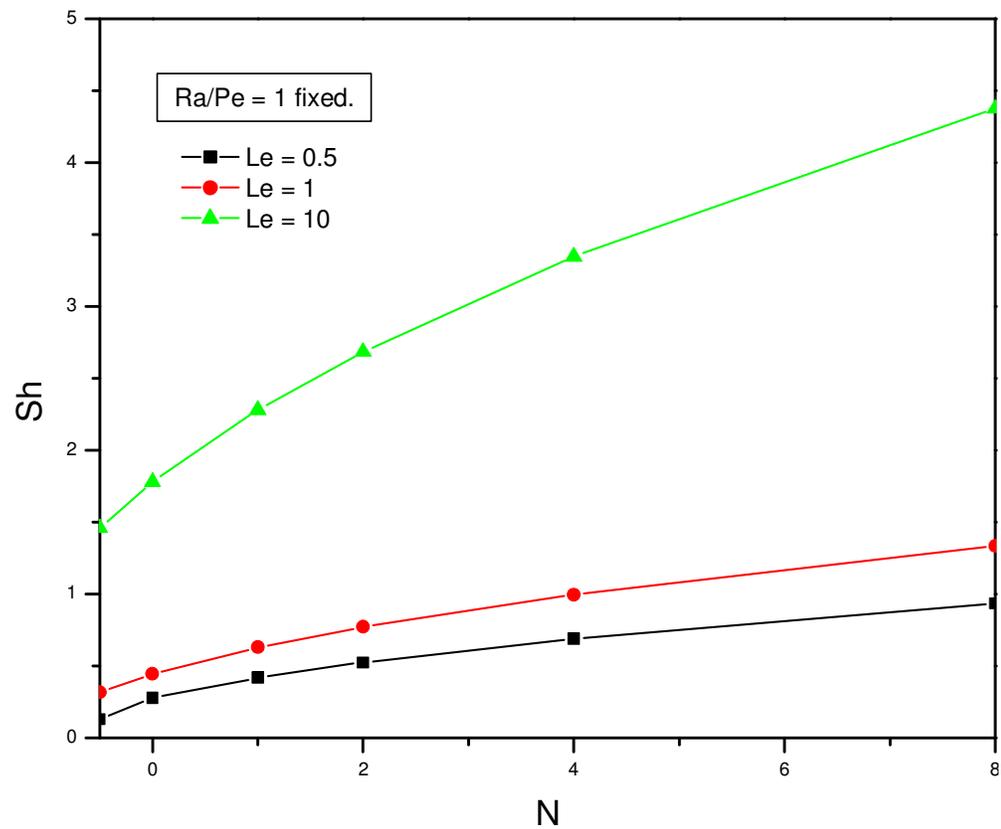
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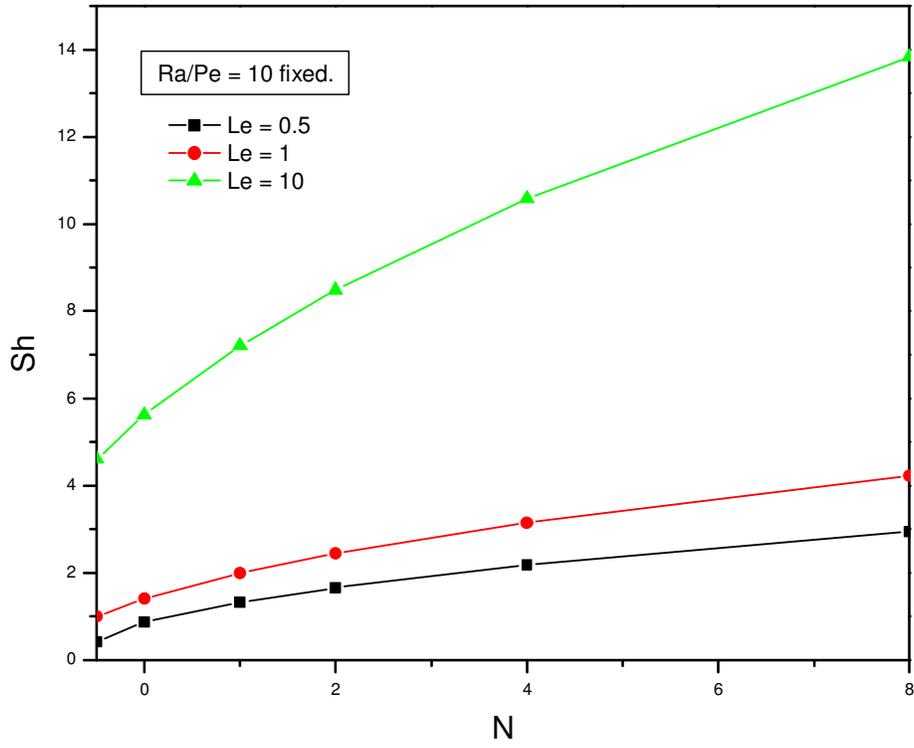
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Figure 4. (a) Mass transfer results as a function of buoyancy ratio when (Ra / Pe) = 0.1 and N = 0.5, 1, 10. (b) Mass transfer results as a function of buoyancy ratio when (Ra / Pe) = 1 and N = 0.5, 1, 10. (c) Mass transfer results as a function of buoyancy ratio when (Ra / Pe) = 10 and N = 0.5, 1, 10.
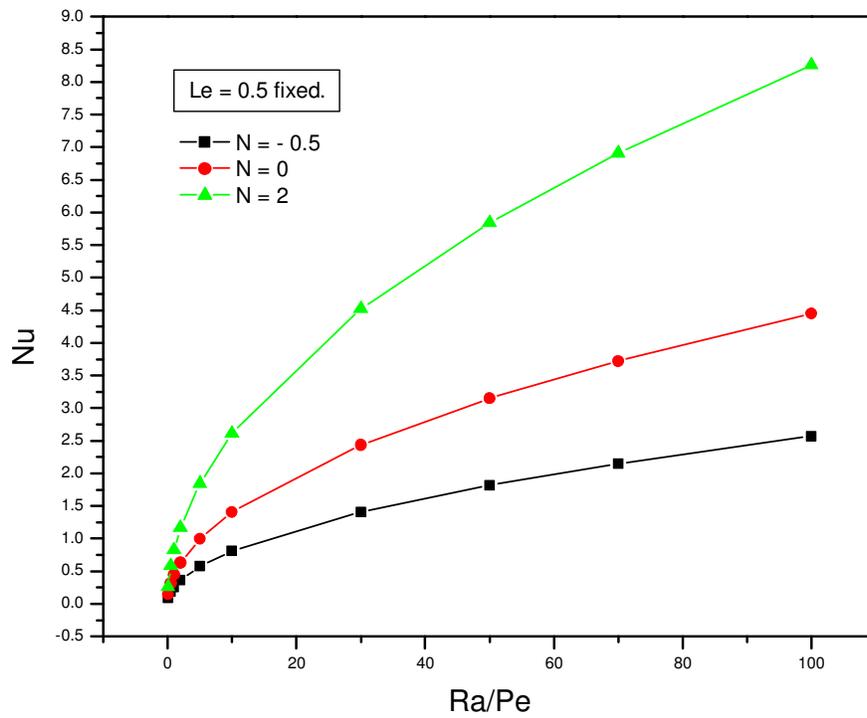
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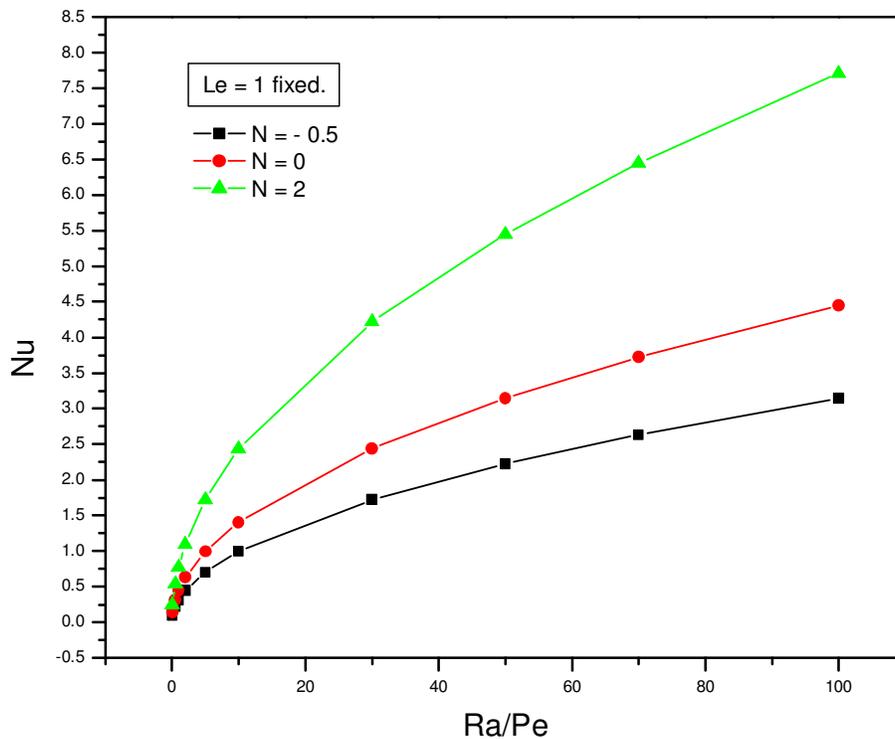
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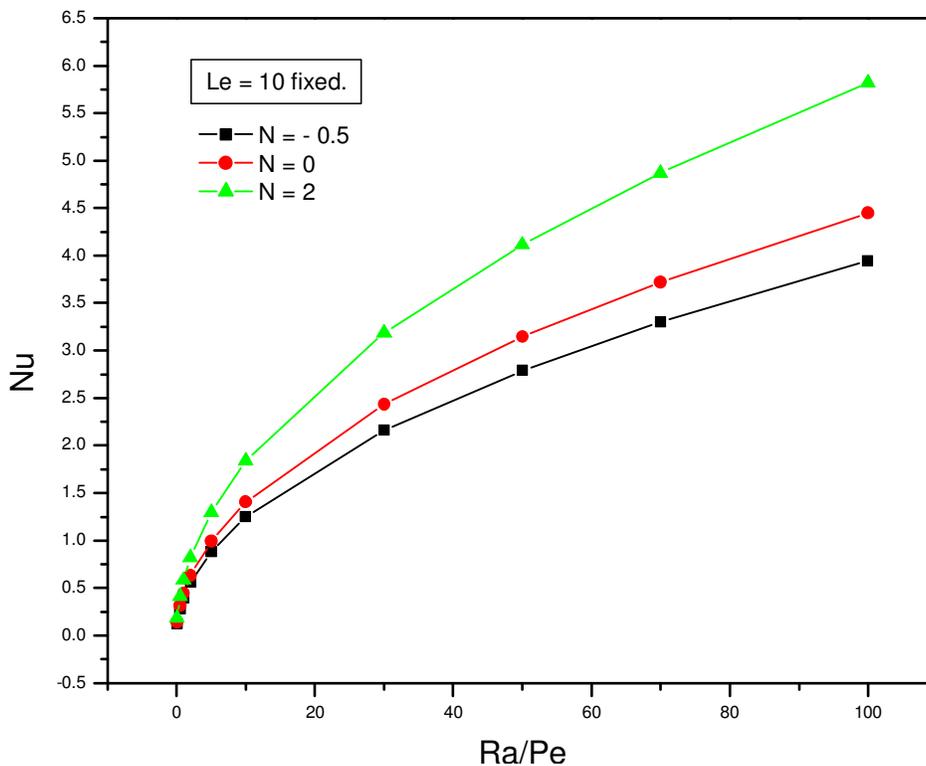
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Figure 5. (a) Heat transfer results as a function of (Ra / Pe) when Le = 0.5 and N = - 0.5, 0, 2. (b) Heat transfer results as a function of (Ra / Pe) when Le = 1 and N = - 0.5, 0, 2. (c) Heat transfer results as a function of (Ra / Pe) when Le = 10 and N = - 0.5, 0, 2
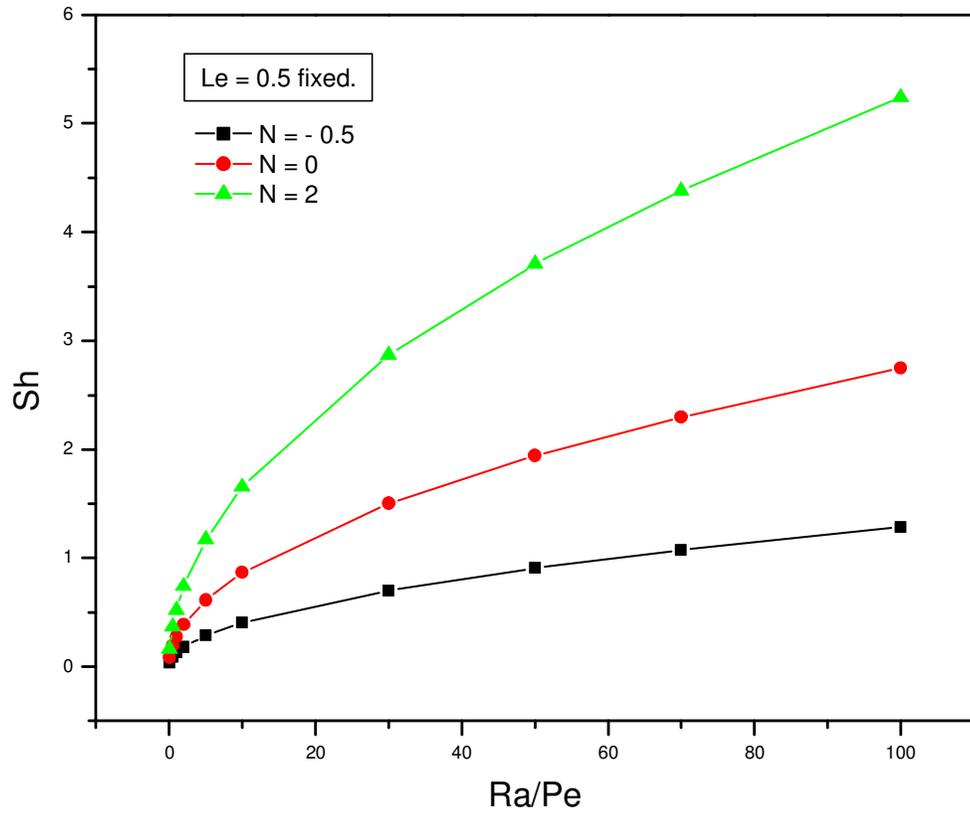
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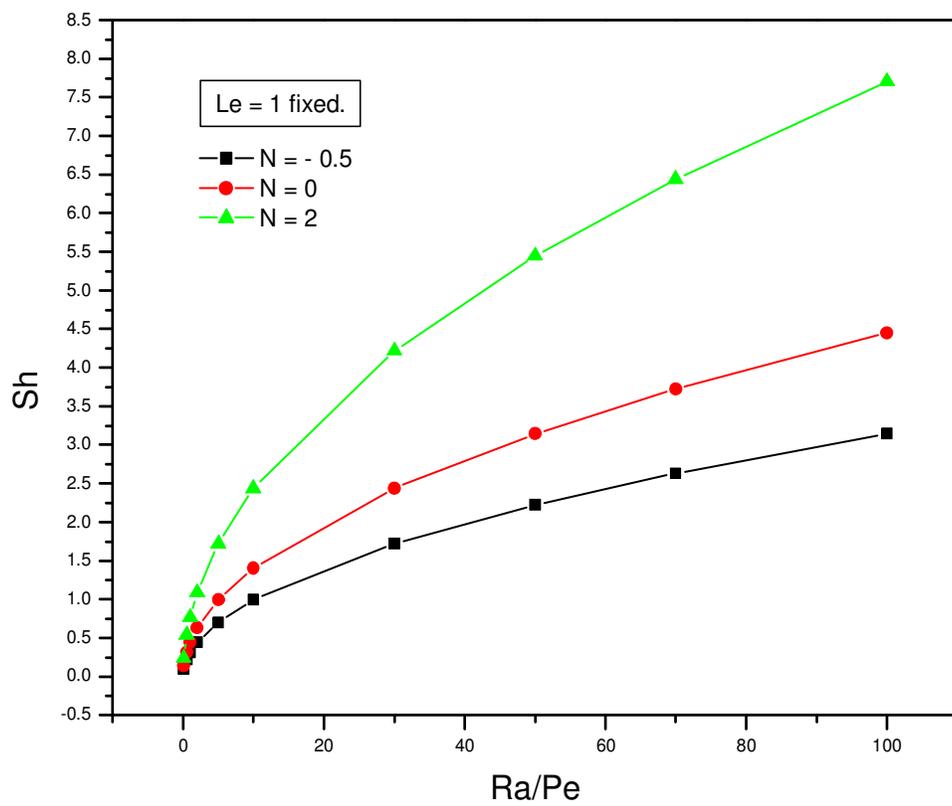
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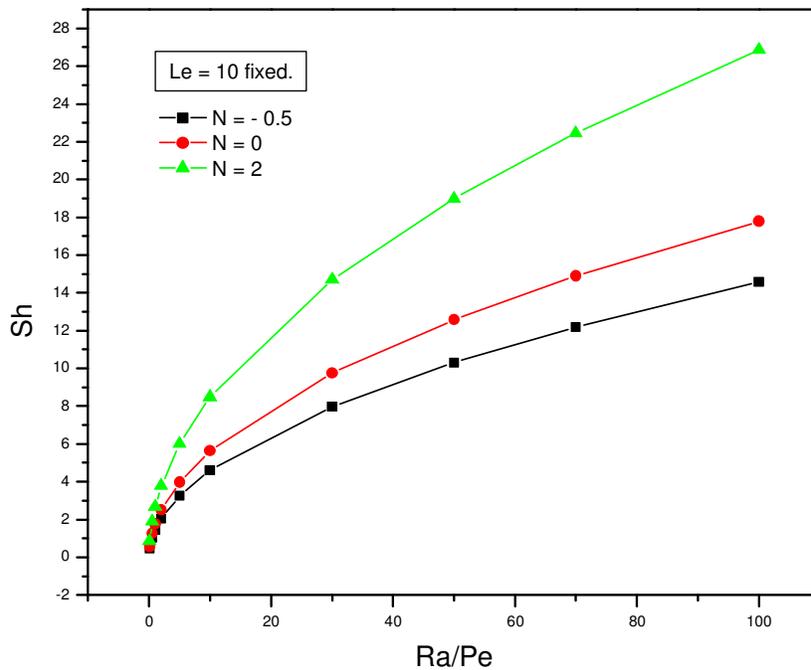
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Figure 6. (a) Mass transfer results as a function of (Ra / Pe) when Le = 0.5 and N = - 0.5, 0, 2. (b) Mass transfer results as a function of (Ra / Pe) when Le = 1 and N = - 0.5, 0, 2. (c) Mass transfer results as a function of (Ra / Pe) when Le = 10 and N = - 0.5, 0, 2.

Table 1. Integral results for Nusselt and Sherwood number

Le	N	Ra/Pe	Nu	Sh
0.5	-0.5	0.1	0.0812	0.0406
		1	0.2568	0.1284
		10	0.8121	0.4060
	0	0.1	0.1406	0.0869
		1	0.4448	0.2749
		10	1.4067	0.8693
	4	0.1	0.3412	0.2174
		1	1.0793	0.9946
		10	3.4130	2.1742
1	-0.5	0.1	0.0994	0.0994
		1	0.3145	0.3145
		10	0.9946	0.9946
	0	0.1	0.1406	0.1406
		1	0.4448	0.4448
		10	1.4067	1.4067
	4	0.1	0.3145	0.3145
		1	0.9947	0.9947
		10	3.1454	3.1454

Table 1. Contd.

		0.1	0.1247	0.4610
	-0.5	1	0.3946	1.4577
		10	1.2479	4.6098
		0.1	0.1406	0.5626
10	0	1	0.4448	1.7793
		10	1.4067	5.6267
		0.1	0.2156	1.0583
	4	1	0.6827	3.3470
		10	2.1588	10.5839

for aiding and opposing flows. Extensive calculations for a wide range of physical parameters are performed. The heat and mass transfer coefficients increases with the increasing value of flow driving parameter ($\frac{Ra}{Pe}$). The Lewis number has a complex impact on the heat and mass transfer mechanism.

REFERENCES

Bansod VJ (2003). The Darcy model for boundary layer flows in a horizontal porous medium induced by combined buoyancy forces, J. Porous Media, 6: 273-281.

Bansod VJ (2005). The effects of blowing and suction on double diffusion by mixed convection over inclined permeable surfaces, J. Transp. Porous Media, 60: 301-317.

Bansod VJ, Jadhav RK (2009). An integral treatment for combined heat and mass transfer by natural convection along a horizontal surface in a porous medium, Int. J. Heat Mass Trans., 52: 2802-2806.

Bansod VJ, Jadhav RK (2010). Effect of double stratification on mixed convection heat and mass transfer from a vertical surface in a fluid saturated porous medium, J. Heat Transf. – Asian Res., 39(6): 378-395.

Beg O A, Beg TA, Bahier AY, Prasad VR (2009). Chemically-reacting mixing convective heat and mass transfer along inclined and vertical plates with soret and Dufour effects: Numerical solutions, Int. J. Appl. Math. Mech., 5(2): 39-57.

Chamkha AJ, Khaled AR (1999). Non-similar hydro-magnetic simultaneous heat and mass transfer by mixed convection from a vertical plate embedded in a uniform porous medium, Numer. Heat Transfer, 36: 327-344.

Chamkha AJ, Khaled AR (2000). Hydro-magnetic simultaneous heat and mass transfer by mixed convection from a vertical plate embedded in a stratified porous medium with thermal dispersion effect, J. Heat Mass Transfer, 36: 63-70.

Cheng P (1977). Combined free and forced convection flow about inclined surfaces in porous media, Int. J. Heat Mass Transfer, 20: 807-814.

Hassan AM, El-Arabawy (2009). Soret and Dufour effects on natural convection flow past a vertical surface in a porous medium with variable surface temperature, J. Math. Stat., 5: 190-198.

Ingham D, Pop I (2002). Transport Phenomena in Porous Media II, Pergamon Press.

Lai FC (1991). Coupled heat and mass transfer by mixed convection from a vertical plate in a saturated porous medium, Int. Comm. Heat Mass Transfer, 18: 83-106.

Lakshmi NPA, Murthy PVSN (2008). Soret and Dufour effects on free convection heat and mass transfer from a horizontal flat plate in a Darcy porous medium, J. Heat Transfer, 130.

Li CT, Lai FC (1997). Coupled heat and mass transfer from a horizontal surface in saturated porous medium, HTD-Vol. 349, ASME National Heat Transfer Conference, 11: 169-176.

Nield DA, Bejan A (2006). Convection in Porous Media, 3rd Edition, Springer Verlag, New York.

Oladapo OP (2010). Dufour and Soret effects of a transient free convective flow with radiative heat transfer past a flat plate moving through a binary mixture, The Pacific J. Sci. Technol. 11, May (Spring).

Postelnicu A (2007). Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects, Int. J. Heat Mass Transfer, 43: 595-602,

Shateyi S (2008). Thermal radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing, J. Appl. Math., Article ID 414830, 12 pages,

Singh P, Quenny, Sharma RN (2002). Influence of lateral mass flux on mixed convection heat and mass transfer over inclined surfaces in porous medium, J. Heat Mass Transfer, 38: 233-242.

Tak SS, Mathur R, Gehlot RK, Khan A (2010). MHD free convection-radiation interaction along a vertical surface embedded in a Darcian porous medium in presence of Soret and Dufour effects, J. Therm. Sci., 14: 137-145.