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Full Length Research Paper

Propagation of plane magneto-thermo-elastic waves in a rotating, electrically conducting and transversely isotropic medium

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In this article, plane wave propagation, in a rotating perfectly electrically conducting unbounded transversely isotropic mediums in the presence of initially applied uniform magnetic field is studied. A general dispersion relation with complex coefficients is obtained for magneto-thermo-elastic waves. Axis of rotation, axis of symmetry and propagation of waves are chosen along z-axis. Four waves propagate in the medium when body is rotating about axis of symmetry. Two of them are elastic waves which depend on the initially applied magnetic field and the frequency of rotation. Remaining two are thermal waves which are independent of magnetic field and rotation. Numerical results for magnesium as a model material are presented.

Key words: Transversely isotropic, dispersion relation, energy dissipation, magneto-thermo-elastic wave, generalized thermo-elasticity.

INTRODUCTION

Elastic theories under the affect of heat have created interest in the last three decades. Theories involving finite speed of thermal signals are known as generalized theories, which involve hyperbolic type instead of parabolic type heat transport equation. Among the generalized theories, other two important models are by Lord and Shulman (1967) and Green and Lindsay (1972). These generalized theories are considered to be more realistic than the conventional theories.

The theories by Green and Naghdi (1991, 1992, 1993) give sufficient development in basic equations which allows treatment of much wider class of heat flow problems labeled as $G-N\,\mathrm{I}, \mathrm{II}$ and III. When these heat equations are linearized, the transport equation of $G-N\,\mathrm{I}$ is the same as the classical heat equation, where $G-N\,\mathrm{II}$ and III admit propagation of thermal signals of finite speed (Green and Naghdi, 1993). An important feature of $G-N\,\mathrm{III}$ is that, it accommodates dissipation of thermal energy. Problems related the mentioned theories have been investigated by many authors (Banik et al., 2007; Kar Avijit and Kanoria, 2007; Kar Avijit and Kanoria, 2007; Kar Avijit and Kanoria, 2006; Mallik and Kanoria, 2006; Mallik and Kanoria, 2007; Das and Kanoria, 2009) and (Othman et al., 2008; Othman and Ya Qin, 2009; Othman and Kumar, 2009) and references therein.

The purpose of this article is to study the effects of heat, electro-magnetic and rotation on the thermo-elastic harmonic waves propagating through transversely isotropic elastic solid by using a thermo-elastic model of $G-N\,\mathrm{III}$ (Green and Naghdi, 1993).

FORMULATION OF THE PROBLEM

In the presence of displacement current and charge density, the electromagnetic field is governed by the

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following Maxwell's equations **(**Das and Kanoria, 2009**)**,

$$\left.\begin{aligned} \boldsymbol{\Delta}\times \boldsymbol{H} &= \boldsymbol{J},\\ \boldsymbol{\Delta}\times \boldsymbol{E} &= -\dot{\boldsymbol{B}},\\ \boldsymbol{\Delta B} &= 0,\\ \boldsymbol{B} &= \mu_e \boldsymbol{H}. \end{aligned}\right\} \qquad (1)$$

The Ohm's law in rotating deformable continua is as follows,

$$J = \sigma\left(\boldsymbol{E} + \dot{\boldsymbol{U}}\times \boldsymbol{B} + (\boldsymbol{\Omega}.\boldsymbol{B})\boldsymbol{U} - (\boldsymbol{U}.\boldsymbol{B})\boldsymbol{\Omega}\right). \qquad (2)$$

The medium is an infinite homogeneous and perfectly electrically conducting elastic solid permeated by a primary magnetic field of intensity $\boldsymbol{H}_0$ and is rotating with an angular velocity $\boldsymbol{\Omega}$. The equation of motion becomes:

$$\sigma_{ij,j} + (J\times B)_i = \rho\left(\ddot{U}_i + (\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times \boldsymbol{U}))_i + 2\left(\boldsymbol{\Omega}\times\dot{\boldsymbol{U}}\right)_i\right), \qquad (3)$$

Where, Hook's law for heat conducting material is,

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - \gamma T\delta_{ij}$$

In this equation, we selected elastic coefficients (stiffness matrix) for selected material that is, transversely isotropic. The heat transport equation in the absence of heat source (Green and Naghdi, 1993) is:

$$\rho c_v \ddot{T} + \gamma T_0 \ddot{U}_{i,i} = K\dot{T}_{,ii} + K^{*}T_{,ii}, \qquad (4)$$

Set $\boldsymbol{H} = \boldsymbol{H}_0 + \boldsymbol{h}$ and $\boldsymbol{\Omega} = \Omega(0,0,1)$ where $\boldsymbol{H}_0 = (0,0,H_0)$. The perturbed magnetic field $\boldsymbol{h}$ is so small that the product of $\boldsymbol{U}$, $\boldsymbol{h}$, and derivatives of $\boldsymbol{h}$ can be neglected when linearizing the field equations (Das and Kanoria, 2009). Equating Equations 1 and 2, implies,

$$\left.\begin{aligned} \sigma\left(E_1 + \mu_e H_3\dot{U}_2 - \mu_e\dot{U}_3 H_2 + \mu_e\Omega H_3 U_1\right) &= H_{3,2} - H_{2,3},\\ \sigma\left(E_2 - \mu_e H_3\dot{U}_1 + \mu_e\dot{U}_3 H_1 + \mu_e\Omega H_3 U_2\right) &= H_{1,3} - H_{3,1},\\ \sigma\left(E_3 - \mu_e H_1\dot{U}_2 + \mu_e\dot{U}_1 H_2 - \mu_e H_1 U_1\Omega - \mu_e\Omega H_2 U_2\right) &= H_{2,1} - H_{1,2}. \end{aligned}\right\} \qquad (5)$$

By applying the value of $\boldsymbol{H}$, it becomes:

$$\left.\begin{aligned} \sigma\left(E_1 + \mu_e H_0\dot{U}_2 + \mu_e\Omega H_0 U_1\right) &= h_{3,2} - h_{2,3},\\ \sigma\left(E_2 - \mu_e H_0\dot{U}_1 + \mu_e\Omega H_0 U_2\right) &= h_{1,3} - h_{3,1},\\ \sigma E_3 &= h_{2,1} - h_{1,2}. \end{aligned}\right\} \qquad (6)$$

By using Equation 1,

$$\sigma\mu_e\left(-\dot{h}_1 + H_0\dot{U}_{1,3} - \Omega H_0 U_{2,3}\right) = h_{2,12} - h_{1,22} - h_{1,33} - h_{3,13}$$

.

Implies that,

$$\left.\begin{aligned} &-\dot{h}_1 + H_0\dot{U}_{1,3} - \Omega H_0 U_{2,3} = \nu_H\left(h_{2,12} - h_{1,22} - h_{1,33} - h_{3,13}\right),\\ &\text{Similarly,}\\ &-\dot{h}_2 + H_0\dot{U}_{2,3} + \Omega H_0 U_{1,3} = \nu_H\left(h_{3,32} - h_{2,33} - h_{2,11} + h_{1,12}\right),\\ &\text{and}\\ &-\dot{h}_3 - H_0\dot{U}_{1,1} + \Omega H_0 U_{2,1} - H_0\dot{U}_{2,2} - \Omega H_0 U_{1,2} = \nu_H\left(h_{1,13} - h_{3,11} - h_{3,22} + h_{2,23}\right), \end{aligned}\right\} \qquad (7)$$

Where $\nu_H = \dfrac{1}{\mu_e\sigma}$, representing the magnetic viscosity (Das and Kanoria, 2009). For perfectly conducting material ν_H approaches to zero, that implies,

$$\left.\begin{aligned} \dot{h}_1 &= H_0\left(\dot{U}_{1,3} - \Omega U_{2,3}\right),\\ \dot{h}_2 &= H_0\left(\dot{U}_{2,3} + \Omega U_{1,3}\right),\\ \dot{h}_3 &= -H_0\left(\dot{U}_{1,1} + \dot{U}_{2,2} + \Omega\left(U_{1,2} - U_{2,1}\right)\right). \end{aligned}\right\}$$

From Equation 3, for $i = 1$

$$\begin{aligned} &C_{11}U_{1,11} + C_{12}U_{2,21} + C_{13}U_{3,31} + \left(\frac{C_{11} - C_{12}}{2}\right)\left(U_{1,22} + U_{2,12}\right) + C_{44}\left(U_{1,33} + U_{3,13}\right)\\ &\quad - \mu_e H_0\left(h_{1,3} - h_{3,1}\right) - \gamma T_{,1} = \rho\left(\ddot{U}_1 - \Omega^2 U_1 - 2\Omega\dot{U}_2\right), \end{aligned} \qquad (8)$$

Equation 8 implies that,

$$\begin{aligned} &C_1^{2}\dot{U}_{1,11} + \left(C_1^{2} - 2C_3^{2}\right)\dot{U}_{2,12} + C_5^{2}\dot{U}_{3,13} + C_3^{2}\left(\dot{U}_{1,22} + \dot{U}_{2,12}\right) + C_2^{2}\left(\dot{U}_{1,33} + \dot{U}_{3,13}\right)\\ &\quad + V_A^{2}\left(-\Omega U_{2,33} + \dot{U}_{1,33} + \dot{U}_{1,11} + \dot{U}_{2,12} - \Omega U_{2,11} + \Omega U_{1,12}\right) - \frac{\gamma}{\rho}\dot{T}_{,1} = \dddot{U}_1 - \Omega^2\dot{U}_1 - 2\Omega\ddot{U}_2, \end{aligned}$$

where $C_1^{2} = \dfrac{C_{11}}{\rho}, C_2^{2} = \dfrac{C_{44}}{\rho}, C_3^{2} = \dfrac{C_{11} - C_{12}}{2\rho}, C_4^{2} = \dfrac{C_{33}}{\rho},$ $C_5^{2} = \dfrac{C_{13}}{\rho}$ and $V_A^{2} = \dfrac{\mu_e H_0^{2}}{\rho}$ is the Alf'ven wave velocity (Alfven, 1942). Above equation can also be written as,

$$1+R_H\ \dot{U}_{1,11}+V_4\dot{U}_{1,22}+\ V_3+R_H\ \dot{U}_{1,33}+\ 1-V_4+R_H\ \dot{U}_{2,12}+V_1\dot{U}_{3,13}+V_3\dot{U}_{3,13}$$
$$-R_H\,\Omega\ U_{2,33}+U_{2,11}-U_{1,12}\ -\frac{\gamma}{C_1^2\rho}\dot{T}_{,1}=\frac{1}{C_1^2}\ \dddot{U}_1-\Omega^2\dot{U}_1-2\Omega\ddot{U}_2\ , \quad (9)$$

Where, $V_1=\dfrac{C_5{}^2}{C_1{}^2}, V_2=\dfrac{C_4{}^2}{C_1{}^2}, V_3=\dfrac{C_2{}^2}{C_1{}^2}, V_4=\dfrac{C_3{}^2}{C_1{}^2}$ are just representing ratio between the velocity components of the wave in transversely isotropic medium, and $R_H=\dfrac{V_A{}^2}{C_1{}^2}$ is the coefficient representing the affect of external magnetic field. The non-dimensional transformation is,

$$u_i=\frac{C_{11}}{\gamma T_0\,l}U_i\,,\xi_i=\frac{x_i}{l},\eta=\frac{C_1 t}{l},\ \Omega_0=\Omega t_0\ ,$$

$$\theta=\frac{T}{T_0}\ C_T{}^2=\frac{K^*}{\rho c_v C_1{}^2}=\frac{c_3{}^2}{c_1{}^2}, \qquad \varepsilon_T=\frac{\gamma^2 T_0}{\rho^2 c_v C_1{}^2},$$

$$k_0=\frac{k}{l\,C_1}\ \text{and}\ \kappa=\frac{K}{\rho c_v},$$

Where, η is dimensionless time, l and t_0 are some standard length and time respectively, k is the thermal diffusivity, k_0 is the non thermal diffusivity. Equation 9 becomes:

$$1+R_H\ \dot{u}_{1,11}+V_4\dot{u}_{1,22}+\ V_3+R_H\ \dot{u}_{1,33}+\ 1-V_4+R_H\ \dot{u}_{2,12}+V_1\dot{u}_{3,13}+V_3\dot{u}_{3,13}\ -R_H a\ u_{2,33}+u_{2,11}-u_{1,12}$$
$$-\dot{\theta}_{,1}=\dddot{u}_1-a^2\dot{u}_1-2a\ddot{u}_2, \quad (10a)$$

similarly equation (3) for i=2 and 3

for i=2

$$1-V_4+R_H\ \dot{u}_{1,12}+V_4\dot{u}_{2,11}+\ 1+R_H\ \dot{u}_{2,22}+\ V_3+R_H\ \dot{u}_{2,33}+\left(V_1+V_3\right)u_{3,23}\ -R_H a\left(u_{2,12}-u_{1,22}-u_{1,33}\right)$$
$$-\dot{\theta}_{,2}=\dddot{u}_2-a^2\dot{u}_2+2a\ddot{u}_1,$$

and

for i=3

$$V_3u_{3,11}+V_3u_{3,22}+V_2u_{3,33}+\ V_1+V_3\ u_{1,13}+\ V_1+V_3\ u_{2,23}-\theta_{,3}=\ddot{u}_3, \quad (10b)$$

and the heat equation in component form can be represented as,

$$\ddot{\theta}+\varepsilon_T\ \ddot{u}_{1,1}+\ddot{u}_{2,2}+\ddot{u}_{3,3}\ =K_0\frac{\partial}{\partial\eta}\ \dot{\theta}_{,11}+\dot{\theta}_{,22}+\dot{\theta}_{,33}\ +c_T{}^2\ \theta_{,11}+\theta_{,22}+\theta_{,33}\ ,$$

where $a=\dfrac{\Omega_0 l}{t_0 C_1}$.

SOLUTION OF THE PROBLEM

Consider the plane wave solution of the form:

$$u_i=e^{i\ k\,\xi_i.n_i-\omega t}\ p_i \quad \text{and} \quad \theta=\theta_0 e^{i\ k\,\xi_i.n_i-\omega t}\ .$$

Where, k is the wave number, ω is representing the angular frequency of the wave and $\dfrac{\omega}{k}=c$, is the wave speed. Equations 10 become:

$$\left(\omega\left(-n_1{}^2-V_4n_2{}^2-V_3n_3{}^2-R_H\ n_1{}^2-R_H\ n_3{}^2+\left(\frac{a}{k}\right)^2\right)-R_H i\,n_1n_2a+\frac{\omega^3}{k^2}\right)p_1+$$
$$\left(\omega\ -\ 1-V_4\ \ n_1n_2-R_Hn_1n_2\ \ +iR_Hn_3{}^2a+iR_Hn_1{}^2a-2i\left(\frac{\omega}{k}\right)^2 a\right)p_2$$
$$+\ -\omega\ V_1+V_3\ \ n_1n_3\ \ p_3-i\frac{\omega}{k}n_1\theta_0=0, \quad (11a)$$
$$\left(\omega\ -\ 1-V_4\ \ n_1n_2-R_Hn_1n_2\ \ -iR_Hn_3{}^2a-iR_Hn_2{}^2a+2ia\left(\frac{\omega}{k}\right)^2\right)p_1$$
$$+\left(\omega\left(-V_4n_2{}^2-\frac{\omega}{k}n_1{}^2-\frac{\omega}{k}V_3n_3{}^2-R_H\frac{\omega}{k}n_2{}^2-R_H\frac{\omega}{k}n_3{}^2+\left(\frac{a}{k}\right)^2\right)+R_H i\,n_1n_2a+\frac{\omega^3}{k^2}\right)p_2$$
$$+\left(-\omega\left(V_1+V_3\right)n_2n_3\right)p_3-i\frac{\omega}{k}n_2\theta_0=0,$$

and

$$V_1+V_3\ \ n_1n_3p_1+\ V_1+V_3\ \ n_2n_3p_2+\left(V_3n_1{}^2+V_3n_2{}^2+V_2n_3{}^2-\left(\frac{\omega}{k}\right)^2\right)p_3+\left(i\frac{n_3}{k}\right)\theta_0=0, \quad (11b)$$

heat equation is represented as,

$$\varepsilon_Tn_1\omega^2\ \ p_1+\ \varepsilon_Tn_2\omega^2\ \ p_2+\ \varepsilon_Tn_3\omega^2\ \ p_3+\left(K_0\omega k+i\,kc_T-\frac{i\,\omega^2}{k}\right)\theta_0=0$$

For non trivial solution of the equations we have,

$$\begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{vmatrix}=0 \quad (12)$$

where,

$$A_{11}=\omega\left(-n_1{}^2-V_4n_2{}^2-V_3n_3{}^2-R_H\ n_1{}^2-R_H\ n_3{}^2+\left(\frac{a}{k}\right)^2\right)$$
$$-R_H i\,n_1n_2a+\frac{\omega^3}{k^2}\ ,$$

,

$$A_{13}=-\omega\ \ V_1+V_3\ \ n_1n_3\ ,$$

$$A_{12}=\omega\ -\ 1-V_4\ \ n_1n_2-R_Hn_1n_2\ \ +iR_Hn_3{}^2a$$
$$+iR_Hn_1{}^2a-2i\left(\frac{\omega}{k}\right)^2 a$$

$$A_{14}=-i\frac{\omega}{k}n_1\ ,$$

$$A_{21} = \omega \;-\; 1 - V_4 \;\; n_1 n_2 - R_H n_1 n_2 \;\; -iR_H n_3{}^2 a$$
$$-iR_H n_2{}^2 a + 2ia\left(\frac{\omega}{k}\right)^2 ,$$

$$A_{23} = -\omega \;\; V_1 + V_3 \;\; n_2 n_3$$

$$A_{22} = \omega\left(-V_4 n_2{}^2 - \frac{\omega}{k} n_1{}^2 - \frac{\omega}{k} V_3 n_3{}^2 - R_H \frac{\omega}{k} n_2{}^2 - R_H \frac{\omega}{k} n_3{}^2 + \left(\frac{a}{k}\right)^2 \right)$$
$$+ R_H i\, n_1 n_2 a + \frac{\omega}{k^2}^{3} ,$$

$$A_{24} = -i\frac{\omega}{k} n_2 ,$$

$$A_{31} = \;\; V_1 + V_3 \;\; n_1 n_3 ,$$

$$A_{32} = \;\; V_1 + V_3 \;\; n_2 n_3 ,$$

$$A_{33} = V_3 n_1{}^2 + V_3 n_2{}^2 + V_2 n_3{}^2 - \left(\frac{\omega}{k}\right)^2 ,$$

$$A_{34} = i\frac{n_3}{k} ,$$

$$A_{41} = \varepsilon_T n_1 \omega^2 ,$$

$$A_{42} = \varepsilon_T n_2 \omega^2 ,$$

$$A_{43} = \varepsilon_T n_3 \omega^2 ,$$

$$A_{44} = K_0 \omega k + i\, k c_T - \frac{i\,\omega^2}{k} ,$$

For simplicity choose the propagation vector along x_3 axis that is, $\boldsymbol{n} = \;\; 0,0,1$, the dispersion relation of Equation 12 can be converted to the following form:

$$\left[-\omega k^2 \;\; V_3^2 + R_H \;\; + \omega^3 + \omega a^2 \;\; {}^2 - \;\; R_H a k^2 - 2\omega^2 a \;\; {}^2 \right]\left[k^4 \;\; i c_T{}^2 + \omega k_0 \;\; + k^2 \;\; -i\omega^2 \;\; V_2 + c_T{}^2 + \varepsilon_T \;\; -\omega^3 k_0 \;\; + i\omega^4 \right] = 0 \qquad (13)$$

The dispersion relation has two factors, 1st one depends on electro-magnetic field, and frequency of rotation of the medium and 2nd factor of the dispersion relation is totally representing heat affect.

$$\left[\;\; -\omega k^2 \;\; V_3^2 + R_H \;\; + \omega^3 + \omega a^2 \;\; {}^2 - \;\; R_H a k^2 - 2\omega^2 a \;\; {}^2 \right] = 0 \qquad (14)$$

This gives two different values for velocity of the wave. The values of these velocities are depending on the strength of magnetic field and frequency of rotation.

PRACTICAL APPLICATION

For particular case, let us consider magnesium as a model material, the physical data for magnesium is as follows (Mallik and Kanoria, 2007):

$$\rho = 1.74\times10^3 \, {}^{kg}\!/\!_{m^3}, C_{11} = 5.974\times10^{10} \, {}^{N}\!/\!_{m^2}, C_{12} = 2.624\times10^{10} \, {}^{N}\!/\!_{m^2}, C_{13} = 2.17\times10^{10} \, {}^{N}\!/\!_{m^2}$$
$$C_{33} = 6.17\times10^{10} \, {}^{N}\!/\!_{m^2}\; C_{44} = 1.510\times10^{10} \, {}^{N}\!/\!_{m^2}$$

where magnetic field is represented by the following relation as, $B = \mu_0 (1 + x_m) H$, where $\mu_e = \mu_0 (1 + x_m)$ is the magnetic permeability, x_m is representing the constant for magnetic susceptibility whose value depends on the nature of the material, μ_0 is the free space magnetic permeability. For magnesium x_m is 2.3×10^{-5} and μ_0 is $4\pi\times10^{-7}\,Hm^{-1}$ (hyperphysics.phy-astr.gsu.edu/hbase/tables/magprop.html.; www.magnesium.com/w3/data-bank/index.php?mgco=153) gives relative permeability equal to 1. Figures 1 and 2 show velocity verses frequency of rotation for different intensity of magnetic fields

In both Figures 1 and 2, velocity (c) is taken along vertical coordinate and rotation is stretched along horizontal axis, and different values of magnetic field are shown.

From Equation 13,

$$\left[i\omega^4 + k^4 \;\; i c_T{}^2 + \omega k_0 \;\; + k^2 \;\; -i\omega^2 \;\; 1 + c_T{}^2 + \varepsilon_T \;\; -\omega^3 k_0 \;\; \right] = 0 \qquad (15)$$

This is a quadratic equation in k^2 having two roots, which shows existence of two thermal waves. Figures 3 and 4 give for one wave (Figures 5 and 6 for 2nd wave) and are representing relation c_T and c is representing velocity of thermal wave propagation through the medium, for different values of diffusivity and coupling constant, respectively.

In both figures, velocity (c) is taken along vertical coordinate and thermal velocity (c_T) is stretched along horizontal axis

CONCLUSION

Figures 1 and 2 show the effect of rotational frequency on velocity of waves for different fixed value of initially applied magnetic field. Velocity of one wave increases

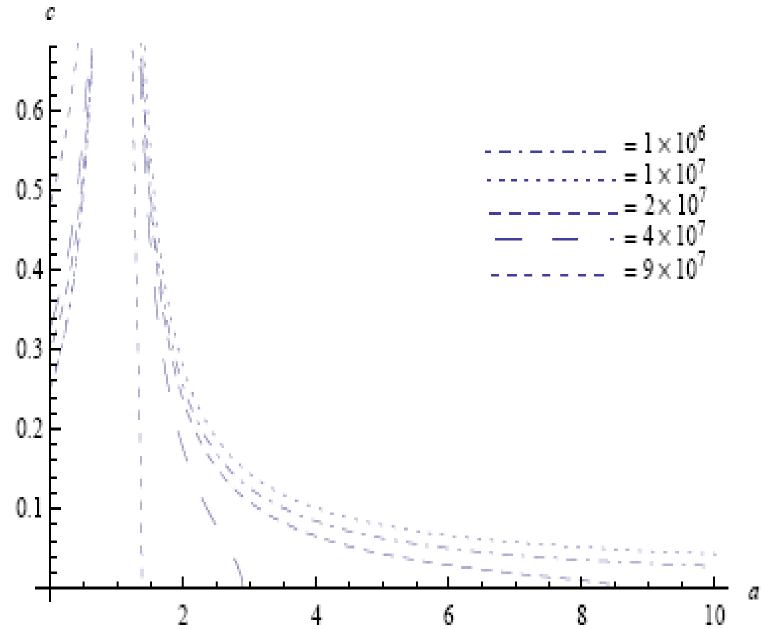

Figure 1. Relation between wave velocity and rotation (a) for different values of H_0. ($\omega = 1$).

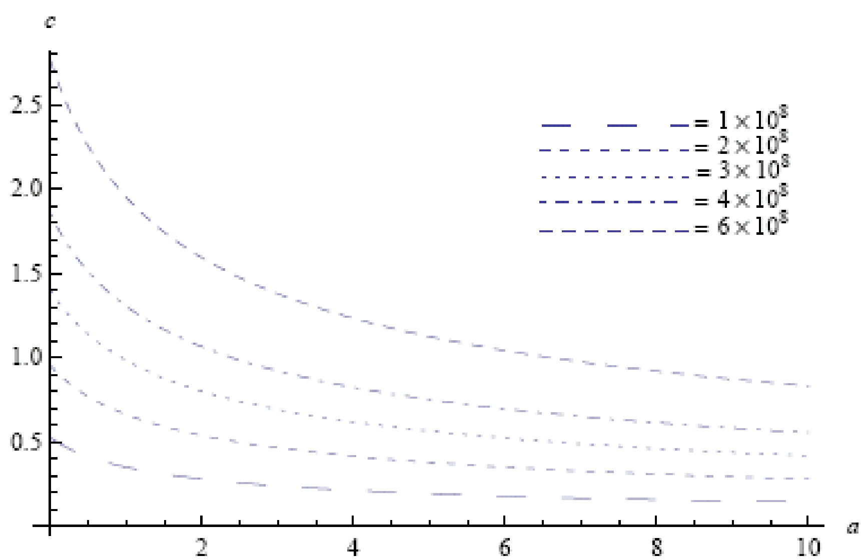

Figure 2. Relation between wave velocity and rotation (a) for different values of H_0. ($\omega = 1$)

exponentially with the increase in rotational and approaches to infinity when $(\Omega_0 l/t_0 C_1) \to 1.4$. In this small range, magnetic field is having increasing effect, but when $(\Omega_0 l/t_0 C_1) > 1.4$ behavior of velocity against rotation is opposite that is, when rotation increases, the velocity of the wave decreases, effect of magnetic field also decrease the velocity of wave propagating. Velocity of Figure 2 wave decreases with the increase of rotation and initially applied magnetic field is having increasing effect on this wave.

From equations, it can be seen that there are two types of waves which appear in the medium and are independent of initially applied magnetic field. These

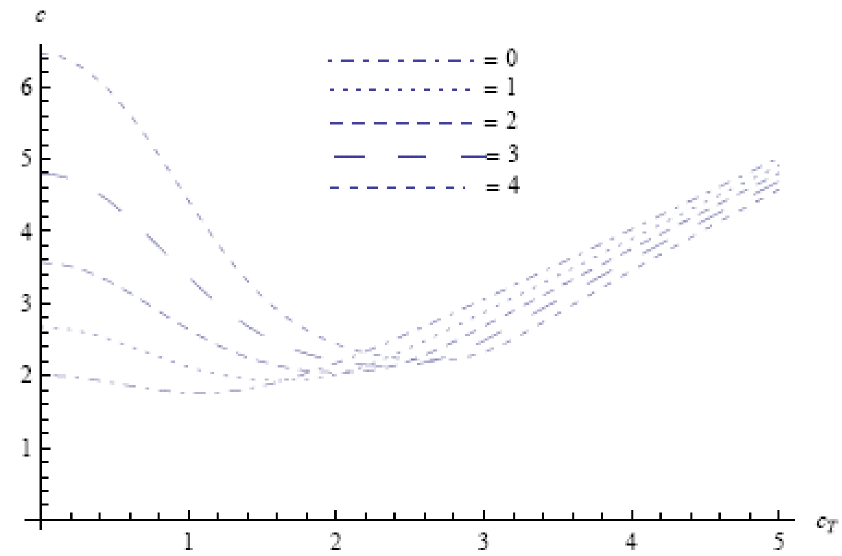

Figure 3. Relation between thermal velocity and velocity of wave, for different values of coupling factor ε_T , where diffusivity is kept constant that is, k_0 =2 (ω =1).

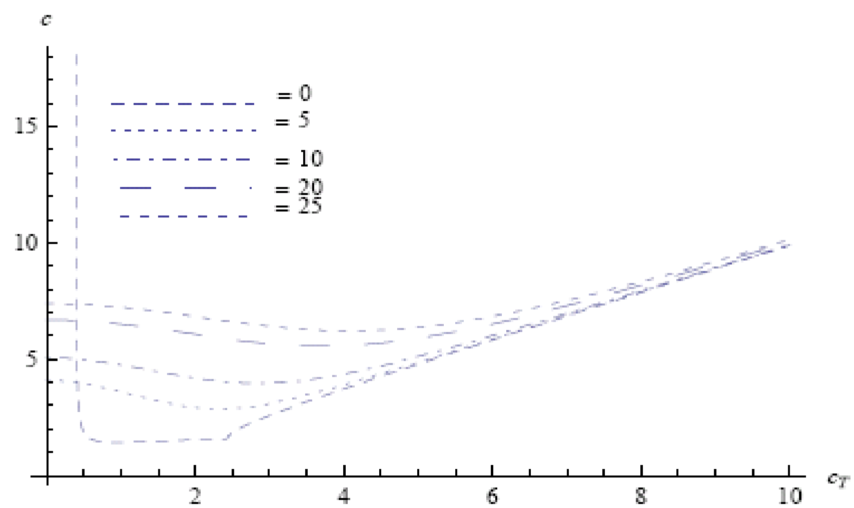

Figure 4. Relation between c_T and velocity of wave (c), for different values of thermal diffusivity k_0 while coupling constant is kept fixed ε_T =2 (ω =1).

velocities are depending on thermal factors present in equations. Graphical representations of these velocities against thermal factors that is, diffusivity and coupling constant are shown in Figures 3 and 4. In Figure 3, diffusivity constant k_0 is kept constant and we studied the variation in wave velocity against thermal velocity (c_T). It can be observed from Figure 3 that for some initial values of thermal velocity, wave velocity is inversely proportional to c_T, and this range of small initial value is depending on value of coupling constant, for greater value of coupling constant the range of this relation between velocities is greater. For higher values of thermal velocity all relation are reversed that is, wave velocity and thermal velocity are directly proportional to each other and coupling constant is having decreasing effect on wave velocity against thermal velocity. Same type of relation is shown in Figure 4, but coupling constant is kept constant and relation is studied for different values of thermal coefficient. When diffusivity is neglected that is, the case

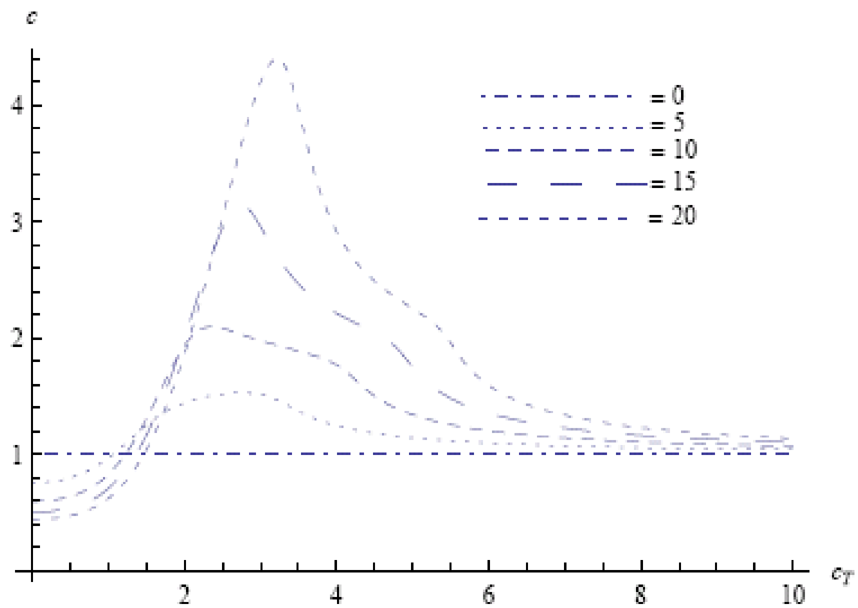

Figure 5. Changes in velocity (c-along Y axis) against thermal velocity (c_T-along X-axis) for different values of coupling factor ε_T with $k_0 = 2$ (ω =1).

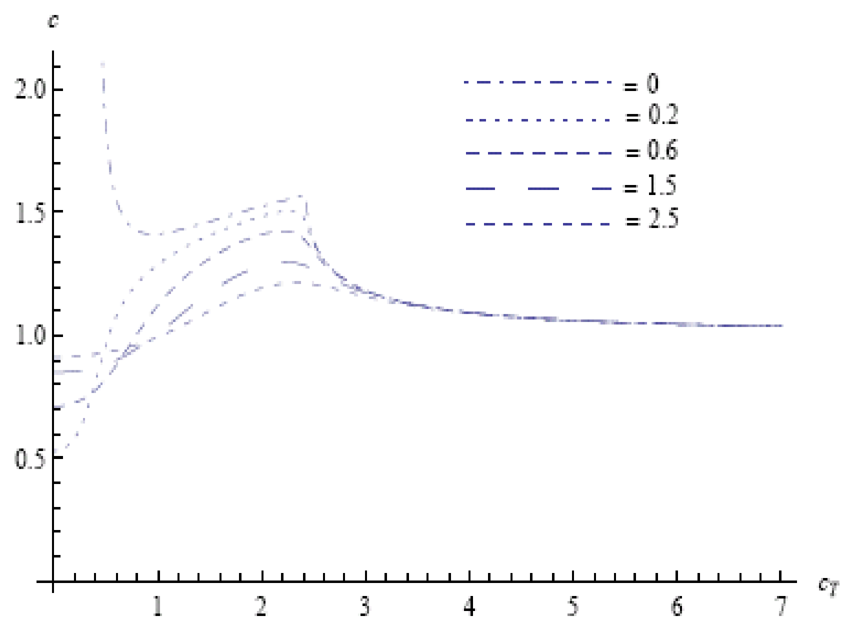

Figure 6. Changes in velocity (c-along Y axis) against thermal velocity (c_T-along X-axis) for different values of coupling factor k_0 with $\varepsilon_T = 2$ (ω =1).

of energy dissipation, it can be observed that energy dissipation totally change the nature of curve. Diffusivity is having positive effect on elastic wave velocity against thermal wave velocity. For higher value of thermal velocity, elasticity wave velocity become independent of diffusivity that is, all curves move with same result. Second velocity depending on thermal properties is shown in Figures 5 and 6. These Figures 5 and 6 are having dual type nature for diffusivity and thermal coupling constant. For simplicity, we have fixed angular frequency equal to unity that is, $\omega = 1$.

Nomenclature

$\boldsymbol{U}$, Displacement vector;	σ, Electric conductivity of the medium;
T, Small temperature increase above the reference temperature T_0;	c_v, Specific heat of the medium at constant strain;
$\boldsymbol{H}$, Total magnetic field vector at any time;	$\boldsymbol{B}$, Magnetic induction vector;
κ, Thermal diffusivity;	k Wave number;
$\boldsymbol{E}$, Electric field vector;	$\boldsymbol{\Omega}$, Angular velocity of rotating medium;
ρ, Constant mass density of the medium;	γ, Thermal modulus;
μ_e, Magnetic permeability of the medium;	α_t, Coefficient of linear thermal expansion;
$\boldsymbol{J}$, Electric current density vector;	T_0, Uniform reference temperature;
K^*, A material constant characteristic for the $G-N$ theory;	c_T, Non dimensional finite thermal wave speed of $G-N$ theory of thermo-elasticity II
ε_T, Thermo-elastic coupling constant;	K, Thermal conductivity;
C_{ijkl}, Elastic coefficients	

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